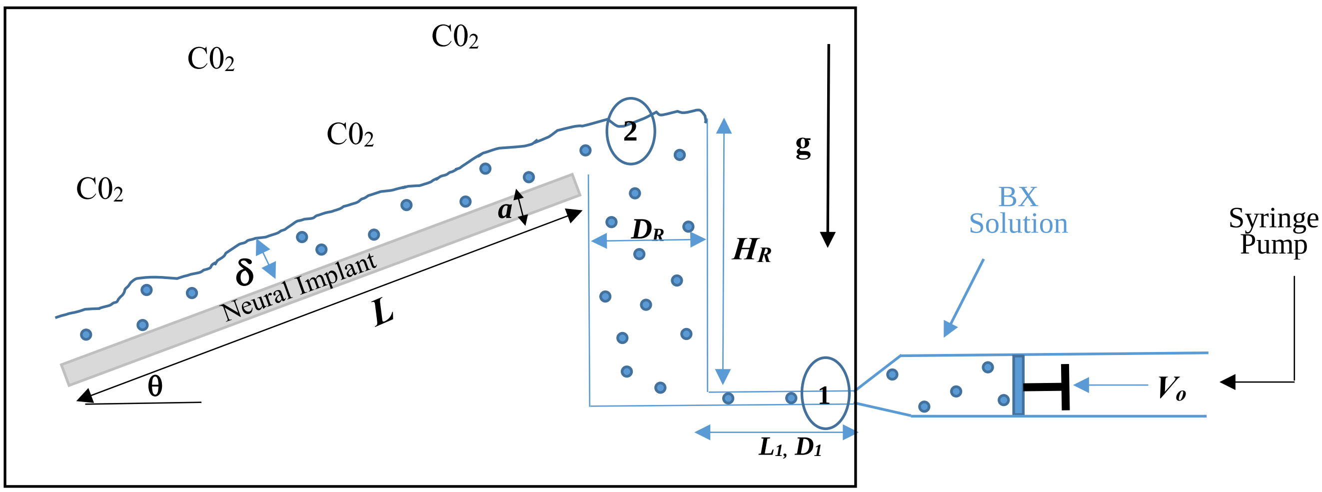
Your startup company has developed a new biomaterial, BX, used to coat the outer surfaces of implants inserted into brain tissue. You have designed the experiment below to examine the flow properties of viscous BX solutions (circles, known ρ_{BX}, μ_{BX}) during coating within a controlled environment of carbon dioxide (CO_2, known ρ_{CO2} and very low viscosity μ_{CO2}).

The implant is of length L, height a, width W (into the page), and is coated at a $\theta = 45^o$ incline to better mimic the insertion angle during surgery. A syringe pump inputs a velocity of V_o that drives solutions of BX from the syringe bore through a microcapillary that enters an enclosed CO_2 environment. The microcapillary diameter, D_1, is one-tenth the bore diameter and has a length, L_1, much smaller than the implant length, L. The pump continues to drive BX solutions through a volumetric reservoir of height H_R and diameter D_R, such that BX solutions flow along the surface of the implant with an approximate thickness of δ.



1. Determine the **inlet velocity of the BX solution** at <u>Point 2</u>, just as it comes into contact with the implant. Note that the answer must be in terms of known variables, only. [**15pts**]

2. **Draw the control volume and coordinate axes** (in your booklet!) that will be used to analyze the flow of the BX along the implant surface and <u>justify this selection</u>. What is the principal direction of flow in your selected coordinate system? **[15 Pts]**

3. Derive the **governing equation** for the flow of BX along the implant surface and identify the **appropriate boundary conditions**. **[15 Pts]**

4. Develop an expression that describes the **velocity profile** of this BX flow and sketch. **[20 Pts]**

5. Is the velocity profile linear? Should it be? Justify your answer [**10pts**].

6. The manufacturer data from the syringe pump suggests that the pressure gradient will remain approximately constant and equal to a value, $-G_p$, during the conditions used for the coating process. Using a known $-G_p$, derive an **expression for the thickness** of the flow, δ. **[25 Pts]**

$$Re = \frac{\rho\, V\, L_C}{\mu}$$

$$\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

$$\tau_{xy} = \mu \frac{\partial u_y}{\partial x}$$

$$\rho\left(\frac{\partial u}{\partial t} + u \cdot \nabla u\right) = -\nabla p + \mu \nabla^2 u + \rho \mathrm{g}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \frac{-\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho \mathrm{g}_x$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \frac{-\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho \mathrm{g}_y$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \frac{-\partial P}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho \mathrm{g}_z$$

$$r\left[\frac{\partial \mathrm{v}_r}{\partial t} + \mathrm{v}_r\frac{\partial \mathrm{v}_r}{\partial r} + \frac{\mathrm{v}_q}{r}\frac{\partial \mathrm{v}_r}{\partial r} + \mathrm{v}_z\frac{\partial \mathrm{v}_r}{\partial z} - \frac{\mathrm{v}_q^2}{r}\right] = -\frac{\partial \mathrm{p}}{\partial r} + m\left[\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(r\mathrm{v}_r)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 \mathrm{v}_r}{\partial q^2} + \frac{\partial^2 \mathrm{v}_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial \mathrm{v}_q}{\partial q}\right] + r g_r$$

$$r\left[\frac{\partial \mathrm{v}_q}{\partial t} + \mathrm{v}_r\frac{\partial \mathrm{v}_q}{\partial r} + \frac{\mathrm{v}_q}{r}\frac{\partial \mathrm{v}_q}{\partial q} + \mathrm{v}_z\frac{\partial \mathrm{v}_q}{\partial z}\right] = -\frac{1}{r}\frac{\partial \mathrm{p}}{\partial q} + m\left[\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(r\mathrm{v}_q)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 \mathrm{v}_q}{\partial q^2} + \frac{1}{r^2}\frac{\partial^2 \mathrm{v}_q}{\partial z^2} + \frac{2}{r^2}\frac{\partial \mathrm{v}_r}{\partial q}\right] + r g_q$$

$$r\left[\frac{\partial \mathrm{v}_z}{\partial t} + \mathrm{v}_r\frac{\partial \mathrm{v}_z}{\partial r} + \frac{\mathrm{v}_q}{r}\frac{\partial \mathrm{v}_z}{\partial q} + \mathrm{v}_z\frac{\partial \mathrm{v}_z}{\partial z}\right] = -\frac{\partial \mathrm{p}}{\partial z} + m\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \mathrm{v}_z}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 \mathrm{v}_z}{\partial q^2} + \frac{\partial^2 \mathrm{v}_z}{\partial z^2}\right] + r g_z$$