ECE 345: Linear Systems and Signals — Fall 2020
Professors Sarwate and El Rouayheb — Due: 5 PM, November 6, 2020

Homework 5

Problem 1 (Oppenheim, Willsky, and Nawab, 10.8)**.** Let $x[n]$ be a signal whose rational z-transform $X(z)$ contains a pole at $z = 1/2$. Given that

$$x_1[n] = \left(\frac{1}{4}\right)^n x[n] \tag{1}$$

is absolutely summable and

$$x_2[n] = \left(\frac{1}{8}\right)^n x[n] \tag{2}$$

is absolutely summable, determine whether $x[n]$ is left-sided, right-sided, or two-sided.

Problem 2 (Oppenheim, Willsky, and Nawab 10.23)**.** Determine the inverse z-transform for the following signals

(a) $X(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}}$, with ROC $|z| < \frac{1}{2}$

(b) $X(z) = \frac{z^{-1}-\frac{1}{2}}{1-\frac{1}{2}z^{-1}}$, with ROC $|z| < \frac{1}{2}$

(c) $X(z) = \frac{z^{-1}-\frac{1}{2}}{(1-\frac{1}{2}z^{-1})^2}$, with ROC $|z| < \frac{1}{2}$

Problem 3 (SSTA 7.24)**.** A causal LTI system has zeros at $\{3, 4\}$ and poles at $\{1, 2\}$. The transfer function $H(z)$ has $H(0) = 6$. Compute:

(a) The transfer function H(z).

(b) The response to $x[n] = \delta[n] - 3\delta[n-1] + 2\delta[n-2]$.

(c) The impulse response $h[n]$.

(d) The difference equation.

Problem 4 (Final Exam, Fall 2017)**.** When the input to an LTI system is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1] \tag{3}$$

the output is

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]. \tag{4}$$

(a) Find the Z-transforms of $X(z)$ and $Y(z)$. Write them as ratios of factorized polynomials in z^{-1}.

(b) Find the system transfer function $H(z)$. Plot the poles and zeros and indicate the region of convergence.

(c) Find the impulse response $h[n]$ of the system.

(d) Is $H(z)$ stable? Is it causal? Explain why or why not.

(e) Find the inverse system $H^{-1}(z)$. Can you find an ROC so the inverse is stable? For that choice is the inverse system causal? Explain why or why not.

Problem 5 (SSTA 7.27). We observe the two signals which start at $n = 0$:

$$y_1[n] = (1, -13, 86, -322, 693, -945, 500) \tag{5}$$
$$y_2[n] = (1, -13, 88, -338, 777, -1105, 750), \tag{6}$$

where $y_1[n] = h_1[n] * x[n]$, $y_2[n] = h_2[n] * x[n]$, and all of $\{h_1[n], h_2[n], x[n]\}$ are unknown! We know only that all of the signals have finite lengths, are causal, and $x[0] = 1$. Compute $h_1[n]$, $h_2[n]$, and $x[n]$. Hint: Use z-transforms and zeros.

Note: this is a real application in communications. The signals $y_1[n]$ and $y_2[n]$ represent signals at a wireless receiver which has two antennas. The transmitted signal $x[n]$ is the same for both but the *channels* are different to each received antenna: these are the systems $h_1[n]$ and $h_2[n]$. The receiver may not know the channels (since they very in time) and must estimate the transmitted signals and the channels. This problem is called *blind equalization* (since the receiver is trying to equalize/normalize the channel without knowing anything) or *blind deconvolution.*

Use MATLAB if you need to do the calculations.

Problem 6 (Oppenheim, Willsky, and Nawab, 5.21). Compute the DTFTs of the following signals.

(a) $x[n] = u[n-2] - u[n-6]$

(b) $x[n] = \left(\frac{1}{2}\right)^{|n|} \cos\left(\frac{\pi}{8}(n-1)\right)$

(c) $x[n] = n$ for $|n| \leq 3$ and 0 elsewhere

(d) $x[n] = \sin(\frac{\pi}{2}n) + \cos(n)$