ECE 345: Linear Systems and Signals
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Fall 2020
Due: 5 PM, October 9, 2020

# Homework 3



**Problem 1.** Find the convolution of the following two DT sequences:

$$x[k] = \begin{cases} 2 & 0 \leq k \leq 2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$h[k] = \begin{cases} k+1 & 0 \leq k \leq 4 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

**Problem 2** (ECE 345 Fall 2017, Midterm 1)**.** For LTI systems the system is BIBO stable if its impulse response is *absolutely summable*:

$$\sum_{n=-\infty}^{\infty} |h_i[n]| < \infty \qquad \Longrightarrow \qquad \text{BIBO-stable} \tag{3}$$

Suppose we have a DT LTI system defined by the following input-output relation:

$$y[n] = x[n] + \frac{1}{3}x[n-1]. \tag{4}$$

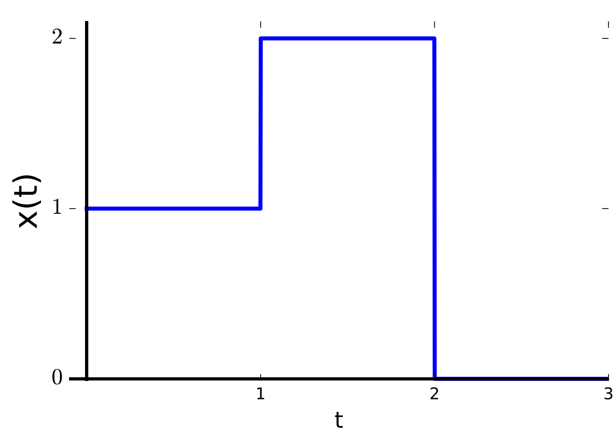
Calculate the *inverse system*. Is it causal? Is it stable? *Hint:* see if you can write $x[n]$ in terms of $y[n]$, $y[n-1]$, etc. and try to find a pattern.

**Problem 3** (*ECE 345 Midterm 1, Fall 2018*)**.** Suppose the impulse response of an LTI system is given by

$$h(t) = e^{-2t}(u(t-1) - u(t-3)). \tag{5}$$

(a) Sketch the impulse response of the signal. Be sure to label all important points.

(b) Suppose we put the following input signal $x(t)$ into the channel.



Write an algebraic expression for $x(t)$ in terms of unit step functions.

(c) For the $x(t)$ in the previous part, compute the output of the LTI system $h(t)$ with input $x(t)$. *Hint:* use linearity.

(d) Suppose we instead apply the input $w(t) = e^{-t}$ into this system $h(t)$. Calculate the output of the system.

**Problem 4.** Suppose a CT LTI system has impulse response

$$h(t) = (3e^{-2t} - 2e^{-4t})u(t) \tag{6}$$

Compute the output of the system with the following inputs:

1. $x(t) = e^{-3t}u(t)$
2. $x(t) = 2e^{-2t}u(t)$

**Problem 5** (SSTA 2.15)**.** It's important to be able to compute convolutions without having to resort to the definition each time. The key is to use convolutions which you have computed before. Here are some to practice on. Remember: draw a picture!

(a) $u(t) * [2u(t) - 2u(t-3)]$

(b) $u(t) * [(t-1)u(t-1)]$

(c) $[\delta(t) + 2\delta(t-1) + 3\delta(t-2)] * [4\delta(t) + 5\delta(t-1)]$

**Problem 6.** Consider the signal

$$x(t) = e^{-5t}u(t)e^{-\beta t}u(t), \tag{7}$$

and denote its Laplace transform by $X(s)$. What are the constraints placed on the real and imaginary parts of $\beta$ if the region of convergence of $X(s)$ is $\mathfrak{Re}(s) > -3$?

**Problem 7** (SSTA 3.8)**.** Determine the Laplace transform of the following functions:

(a) $x_1(t) = \frac{d}{dt}\left(4te^{-2t}\cos(4\pi t + \pi/6)u(t)\right)$

(b) $x_2(t) = e^{-3t}\cos(4t + \pi/6)u(t)$

(c) $x_3(t) = t^2(u(t) - u(t-4))$

(d) $x_4(t) = 10\cos(6\pi t + \pi/6)\delta(t - 0.2)$

**Problem 8** (ECE 345 Fall 2019 Midterm 2)**.** Suppose a causal CT LTI system has bilateral Laplace transform

$$H(s) = \frac{2s - 2}{s^2 + (10/3)s + 1}. \tag{8}$$

(a) Write the linear constant coefficient differential equation (LCCDE) relating a general input $x(t)$ to its corresponding output $y(t)$ of the system corresponding to this transfer function in equation (8).

(b) Plot the pole-zero diagram and indicate the region of convergence for this system. Is this system stable? Explain your answer.

(c) Suppose the input $x(t) = e^{-t}u(t)$. Find the output $y(t)$.