Linear Systems and Signals
Simulating CT LTI systems

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Learning objectives

The learning objectives for this section are:

- use the system toolbox to implement rational transfer functions
- visualize pole-zero diagrams for CT LTI systems
- visualize the impulse response of a CT system
- visualize the frequency response of a CT system

Two example systems

We're going to use the following two systems to illustrate how to deal with CT LTI systems in MATLAB:

$$H_1(s) = \frac{s-3}{s^2+5s+6} = \frac{s-3}{(s+2)(s+3)} \tag{1}$$

$$H_2(s) = \frac{s+5}{s^2+3s+6} \quad \leftarrow \text{complex roots} \tag{2}$$

As an exercise, compute the inverse transform of these two systems and find the corresponding impulse responses. You can then compare those against the plots we will generate.

$$\frac{-3 \pm \sqrt{9-36}}{2}$$

$$= -\frac{3}{2} \pm \frac{3}{2}\sqrt{3}\, j$$

Example

In MATLAB you can use the `tf()` (transfer function) formula to make a system object corresponding to a system with rational Laplace transform:

Code Example 1: defining two transfer functions

```
num1 = [1,-3];
den1 = [1, 5, 6];
num2 = [1, 5];
den2 = [1, 3, 6];
H1 = tf(num1,den1)
H2 = tf(num2,den2)
```

$1s - 3$

$1s^2 + 5s + 6$

$1s + 5$

$1s^2 + 3s + 6$

You should see MATLAB print out the two transfer functions.

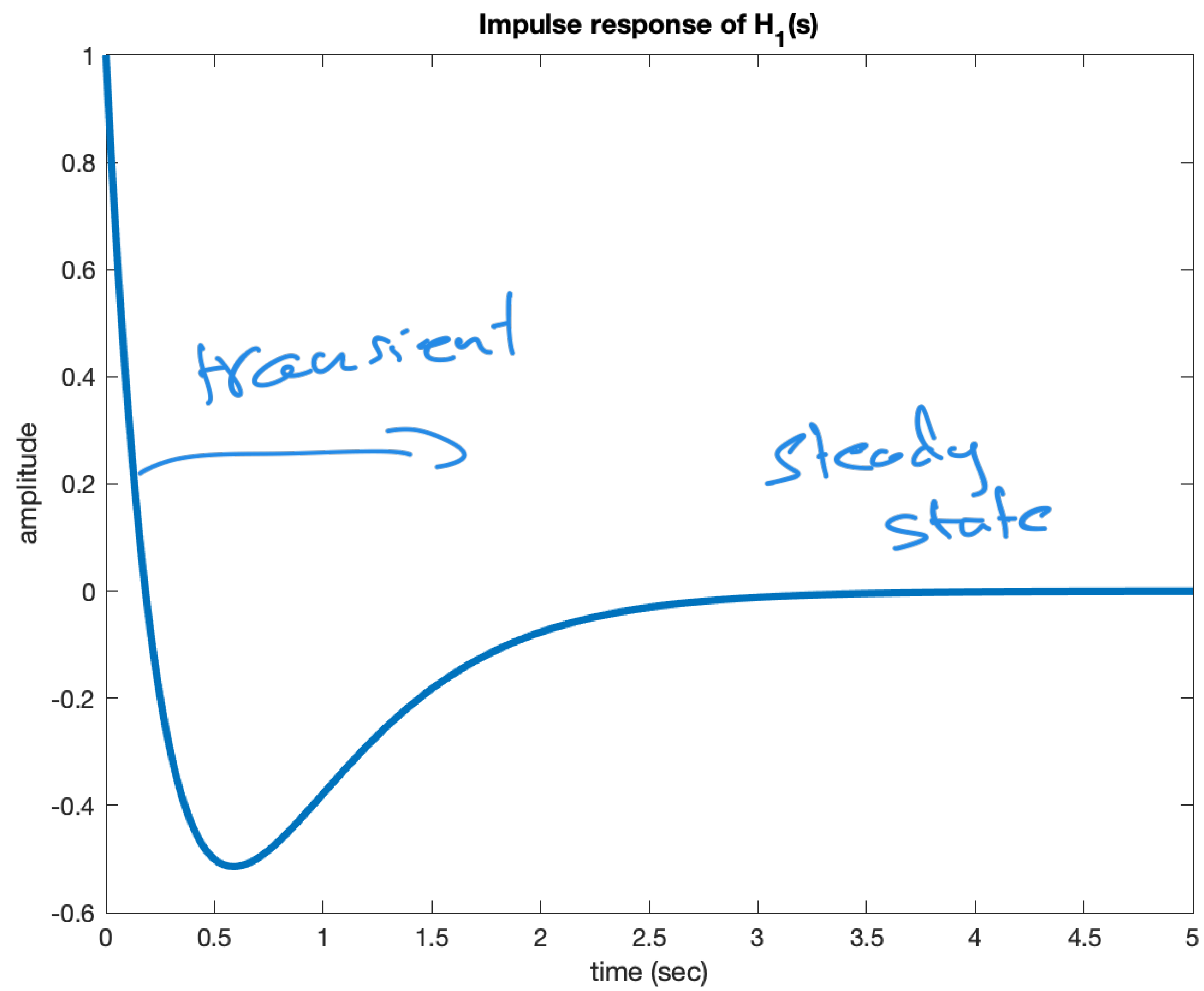
Plotting the impulse response

Suppose we want to plot the impulse response. We can use the `impulse(H,t)` function to produce a plot, or `[h,t] = impulse(H1,t)` to get the time axis and $h(t)$ back. The default time axis is something like 3.5 seconds. It is important for you to keep track of the sampling rate f_s and the time axis you are using. You may be interested in shorter or longer period of time!

Code Example 2: plotting an impulse response

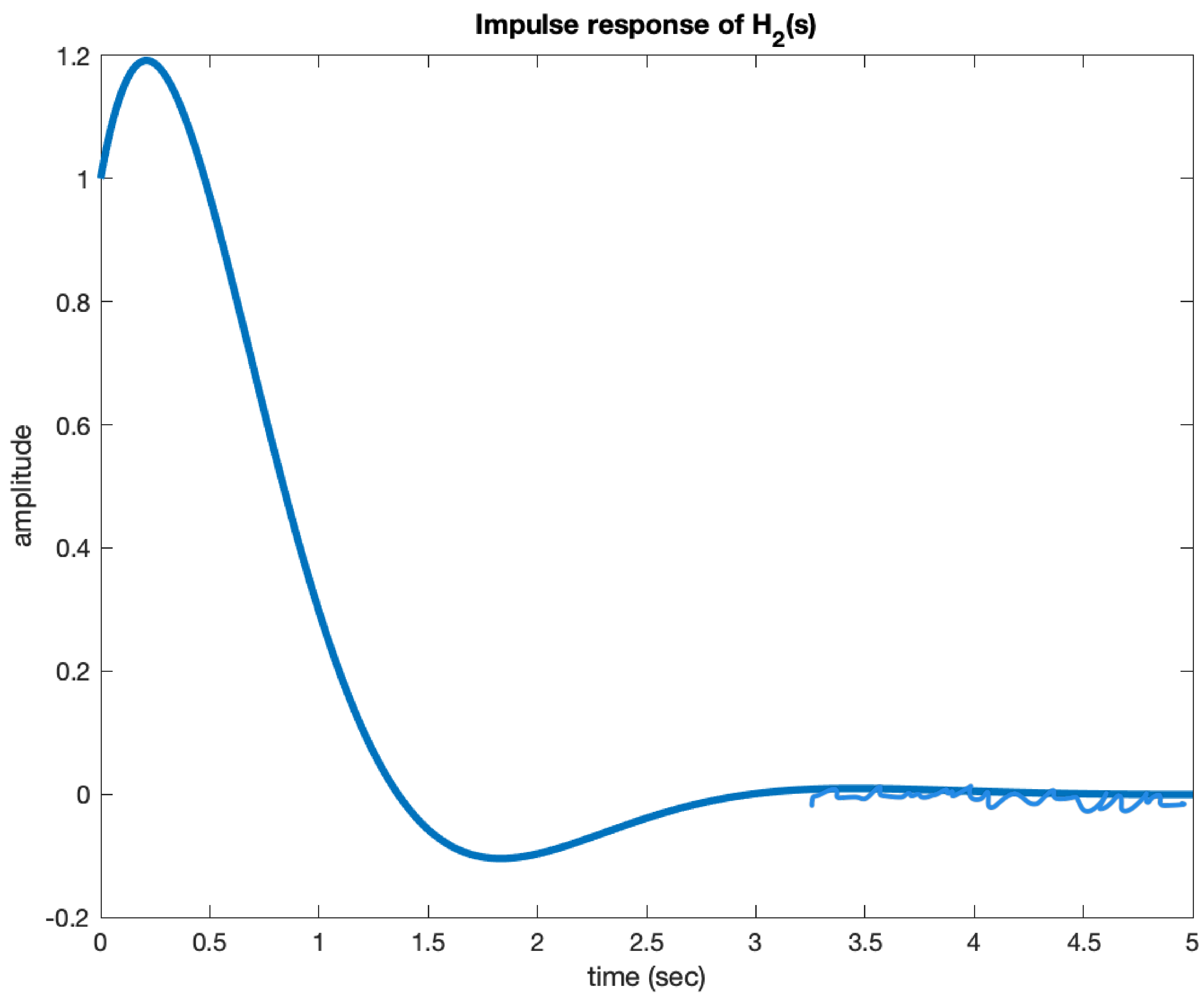
```
fs = 1e5;            % 100 kHz
t = 0:(1/fs):5;      % 5 seconds
[h1,t] = impulse(H1,t);
figure;
plot(t,h1,'LineWidth', 3);
xlabel('time (sec)'); ylabel('amplitude');
title('Impulse response of H_1(s)');
```

Impulse response for system 1



$$H_1(s) = \frac{s-3}{s^2+5s+6} \tag{3}$$

Impulse response for system 2



$$H_2(s) = \frac{s+5}{s^2+3s+6} \tag{4}$$

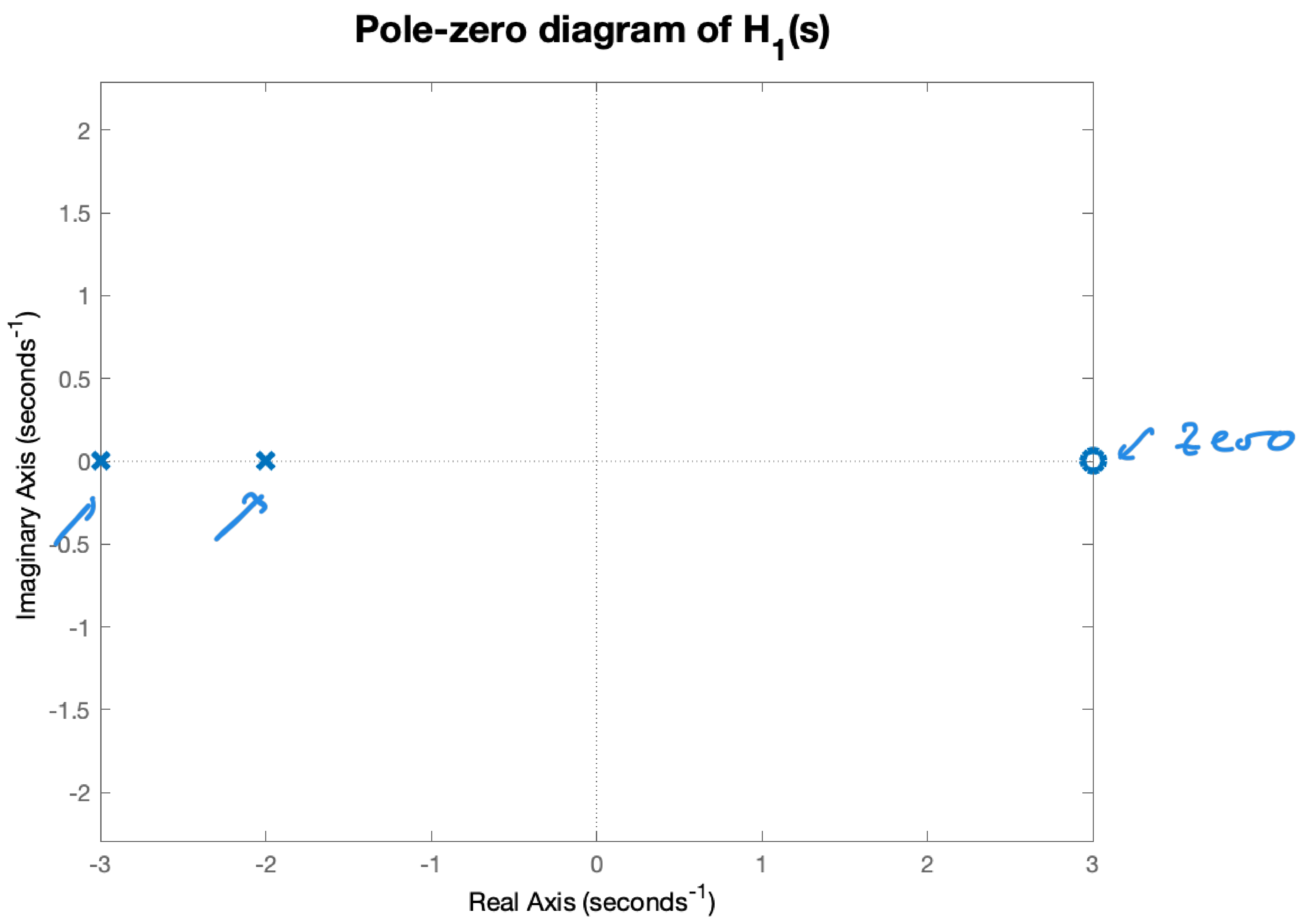
The pole-zero diagram

You can use pzplot() to plot the pole-zero diagram. The default plot is sort of ugly so we can adjust the axes and line properties to make it easier to read/see.

Code Example 3: plotting the pole-zero diagram

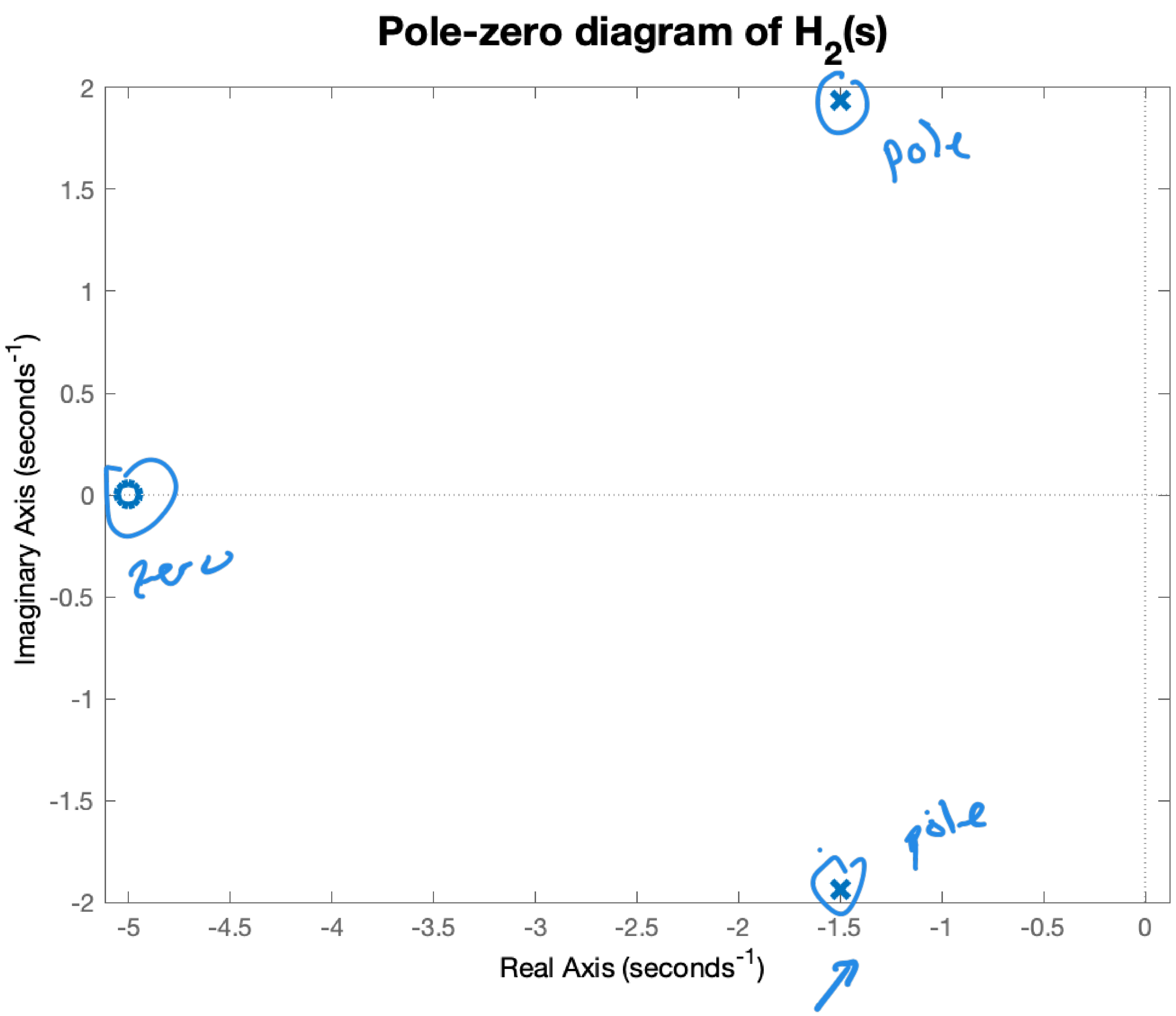
```
figure;
pzplot(H1);
lineobject = findobj(gca, 'type', 'line');
set(lineobject, 'markersize', 9);
set(lineobject, 'linewidth', 3);
axis equal;
title('Pole-zero diagram of H_1(s)','FontSize',16);
```

Pole-zero diagram for system 1



$$H_1(s) = \frac{s-3}{s^2+5s+6} \tag{5}$$

Pole-zero diagram for system 2



$$H_2(s) = \frac{s+5}{s^2+3s+6} \tag{6}$$

The frequency response

Remember the eigenfunction property of CT LTI systems: a complex exponential $x(t) = e^{j\omega_0 t}$ going into a system with impulse response $h(t)$ produces an output

complex coefficient

$$y(t) = H(j\omega_0)e^{j\omega_0 t} \tag{7}$$

where the *frequency response* $H(j\omega)$ is a complex-valued function:

this notation is consistent w/ Laplace

$$\boxed{H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt} \tag{8}$$

H(s) on the imag. axis.

This is just the Laplace transform $H(s)$ when $s = j\omega$!

Frequency response: magnitude and phase

If we input $\cos(\omega_0 t)$ we get as the output

$$y(t) = |H(j\omega_0)| \cos(\omega_0 t + \measuredangle H(j\omega_0)) \tag{9}$$

We are interesting in computing the *magnitude* $|H(j\omega)|$ and phase $\measuredangle H(j\omega)$ functions as a function of ω. These functions tell us the effect the system has on sinusoids at different frequencies:

- a low magnitude means that the system suppresses those frequencies
- a high magnitude means it amplifies those frequencies.

The phase represents the distortion the signal receives. A *linear phase* means $\measuredangle H(j\omega) = -c\omega$, which is just a pure delay:

$$\cos(\omega_0 t + c\omega_0) = \cos(\omega_0 (t - c)) \tag{10}$$

Plotting the frequency response

We can use the `freqs()` function to plot the frequency response.

Code Example 4: plotting a frequency response

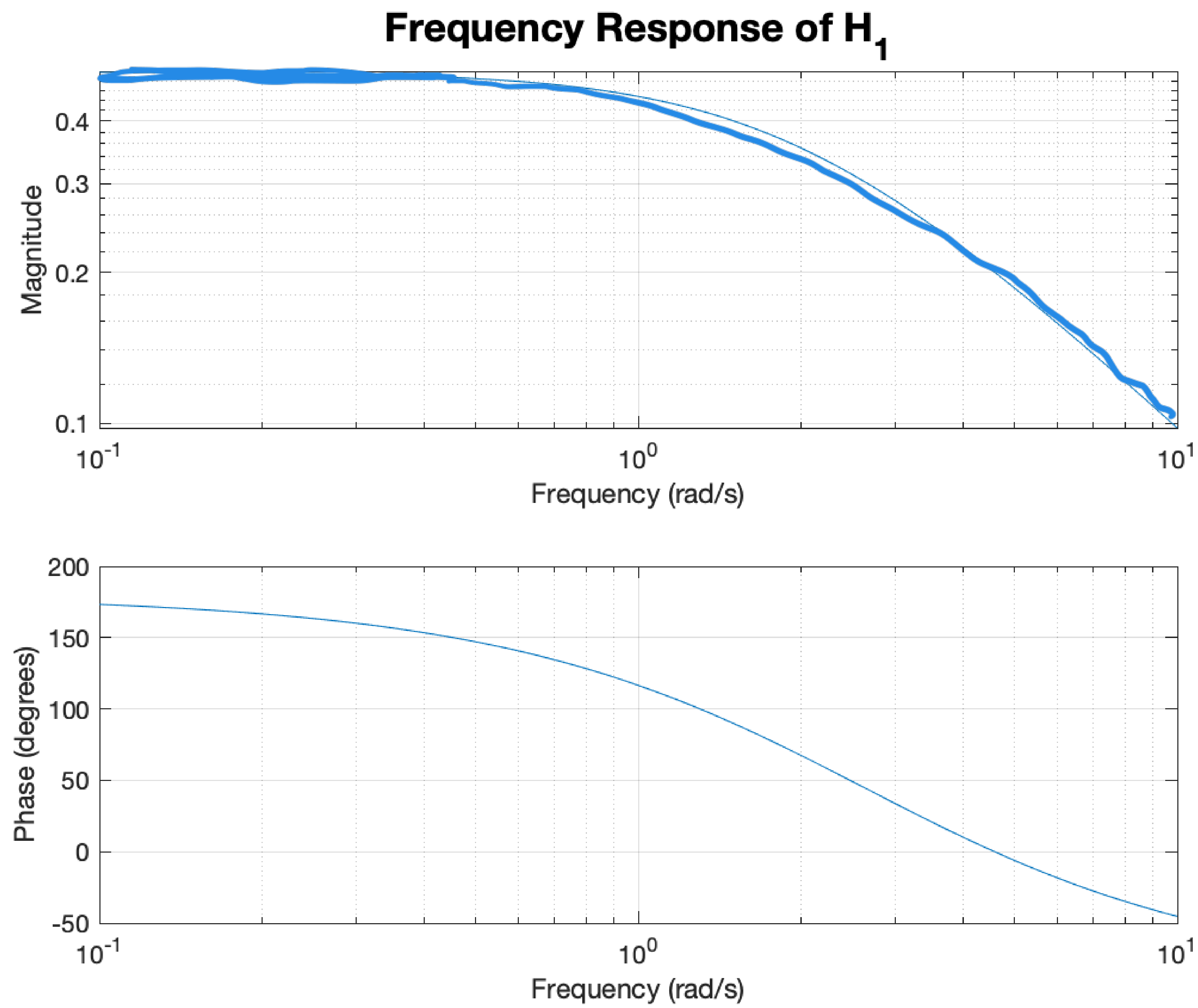
```
figure;
freqs(num1,den1);
title('Frequency Response of H_1','FontSize',16);
```

A few notes:

- You have to explicitly pass the numerator and denominator to `freqs`.
- Magnitude is plotted in log-log scale, so frequencies are increasing exponentially along the time axis and magnitudes are reported proportional to decibels. If you want a dB scale plot you can use `bode(H1)` to produce a *Bode plot*.
- Phase is plotted in degrees (vertical axis) vs. radian frequency[1].

[1] TBH this makes no sense: it's like reporting feet as a function of meters.

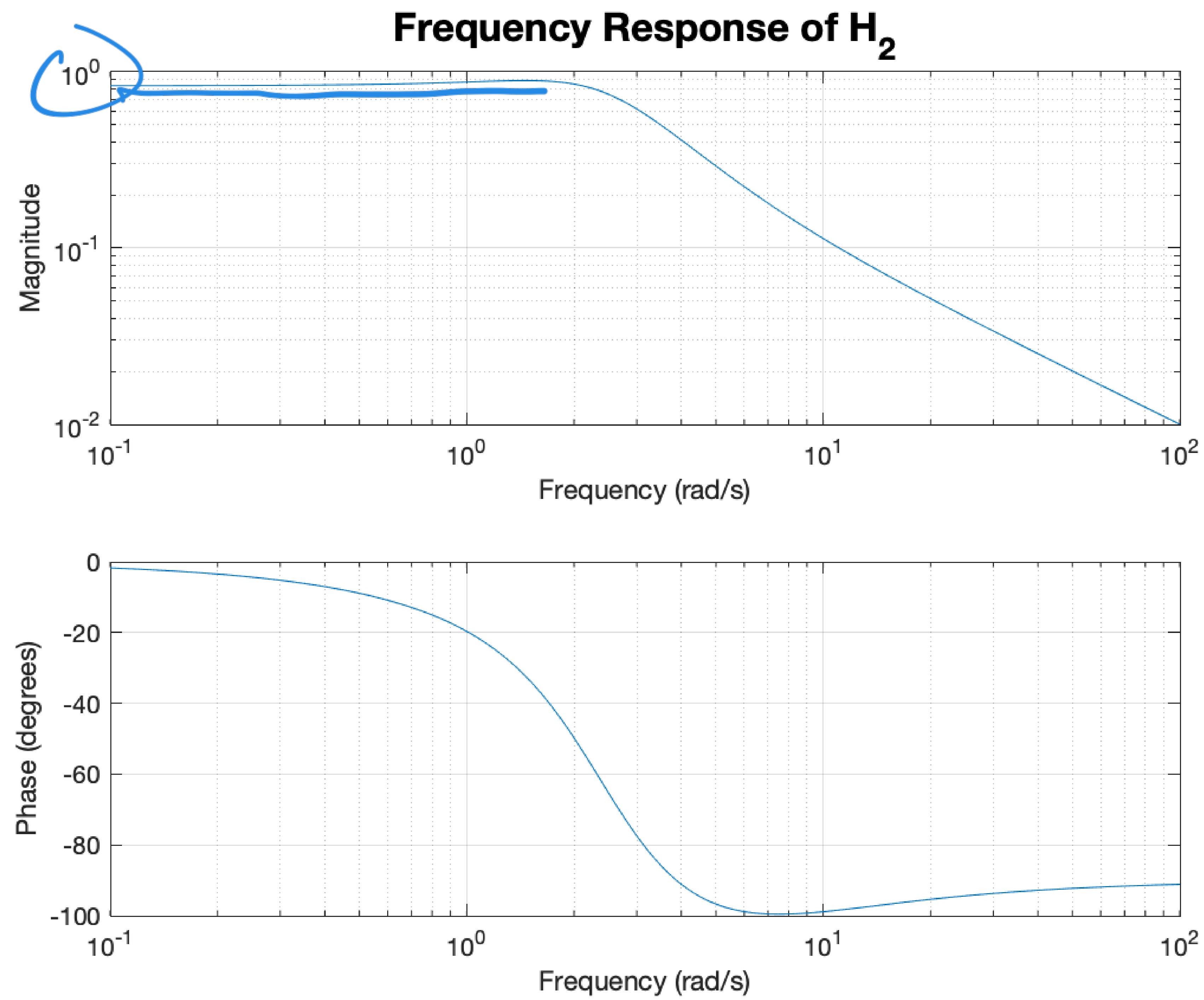
Frequency response for system 1



allows low frequencies to go through
attenuates higher frequencies

$$H_1(s) = \frac{s-3}{s^2+5s+6} \tag{11}$$

Frequency response for system 2



also passes low frequencies w/ gain always 1
drops off & attenuates lower frequencies

$$H_2(s) = \frac{s+5}{s^2+3s+6} \tag{12}$$

Cascading systems

We can put one system after the other to get a "cascade." Since convolution in time is multiplication in the Laplace domain:

$x(t) * h_1(t) * h_2(t)$

$$H_3(s) = H_1(s) H_2(s) = \boxed{\frac{s^2 + 2s - 15}{s^4 + 8s^3 + 27s^2 + 48s + 36}} \tag{13}$$

In MATLAB we can just do this using the system toolbox and then extract out the numerator and denominator.

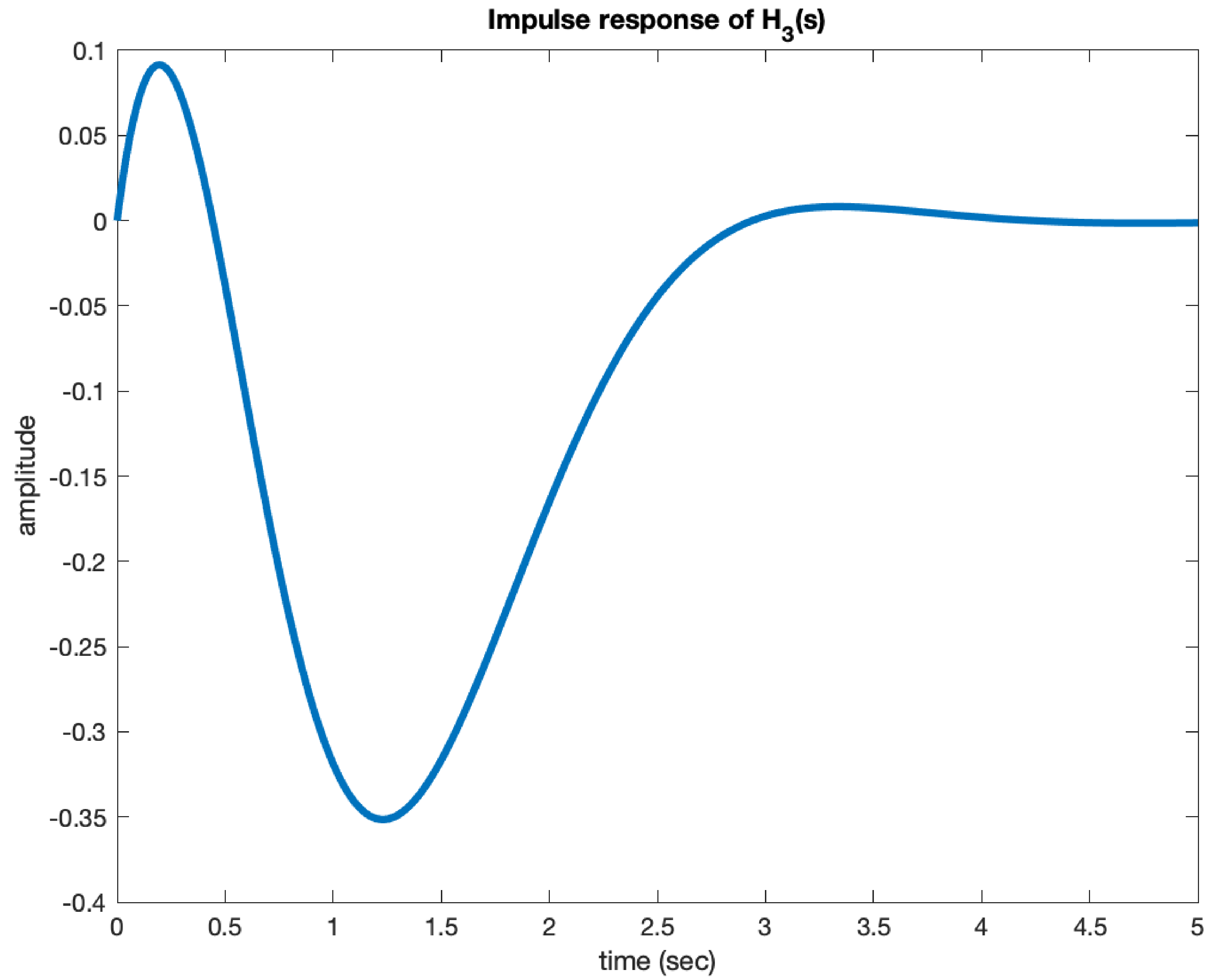
Code Example 5: finding a cascade system

```
H3 = H1 * H2
num3 = cell2mat(get(H1,'Numerator'));
den3 = cell2mat(get(H1,'Denominator'));
```

need this for freqs(.)

Then we can plot the impulse response, pole-zero diagram, and frequency response.

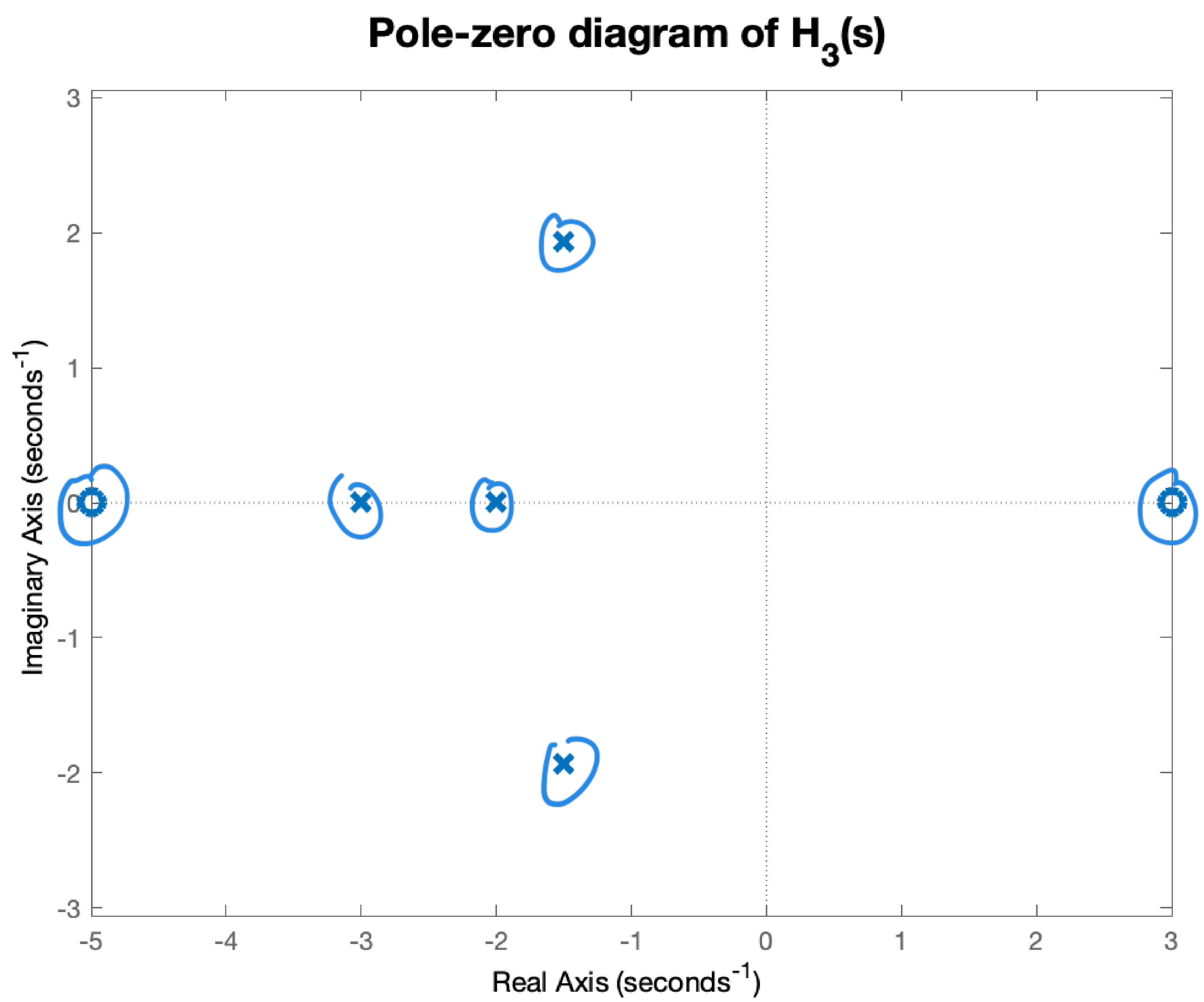
Impulse response for system 3



is ~~looks a like~~ ~~combo~~ convo-lution of the two impulse responses of h_1, h_2

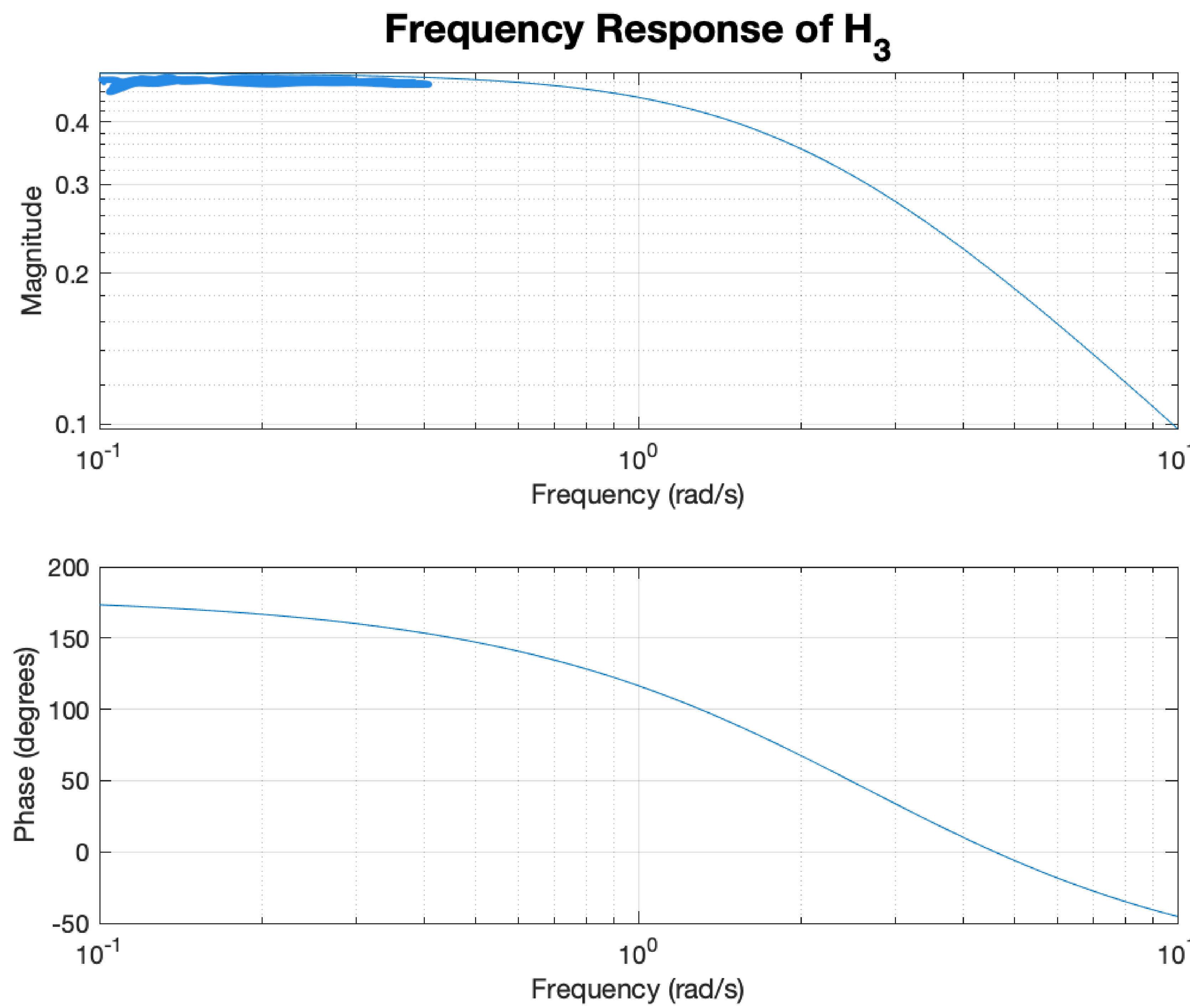
$$H_3(s) = \frac{(s-3)(s+5)}{(s+2)(s+3)(s^2+3s+6)} \tag{14}$$

Pole-zero diagram for system 3



$$H_3(s) = \frac{(s-3)(s+5)}{(s+2)(s+3)(s^2+3s+6)} \tag{15}$$

Frequency response for system 3



still get flat FR @ low frequencies

$$H_3(s) = \frac{(s-3)(s+5)}{(s+2)(s+3)(s^2+3s+6)} \tag{16}$$

Try it yourself

Problem

Take any of the systems we have seen in the previous slides. Plot the impulse response, frequency response, and pole-zero diagram to get a feel for how these objects work computationally.

The cascade produces systems in series. For systems in parallel you can just add them (you can use this to check your partial fraction expansions). See what happens to the impulse response, pole-zero diagram, and frequency response and try to interpret what you see in terms of the original component systems.