Linear Systems and Signals

The frequency response of CT LTI systems

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Learning objectives

The learning objectives for this section are:

- understand the relationship between the frequency response and the Laplace transform
- visualize how the pole-zero placement affects the sinusoidal response

Frequency response

We saw how to computationally visualize the *frequency response* of an LTI system:

Laplace transform

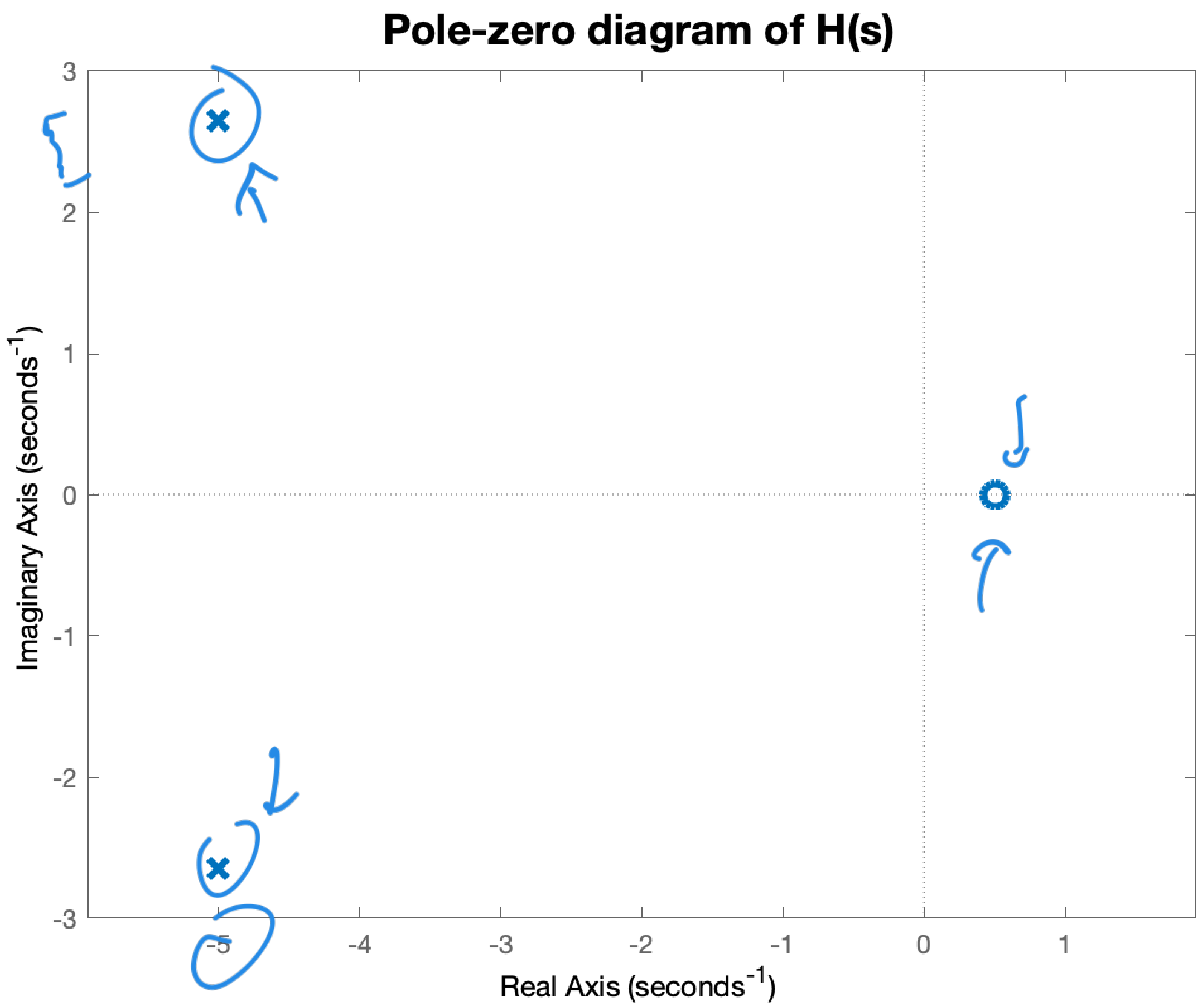
$$H(s)\Big|_{s=j\omega} = H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt \tag{1}$$

How does the location of poles and zeros affect the frequency response of the CT system?

Focus on $|H(j\omega)|$

- poles "pull up" on the surface of $|H(j\omega)|$
- zeros "pin down" the surface of $|H(j\omega)|$

Example: pole-zero diagram



$$\frac{-10 \pm \sqrt{100-128}}{2}$$

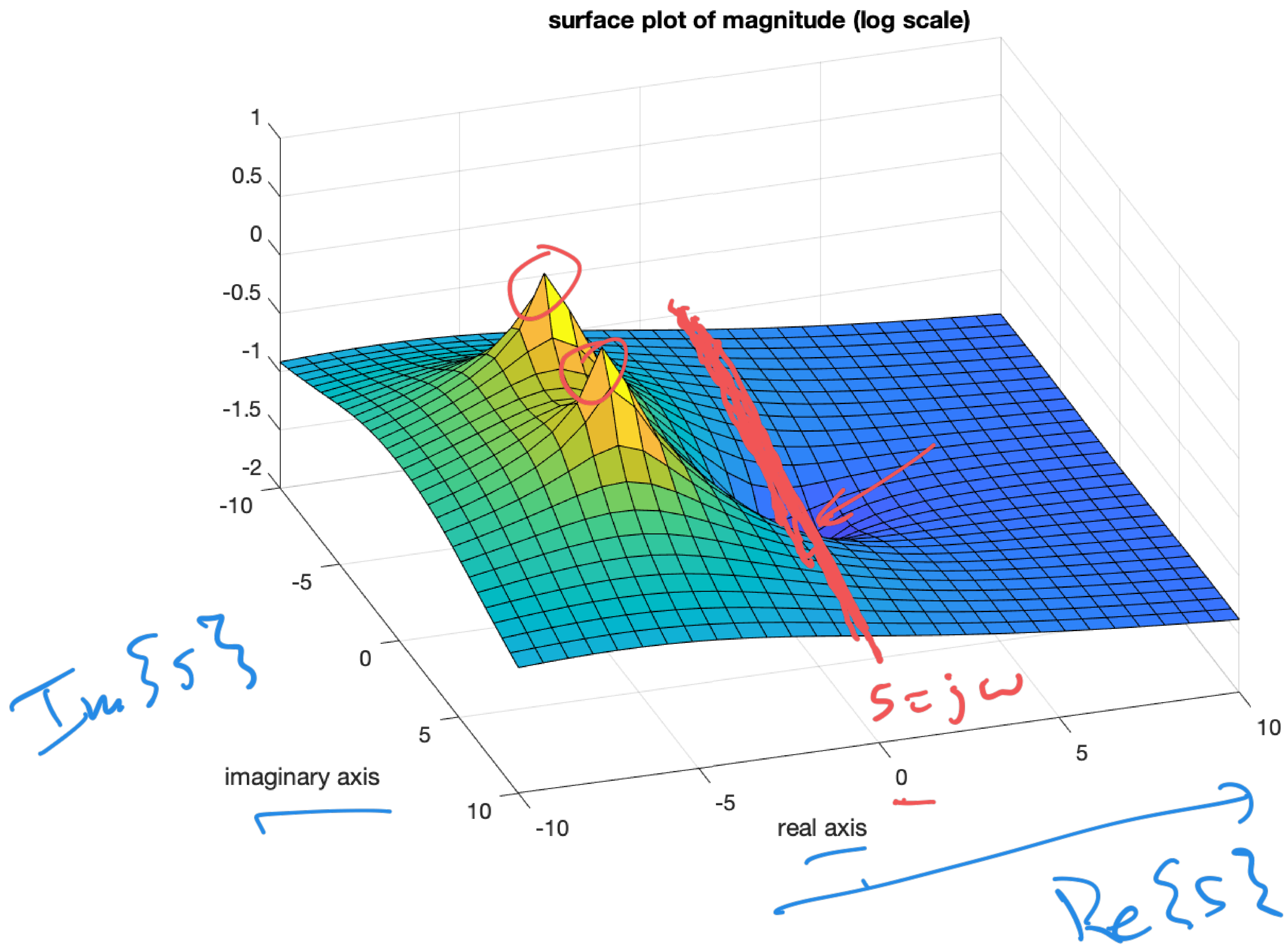
$$-5 \pm \sqrt{7}$$

zero @ 1/2

2 complex poles

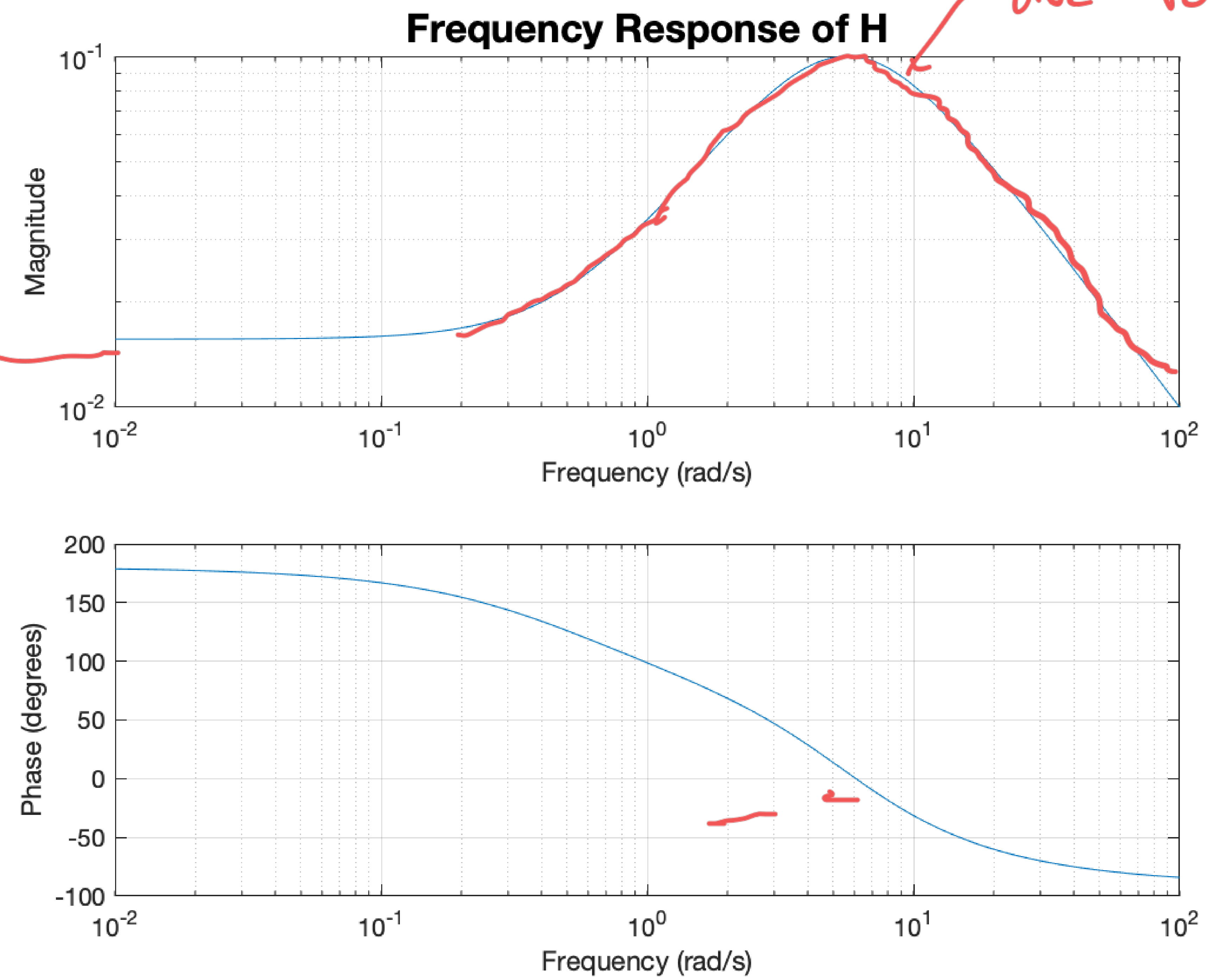
$$H(s) = \frac{s - 1/2}{s^2 + 10s + 32} \quad (2)$$

Example: surface plot of $\log_{10}|H(j\omega)|$



$$H(s) = \frac{s - 1/2}{s^2 + 10s + 32} \tag{3}$$

Example: frequency response



due to the poles

$\cos(\omega t) = \frac{1}{2} e^{j\omega t} + \frac{1}{2} e^{-j\omega t}$

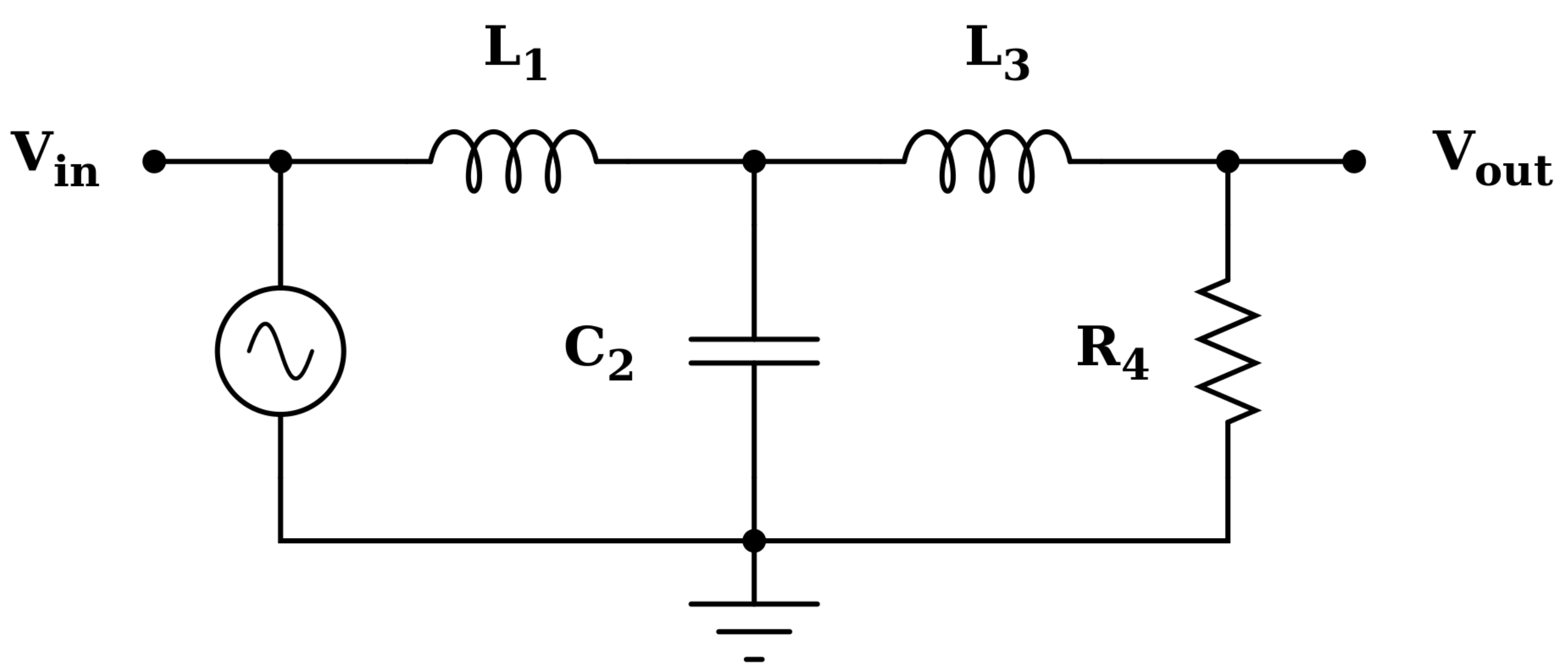
$H(j\omega)$

$$H(s) = \frac{s - 1/2}{s^2 + 10s + 32} \tag{4}$$

Example: Butterworth filter

3 poles

A 3rd-order Butterworth filter (a class of commonly used filters) can be implemented as follows[1]



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_4}{s^3(L_1C_2L_3) + s^2(L_1C_2R_4) + s(L_1 + L_3) + R_4} \quad (5)$$

Using $C_2 = 4/3$ F, $R_4 = 1\ \Omega$, $L_1 = 3/2$ H and $L_3 = 1/2$ H we can get a simple transfer function.

[1] Image courtesy of Wikipedia

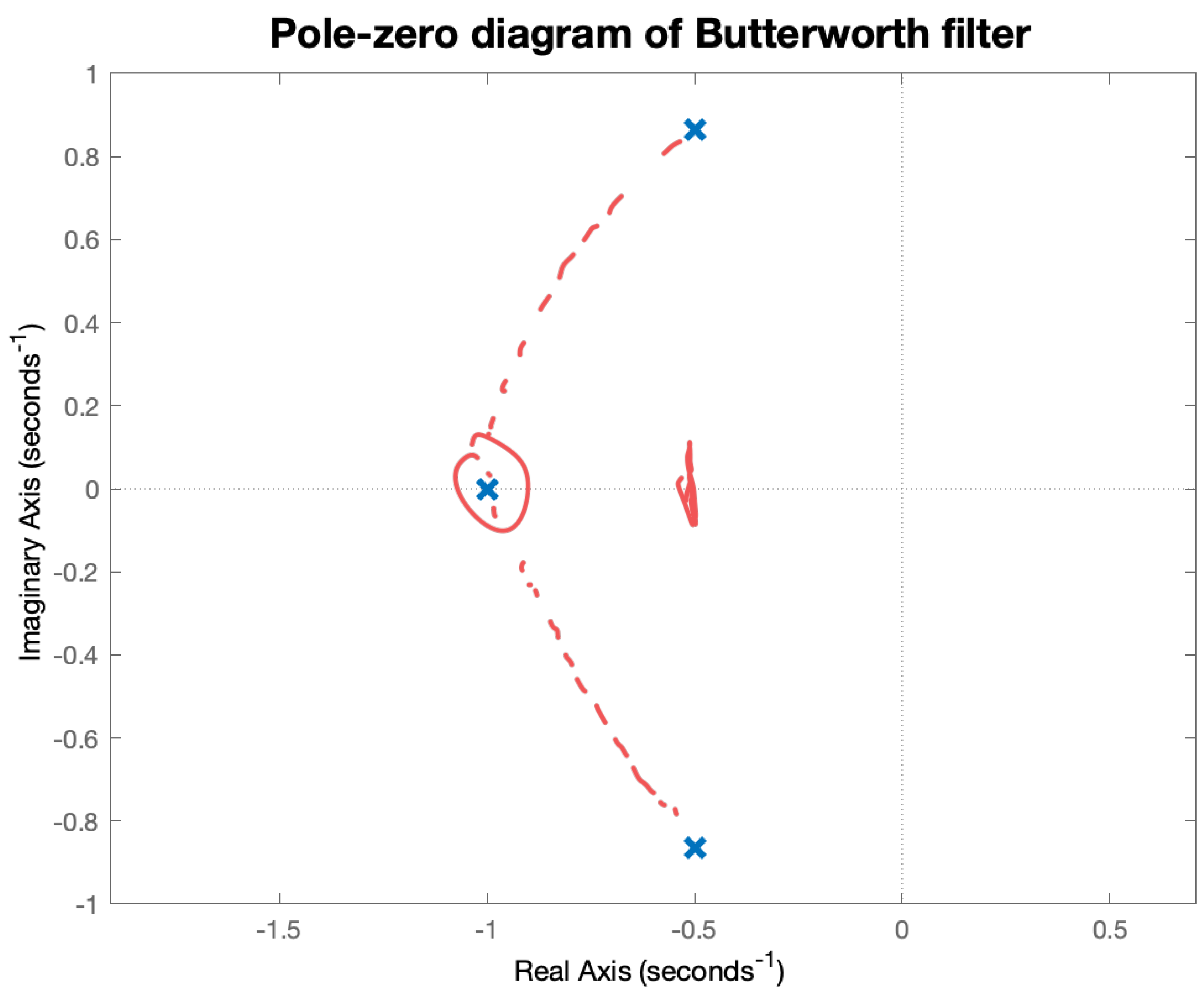
Example: 3rd order Butterworth filter pole-zeros

The transfer function

$$H(s) = \frac{1}{1 + 2s + 2s^2 + s^3} \qquad (6)$$

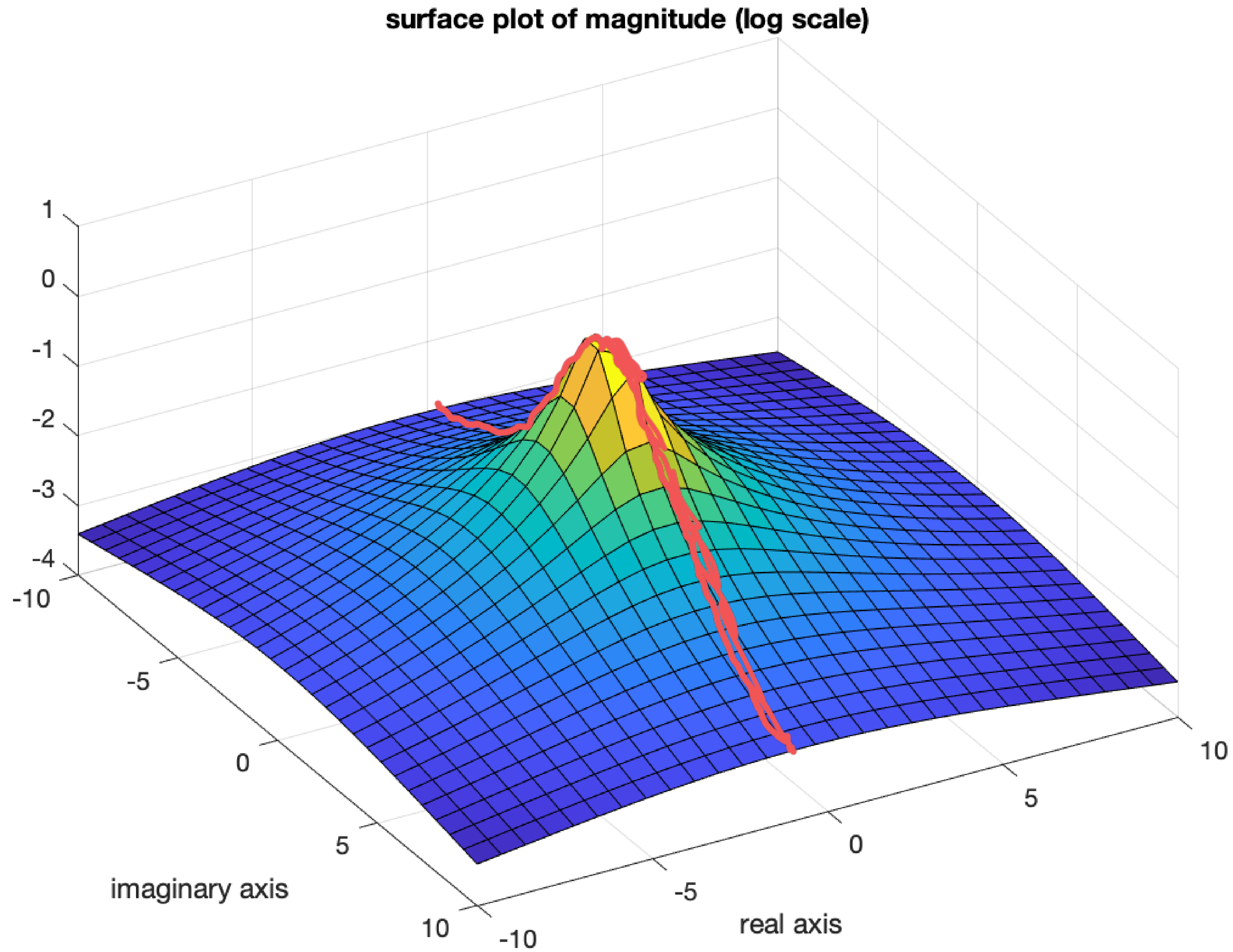
roots([1, 2, 2, 1])

has the pole-zero diagram

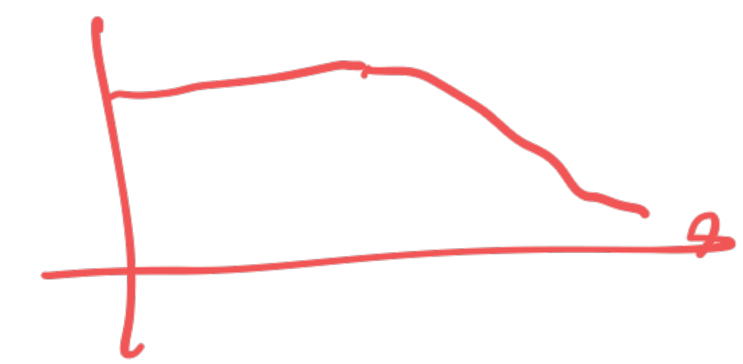


3rd order
$\Rightarrow$ 3 poles

Example: Butterworth surface

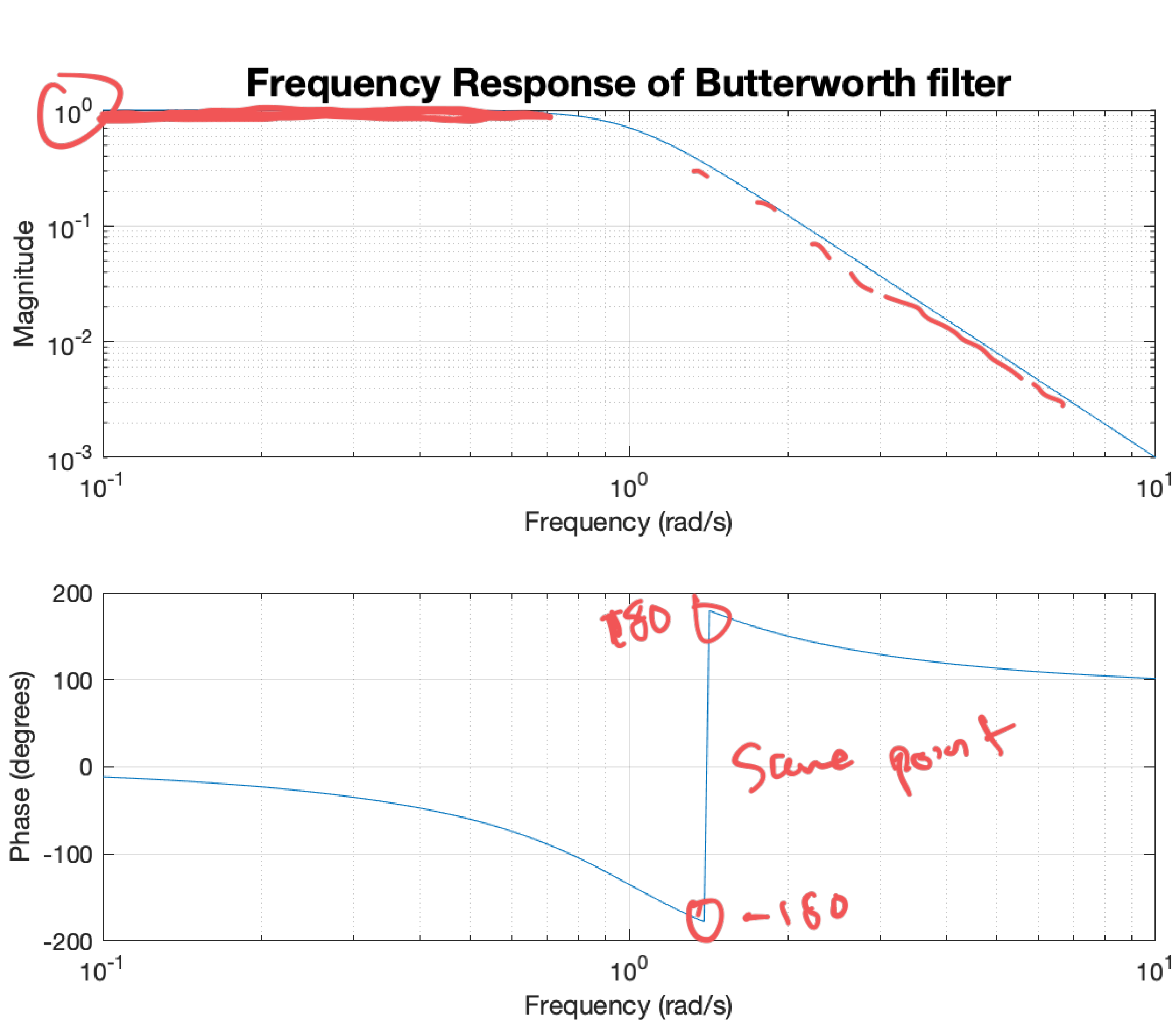


$H(j\omega)$



$$H(s) = \frac{1}{1 + 2s + 2s^2 + s^3} \qquad (7)$$

Example: Butterworth frequency response



allows low freqs to go through w/ gain 1

$\Rightarrow$ low pass filter

$$H(s) = \frac{1}{1 + 2s + 2s^2 + s^3} \tag{8}$$

Try it yourself

Hbutter = tf(num, den) order cutoff give a CT filter

Running `[num,den] = butter(N, Wc, 's')` produces the numerator and denominator for an N-th order analog lowpass Butterworth filter with cutoff frequency `Wc`.

Problem

Try playing around with this function to visualize higher-order Butterworth filters, their pole-zero diagrams, and the frequency response. You can also look for other filter types, like Chebyshev `cheby1`, `cheby2`*, elliptical (*`ellip`*), and Bessel (*`besself`*) filters, but these often require translating a DT filter design to CT.*

$x(t) \rightarrow \boxed{\text{sampler}} \xrightarrow{x[n]} \boxed{h[n]} \xrightarrow{y[n]} \boxed{\text{interp}} \rightarrow y(t)$, DT processing of CT signals