ECE 345: Linear Systems and Signals
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Fall 2020
Due: 5 PM, October 23, 2020

Homework 4

Problem 1. Obtain the inverse Laplace transform of each of the following functions:

(a) $X_1(s) = 2 + \frac{4(s-3)}{s^2+9}$

(b) $X_2(s) = \frac{4}{s} + \frac{4s}{s^2+16}$

(c) $X_3(s) = \frac{(s+5)e^{-2s}}{(s+1)(s+3)}$

Problem 2 (LSS Midterm 2, Fall 2018 Short Answer)**.** (a) A causal and stable system has a rational Laplace transform

$$H(s) = \frac{(s-1)}{(s+(1-2j))(s+(1+2j))}. \tag{1}$$

What is the output $y(t)$ of the system with input $x(t) = 3e^{-3t} - e^{-2t}$?

(b) Consider a system with Laplace transform

$$H(s) = \frac{s+1}{s^2+s-20}. \tag{2}$$

Is this system lowpass or bandpass? Explain your reasoning.

Problem 3. How many signals have a Laplace transform that may be expressed as

$$X(s) = \frac{s-1}{(s+2)(s+3)(s^2+s+1)}. \tag{3}$$

Find the ROC for each of these transforms.

Problem 4. This one is like solving a mystery! We are given five facts about a real signal $x(t)$ with Laplace transform $X(s)$:

1. $X(s)$ has exactly 2 poles.
2. $X(s)$ has no zeros in the finite s-plane.
3. $X(s)$ has a pole at $s = -1 + j$.
4. $e^{2t}x(t)$ is *not* absolutely integrable.
5. $X(0) = 8$.

With these pieces of evidence in hand, find $X(s)$ and specify its region of convergence.

Problem 5. This problem how LTI systems can be used to model all sorts of real phenomena. When we build measurement systems (i.e. sensors), we often cannot control the relationship between the physical thing we want to measure and the output of the sensor. But by using some known inputs (steps, sinusoids, etc.) we can try to characterize the system behavior to post-process the "raw" sensor reading into the signal we actually want to measure.

A pressure gauge that can be modeled as an LTI system has a time response to a unit step input given by $(1 - e^{-t} - te^{-t})u(t)$. For a certain input $x(t)$ the output is observed to be $(2 - 3e^{-t} + e^{-3t})u(t)$.

From this observed measurement, determine the true pressure input to the gauge as a function of time.

Problem 6 (Oppenheim, Willsky, and Nawab 10.21)**.** Determine the z-transform for each for the following sequences:

(a) $\left(-\frac{1}{3}\right)^n u[-n-2]$

(b) $2^n u[-n] + \left(\frac{1}{4}\right)^n u[n-1]$

(c) $\left(\frac{1}{2}\right)^{n+1} u[n+3]$

(d) $\left(\frac{1}{3}\right)^{n-2} u[n-2]$

Problem 7. Find the z-transforms of the following signals (including the ROC).

(a) A chunk of a decaying exponential:

$$x[n] = \begin{cases} a^n, & 0 \leq n \leq N-1, a > 0 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

(b) A two-sided exponential:

$$y[n] = b^{|n|}, \qquad b > 0. \tag{5}$$

Problem 8. Consider the signal

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right), & n \leq 0 \\ 0, & n > 0 \end{cases} \tag{6}$$

Determine the poles and ROC for $X(z)$.