Linear Systems and Signals

A brief introduction to bandpass filters

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Learning objectives

The learning objectives for this section are:

- Understand the frequency response of different bandpass filters
- Explain how different filters can be used for different applications

The shape of the frequency response

$$\cos(\omega_0 t) \longrightarrow \boxed{h(t)} \longrightarrow |H(j\omega_0)| \cos(\omega_0 t + \angle H(j\omega_0))$$

We are interested in the *shape* of the frequency response and what it means for sinusoidal inputs at different frequencies.

- When $|H(j\omega)|$ is > 1, frequencies get amplified.
- When $|H(j\omega)|$ is < 1, frequencies are attenuated.
- When $|H(j\omega)|$ is ≈ 1, frequencies are “passed.”
- When $|H(j\omega)|$ is ≈ 0, frequencies are “rejected.”

Bandpass and other filters

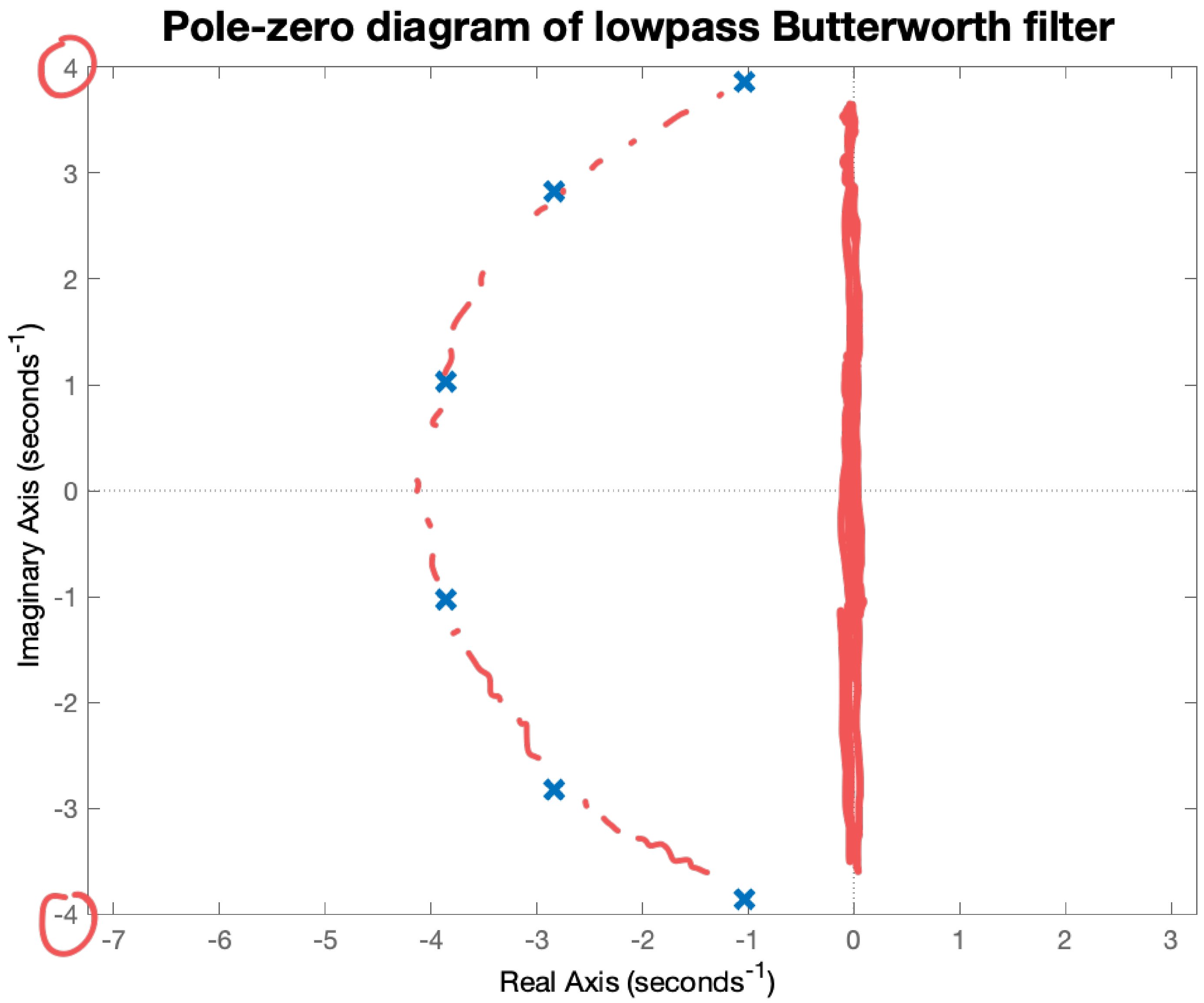
When we talk about filters we usually talk about x-pass filters, where x is "band" or "low" or "high":

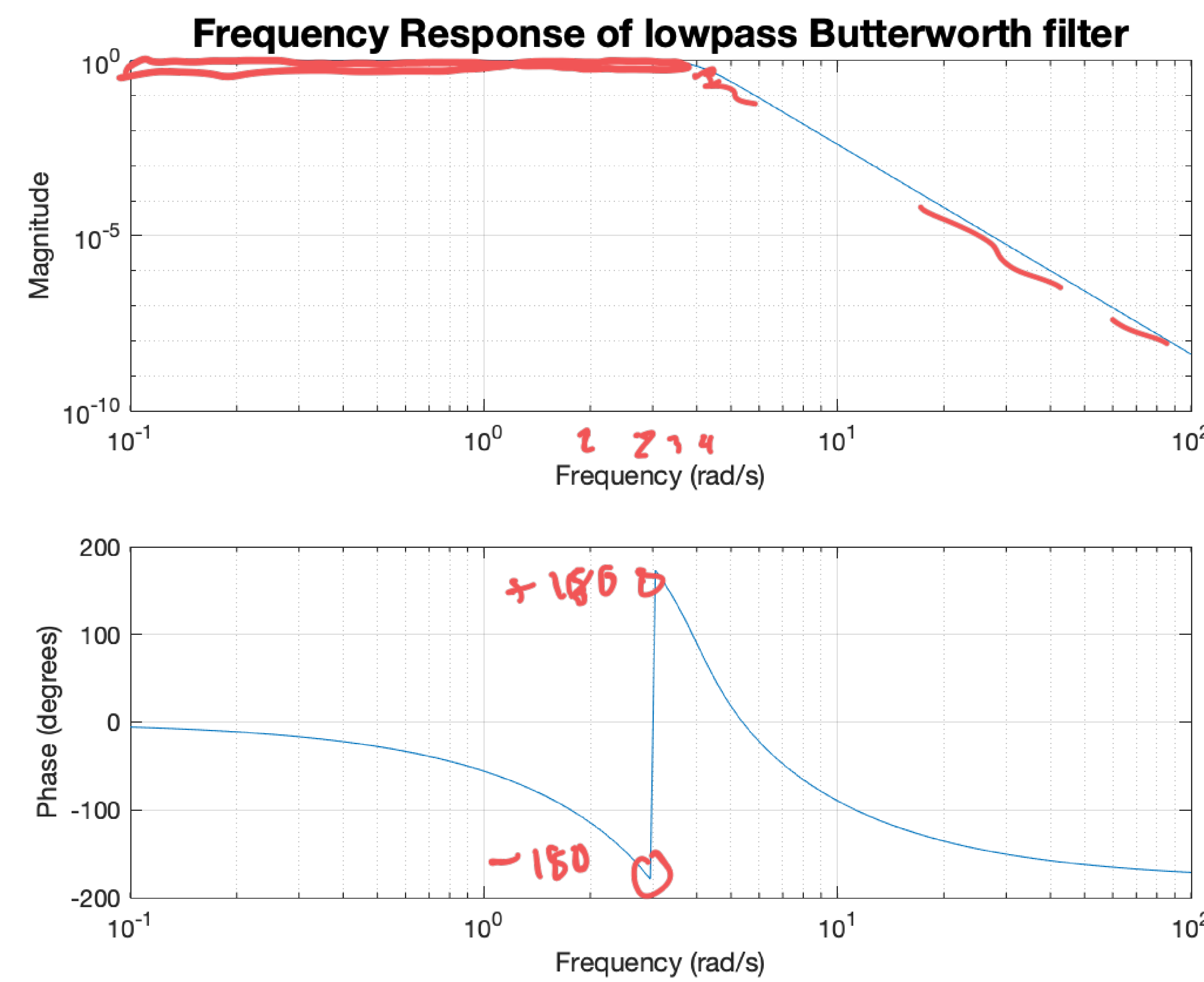
- a lowpass filter has magnitude ≈ 1 from $\omega = 0$ to some cutoff ω_c,
- a highpass filter has magnitude ≈ 1 from ω_c to ∞,
- a bandpass filter has magnitude ≈ 1 from ω_a to ω_b.

There are others:

- a band reject (or band stop) filter has magnitude ≈ 0 from ω_a to ω_b,
- a notch filter has magnitude ≈ 0 for some specific ω_n but otherwise has constant gain.

Example: lowpass filters

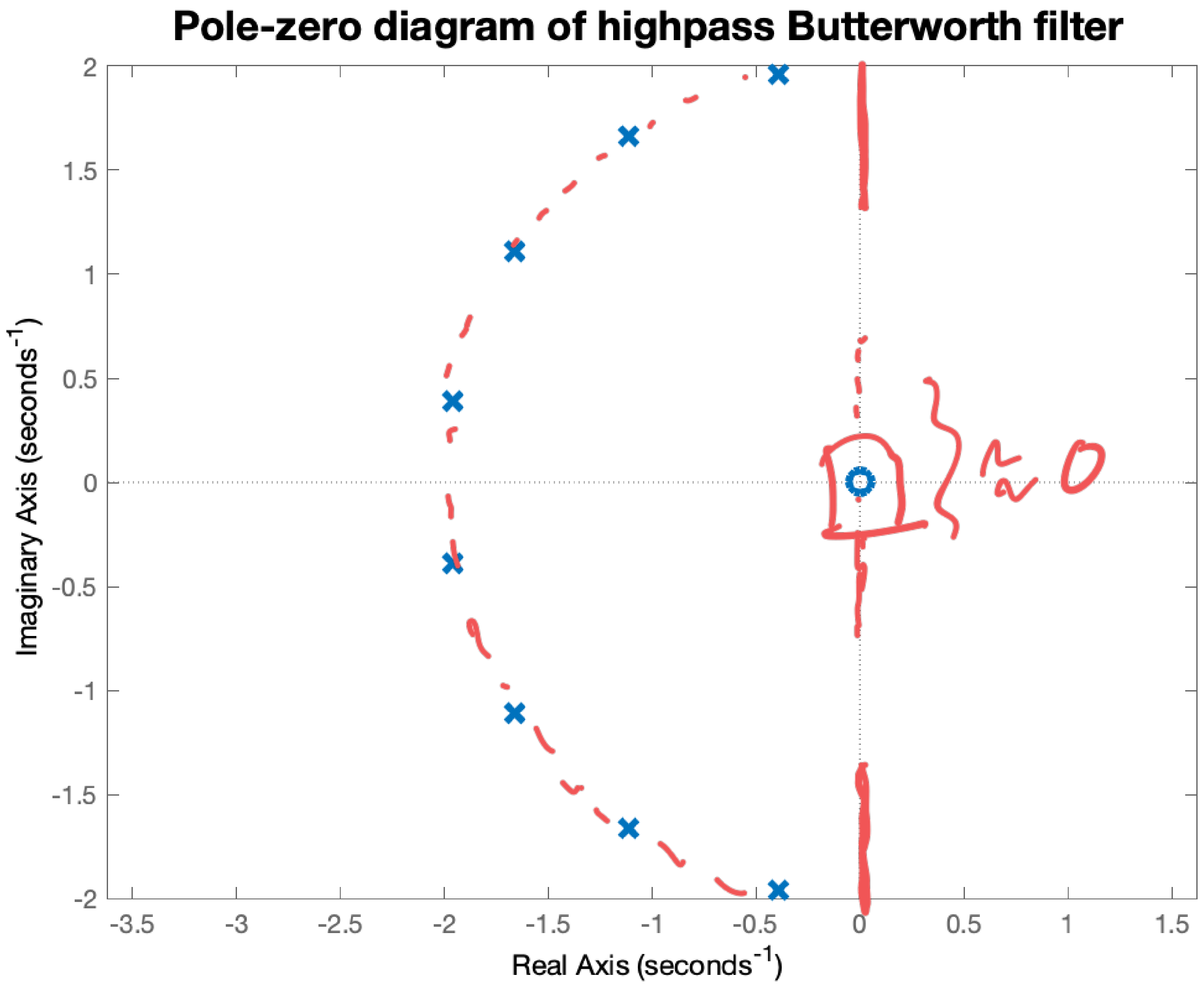


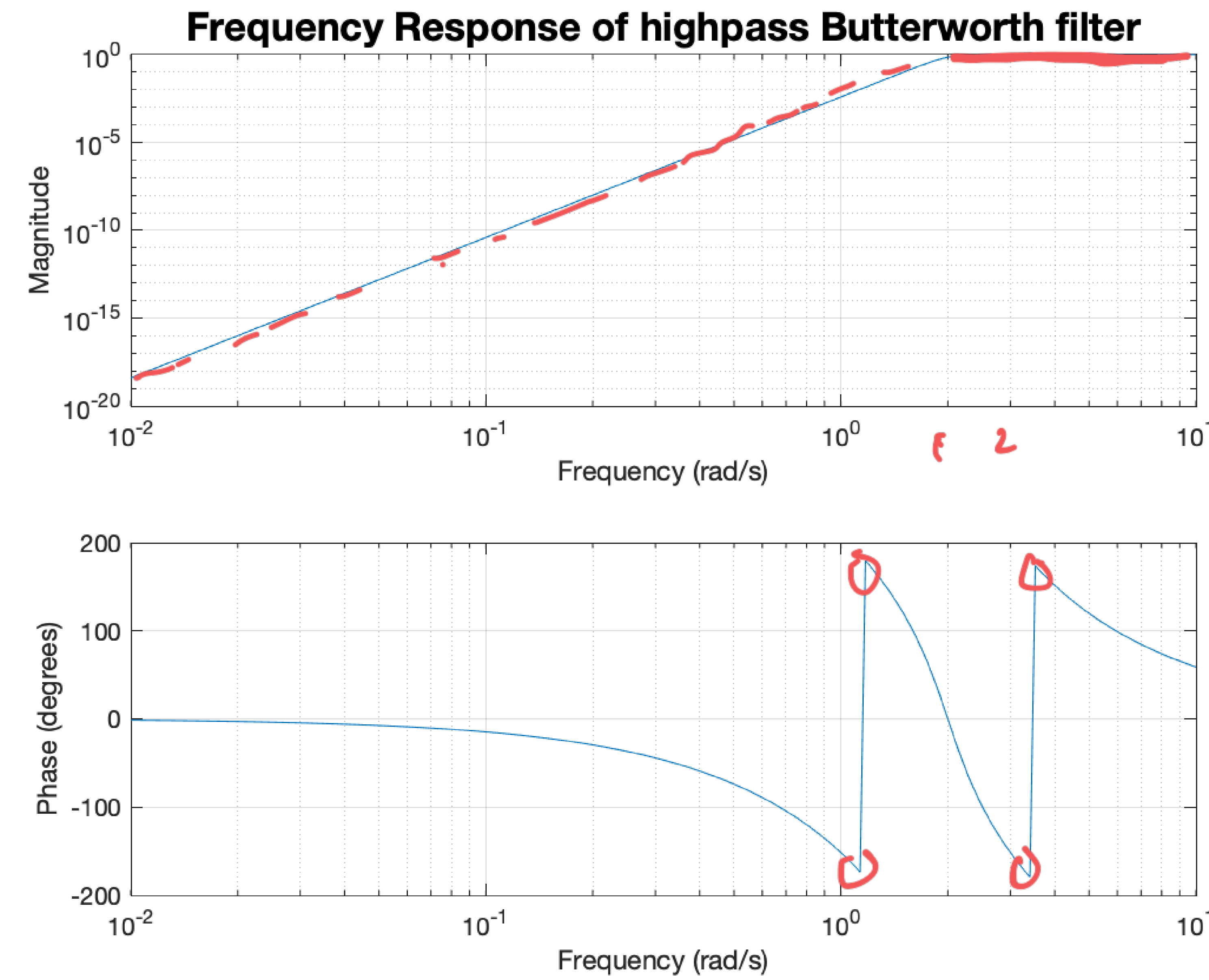


Code Example 1: 6th order lowpass Butterworth filter

```
[B,A] = butter(6,4,'low','s');
Hlow = tf(B,A);
```

Example: highpass filters

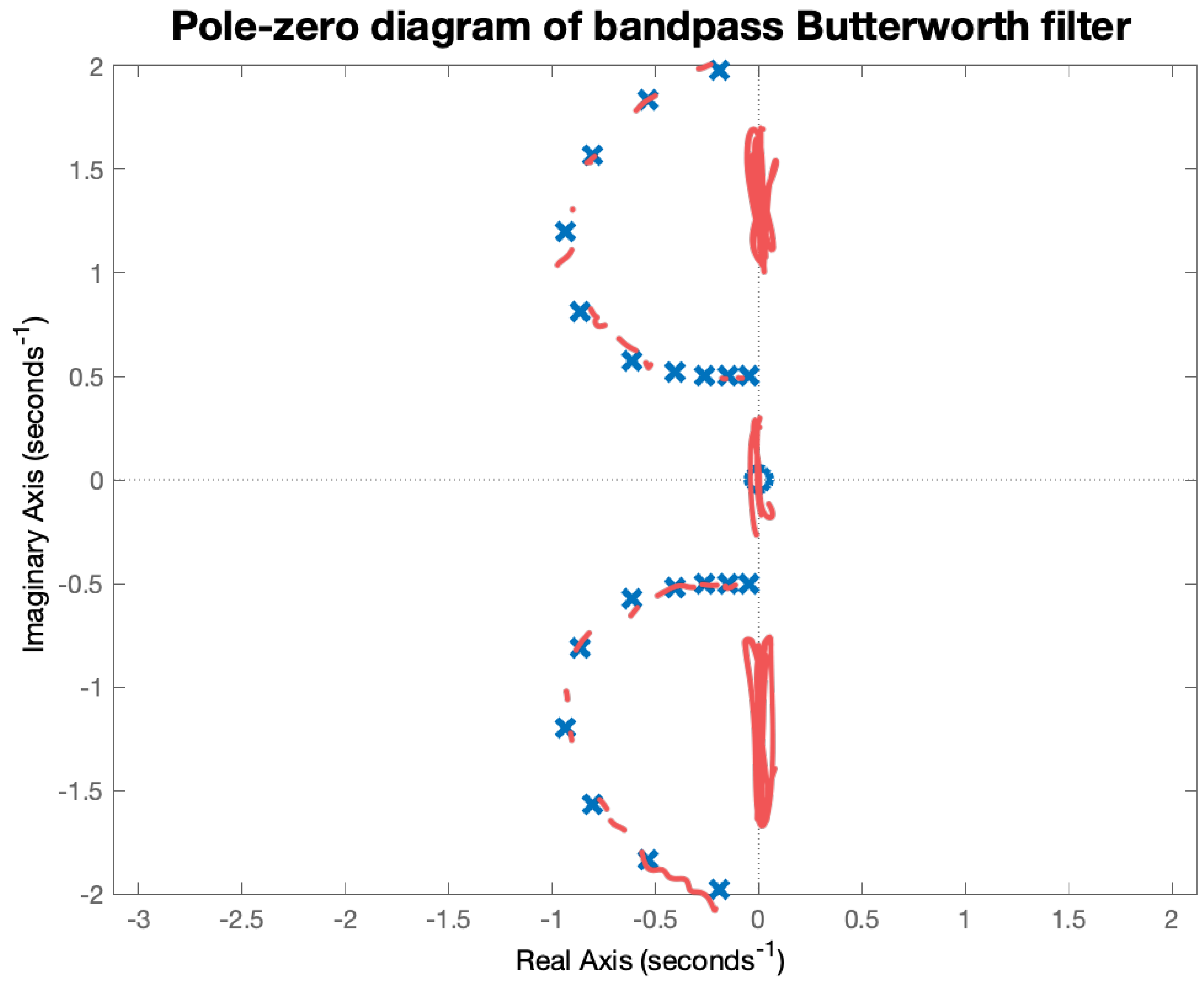


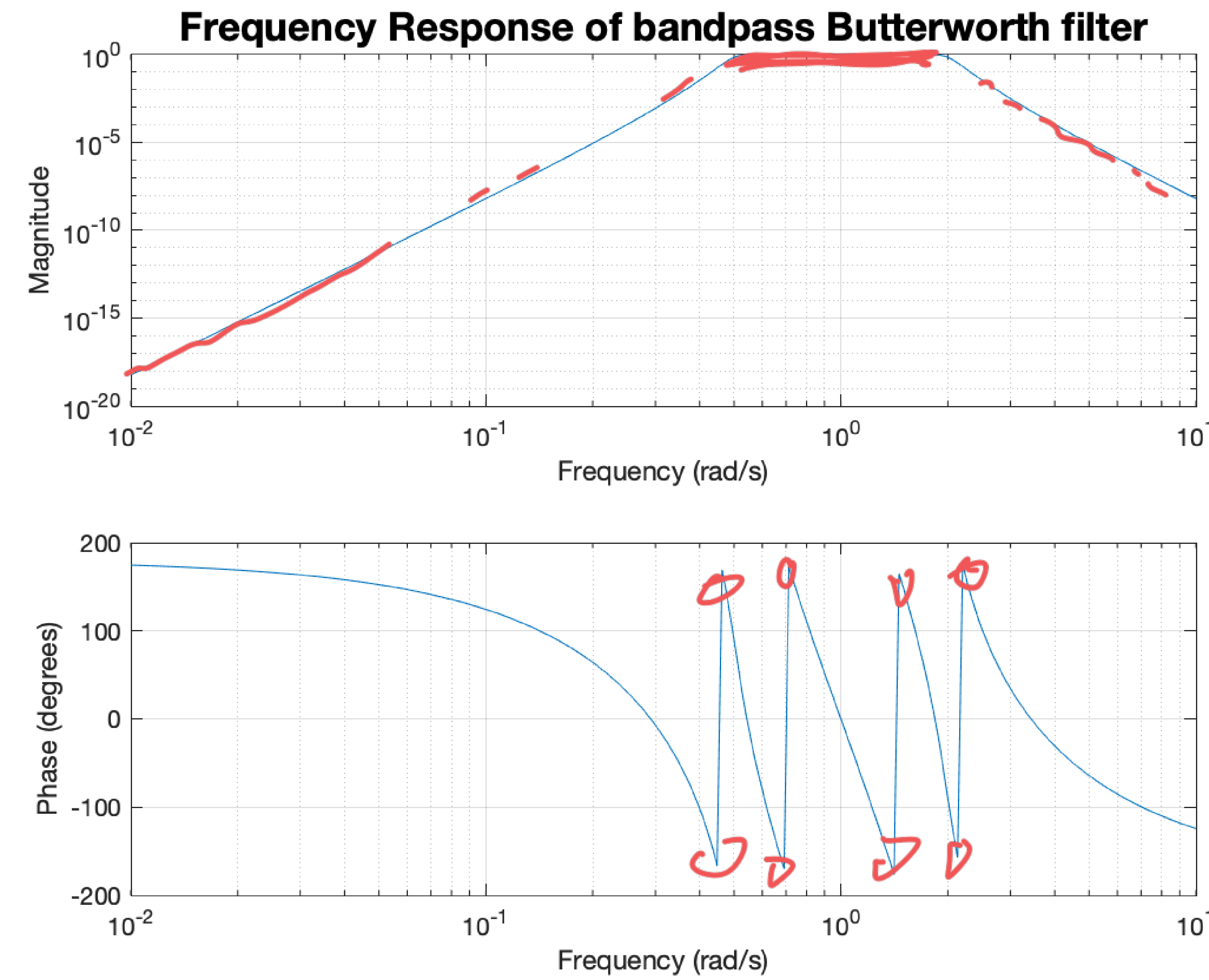


Code Example 2: 8th order highpass Butterworth filter

```
[B,A] = butter(8,2,'high','s');
Hhi = tf(B,A);
```

Example: bandpass filters

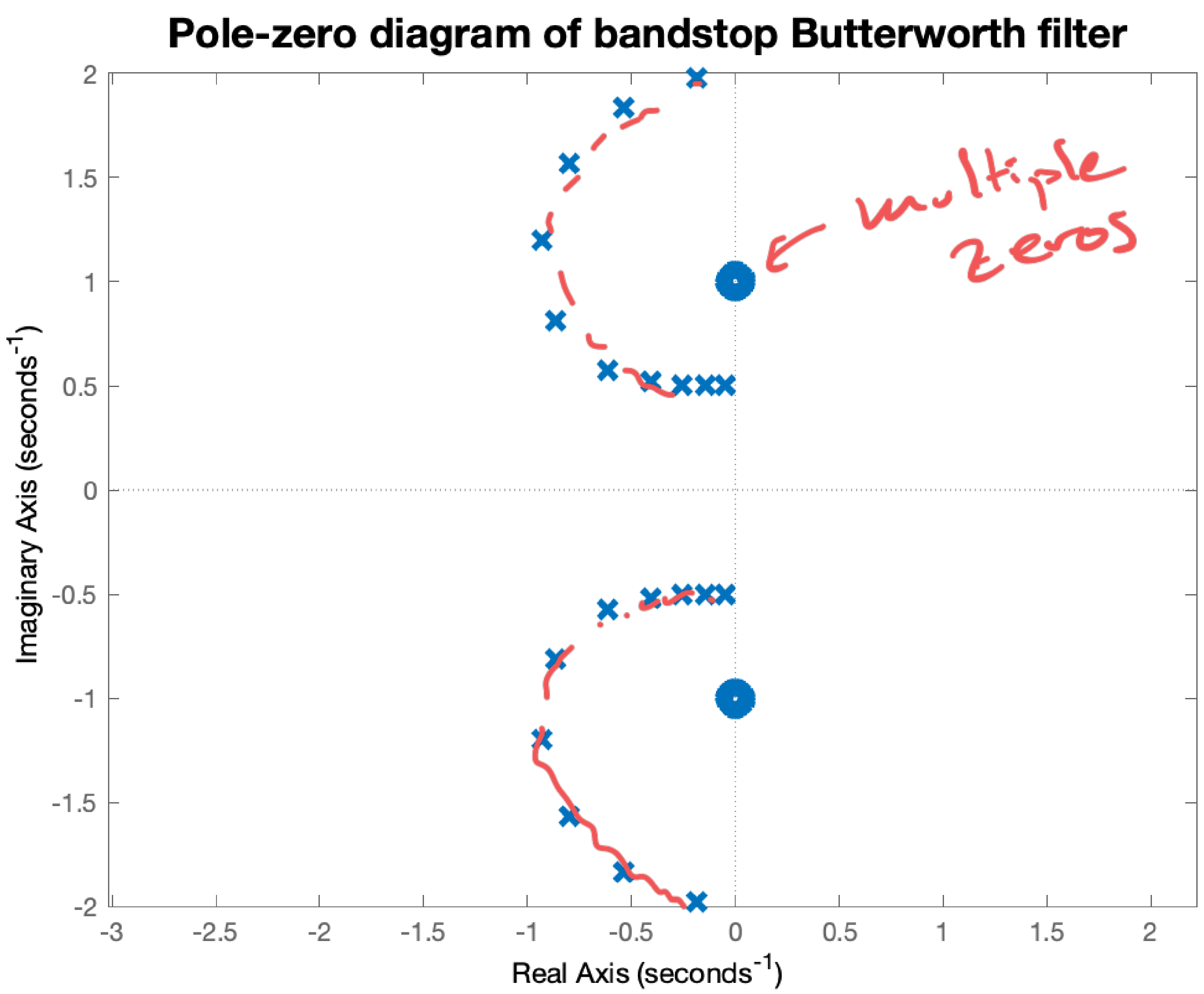


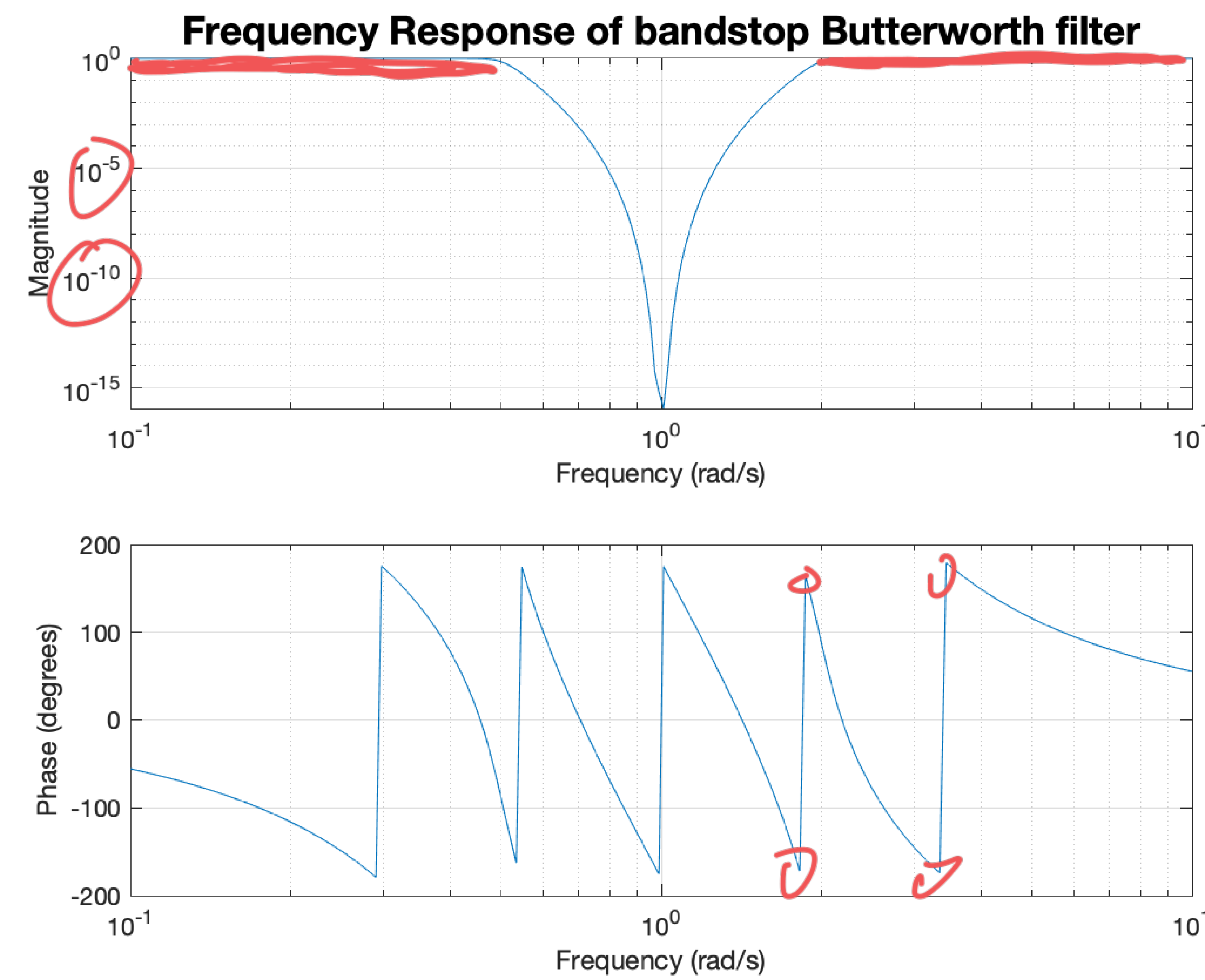


Code Example 3: 10th order bandpass Butterworth filter

```
[B,A] = butter(10,[0.5,2],'s');
Hband = tf(B,A);
```

Example: bandstop filters

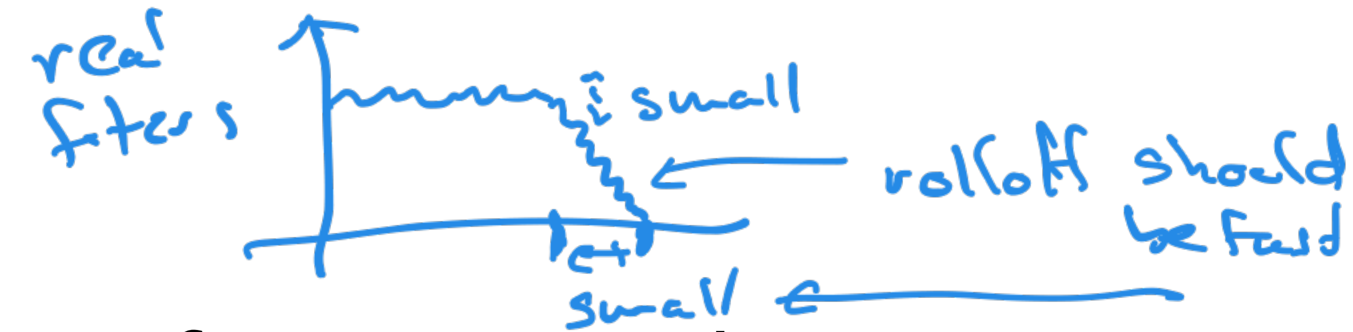




Code Example 4: 10th order bandstop Butterworth filter

```
[B,A] = butter(10,[0.5,2],'stop','s');
Hstop = tf(B,A);
```

Recap and looking forward



We've seen how to design Butterworth filters of various orders to get lowpass, highpass, bandpass, and bandstop frequency characteristics. There are a number of questions left to ask:

- Is this the best we can do? What would an "ideal" lowpass filter look like?
- What about non-Butterworth filters? Many filters are designed in DT for DT processing of CT signals.
- What are the tradeoffs in choosing filter order and other properties in terms of circuit complexity, energy dissipation, and other design constraints?