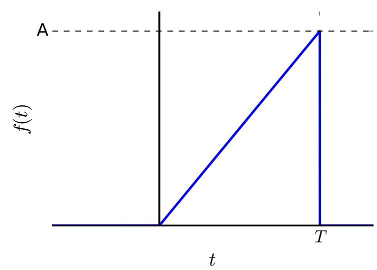
ECE 345: Linear Systems and Signals
Professors Sarwate and El Rouayheb

Fall 2020
Due: Monday, December 7, 2020

Homework 7

Problem 1 (Computing CTFTs from the definition, SSTA 5.39). Find the CTFT of the following signal with $A = 5.0$ and $T = 3.0$:



Problem 2 (Using CTFT properties, OW 4.6). Suppose that $x(t) \overset{\text{CTFT}}{\longleftrightarrow} X(j\omega)$. Then find the CTFT of the following signals using CTFT properties.

(a) $x_1(t) = x(1-t) + x(-1-t)$

(b) $x_2(t) = x(3t-6)$

(c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$

Problem 3 (Using CTFT properties II). Suppose that $x(t) \overset{\text{CTFT}}{\longleftrightarrow} X(j\omega)$. Then find the time domain signals corresponding to the following CTFTs.

(a) $X_1(j\omega) = \frac{4}{3} e^{-j2\omega/3} X(-j\omega/3)$

(b) $X_2(j\omega) = \frac{1}{3} e^{j2(\omega-2)} X^* \left(j \frac{\omega-2}{3}\right)$

Problem 4 (Computing CTFTs, OW 4.21). Compute the CTFT for the following signals.

(a) $e^{-3|t|} \sin(2t)$

(b) $\frac{\sin(\pi t)}{(\pi t)} \cdot \frac{\sin(2\pi(t-1))}{\pi(t-1)}$

(c) $te^{-2t} \sin(4t) u(t)$

Problem 5 (Even and odd parts, OW 4.9). Suppose

$$x(t) = \begin{cases} 0 & |t| > 1 \\ \frac{t+1}{2} & -1 \le t \le 1 \end{cases} \tag{1}$$

First, sketch the function so that you know what you are looking at.

(a) Find the CTFT $X(j\omega)$

(b) Find $x_{\text{even}}(t) = \frac{x(t)+x(-t)}{2}$ and $x_{\text{odd}}(t) = \frac{x(t)-x(-t)}{2}$.

(c) Find the CTFT of the even and odd parts of $x(t)$ and check that they correspond to the real and imaginary parts of $X(j\omega)$

Problem 6 (System identification is not so easy)**.** Consider the following three impulse responses:

$$h_1(t) = u(t) \tag{2}$$
$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t) \tag{3}$$
$$h_3(t) = 2e^{-t}u(t) \tag{4}$$

Using the CTFT, do the following.

1. Find the output of these systems to the input $\cos(t)$.
2. Find the output of these systems to the input $\cos(2t)$.
3. Find another system whose output with input $\cos(t)$ is the same as $h_1(t)$.
4. Find another system whose output with input $\cos(2t)$ is the same as $h_2(t)$.

What this means is that if you have an LTI system with a wire going in and a wire going out, just looking at the output to a single cosine input will not let you figure out ("identify") the system impulse response.

Problem 7 (Filtering)**.** One thing that differs between textbooks is how they define the sinc function. For example, in the textbook (SSTA p.219), they define

$$\mathrm{rect}(t) = \begin{cases} 1 & \frac{-1}{2} \le t \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad \mathrm{sinc}(t) = \frac{\sin(t)}{t} \tag{5}$$

and the CTFT pairs

$$\mathrm{rect}\left(\frac{t}{\tau}\right) \xleftrightarrow{\text{CTFT}} \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right) \tag{6}$$
$$\frac{W}{\pi}\,\mathrm{sinc}(Wt) \xleftrightarrow{\text{CTFT}} \mathrm{rect}\left(\frac{\omega}{2W}\right) \tag{7}$$

Unfortunately, MATLAB's `sinc`(t) function is defined to be $\frac{\sin(\pi t)}{\pi t}$, which leads to all sorts of confusion.

In this problem we will use the above defenitions of rect and sinc. Suppose that you apply the periodic signal

$$x(t) = 1 + 2\cos(5\pi t) + 3\sin(8\pi t) \tag{8}$$

to an LTI system with impulse response

$$h(t) = 8\,\mathrm{sinc}(4t)\cos(4\pi t). \tag{9}$$

(a) Sketch $x(t)$ and $h(t)$.

(b) Compute the fundamental radian frequency of $x(t)$.

(c) Compute the CTFT $X(j\omega)$.

(d) Compute the CTFT $H(j\omega)$.

(e) Sketch $|H(j\omega)|$ over $-10\pi \le \omega \le 10\pi$. Verify your sketch using MATLAB.

(f) Find the output CTFT $Y(j\omega)$ and $y(t)$.

Try to explain in words what the system is doing.

Problem 8 (ECE 345 Fall 2017 Midterm 2)**.** Consider the following signal $x(t)$:

$$x(t) = e^{-3|t|}\cos(\omega_c t). \tag{10}$$

where $\omega_c = 100\pi$.

(a) Calculate the CT Fourier transform $X(j\omega)$ of $x(t)$.

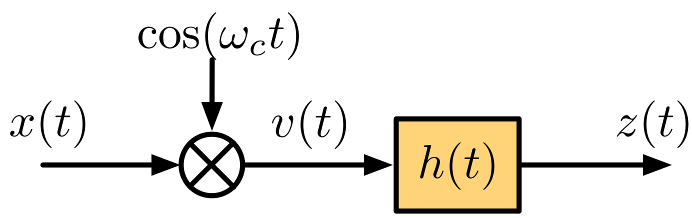
(b) Plot $X(j\omega)$.

(c) Suppose the signal $x(t)$ is passed through an LTI system with impulse response

$$h(t) = \frac{\sin(50\pi t)}{\pi t} \tag{11}$$

What is the output $y(t)$ of the LTI system? You can give your answer in the time or frequency domain.

(d) Instead, suppose you implement the following system, where $h(t)$ is the same LTI filter as (103):



Calculate $V(j\omega)$ and sketch it.

(e) Sketch the output $Y(j\omega)$.

Problem 9 (Nyquiste Rate, OW 7.3)**.** Find the Nyquist rate of the following signals.

(a) $x(t) = 1 + \cos(1000\pi t) + \cos(3000\pi t)$

(b) $x(t) = \frac{\sin(4000\pi t)}{\pi t}$

Problem 10 (Nyquiste Rate, OW 7.4(c))**.** If $x(t)$ has Nyquist rate ω_0, find the Nyquist rate of $y(t) = (x(t))^2$.