ECE 345: Linear Systems and Signals
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Fall 2020
Due: Friday, November 20, 2020

Homework 6

Problem 1 (Computing inverse DTFTs)**.** Compute the time domain signals corresponding to the following DTFTs.

(a) $X(e^{j\omega}) = 1$ for $\pi/4 \leq |\omega| \leq 3\pi/4$ and 0 elsewhere.

(b) $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \leq \omega \leq \pi$

(c) $X(e^{j\omega}) = \cos^2(\omega) + \sin^2(3\omega)$.

(d) $X(e^{j\omega}) = \frac{e^{-j\omega} - \frac{1}{5}}{1 - \frac{1}{5}e^{-j\omega}}$.

Problem 2 (Using DTFT properties)**.** Suppose $x[n] \xleftrightarrow{\text{DTFT}} X(e^{j\omega})$. Use DTFT properties to find the DTFTs of the following signals

(a) $x_1[n] = x[1-n] + x[-1-n]$

(b) $x_3[n] = (n-1)^2 x[n]$

Problem 3 (A mystery signal!)**.** Suppose you know the following four facts about a real signal $x[n]$ with DTFT $X(e^{j\omega})$:

1. $x[n] = 0$ for $n > 0$.
2. $x[0] > 0$
3. $\mathfrak{Im}\{X(e^{j\omega})\} = \sin(\omega) - \sin(2\omega)$
4. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3$

Determine $x[n]$. *Hint: You may want to use Parseval's relation.*

Problem 4 (Filtering)**.** Sincs and rects are important in DT signal processing as well.

(a) Find the DTFT of $h[n] = \frac{\sin(Wn)}{\pi n}$ and sketch it.

(b) Suppose

$$x[n] = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right) \tag{1}$$

Find the DTFT of $x[n]$.

(c) Find the output of the following systems with input $x[n]$:

$$h_1[n] = \frac{\sin(\pi n/6)}{\pi n} \tag{2}$$

$$h_2[n] = \frac{\sin(\pi n/6)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n} \tag{3}$$

$$h_3[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi^2 n^2} \tag{4}$$

$$h_4[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi n} \tag{5}$$

Problem 5. Using the MATLAB function `freqz`, compute and plot the real and imaginary parts as well as magnitude and phase functions of the following DTFTs:

$$X(e^{j\omega}) = \frac{1}{1 - 0.4e^{-j\omega}} \tag{6}$$

$$Y(e^{j\omega}) = \frac{0.2 + 0.4e^{-j\omega} + e^{-2j\omega}}{1 + 0.4e^{-j\omega} + 0.2e^{-2j\omega}} \tag{7}$$

Make sure to use appropriate frequency points while computing and plotting the DTFTs. Also, use the MATLAB function `unwrap` to unwrap the phase functions.