Linear Systems and Signals

Graphical convolution for CT signals

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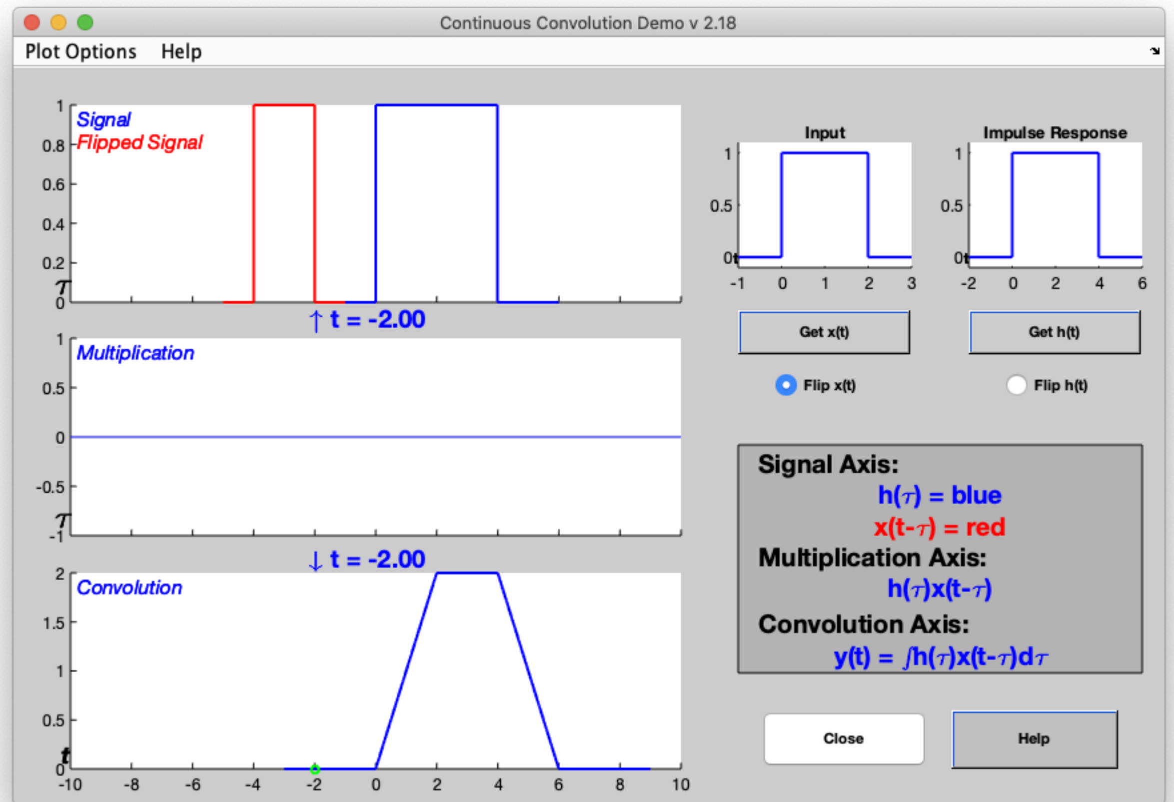
Learning objectives

The learning objectives for this section are:

- describe LTI systems in terms of convolution with the impulse response
- manually compute convolutions in the time domain
- use LTI properties to simplify calculations
- use standard formulas to simplify output calculations

Graphical approach to convolution



The graphical approach to convolution problems interprets the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \qquad (1)$$

as "flipping" or reversing $x(\tau)$ and "sliding" it along by delays of t. The flip makes $x(\tau)$ enter the system $h(\tau)$ starting with earlier times t.

MATLAB demo

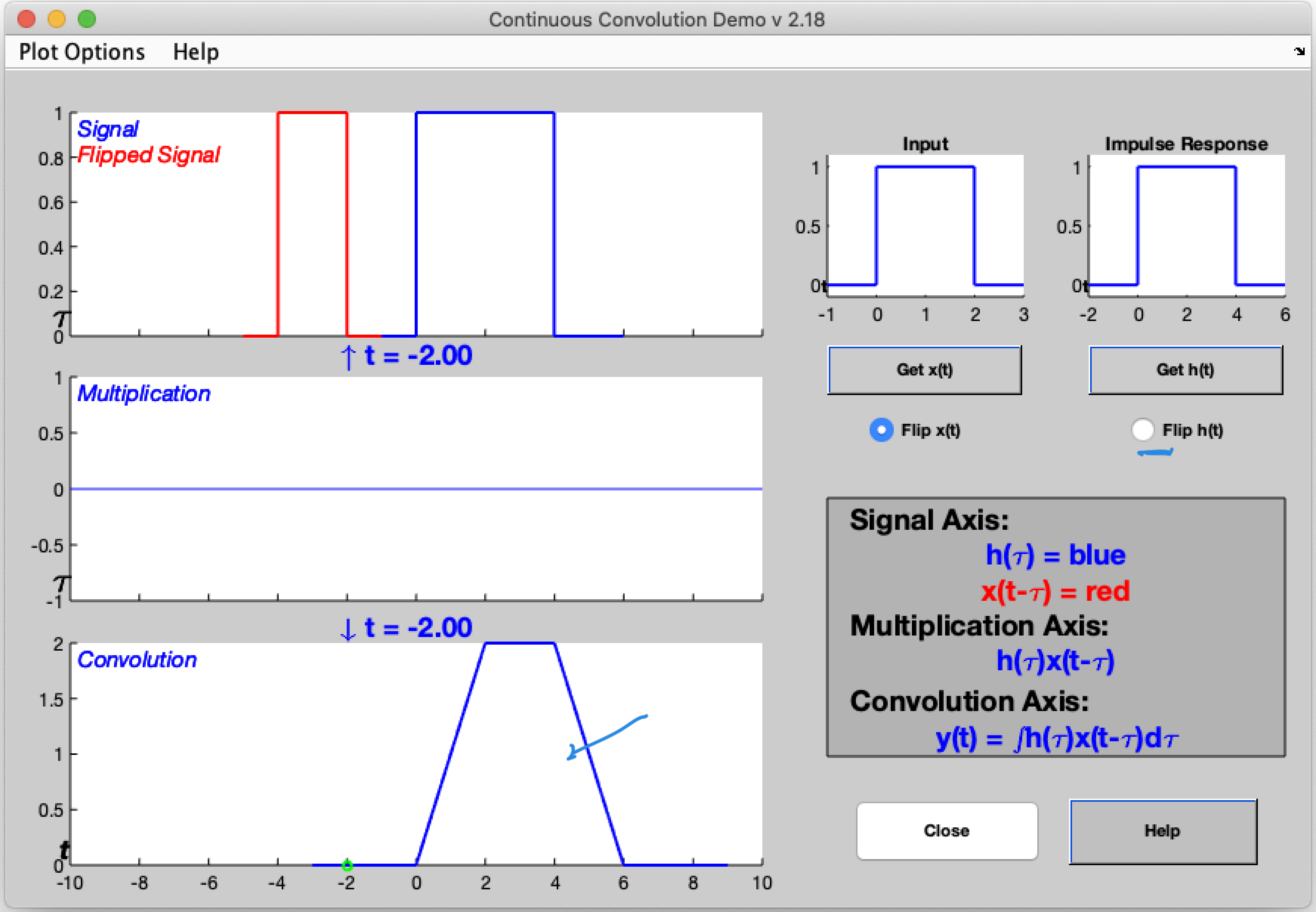
Check out the following demo from Georgia Tech:

`http://dspfirst.gatech.edu/matlab/cconvdemo`

We're going to use it to convolve

$$(u(t) - u(t-4)) * (u(t) - u(t-2)) \tag{2}$$

Convolving two boxes



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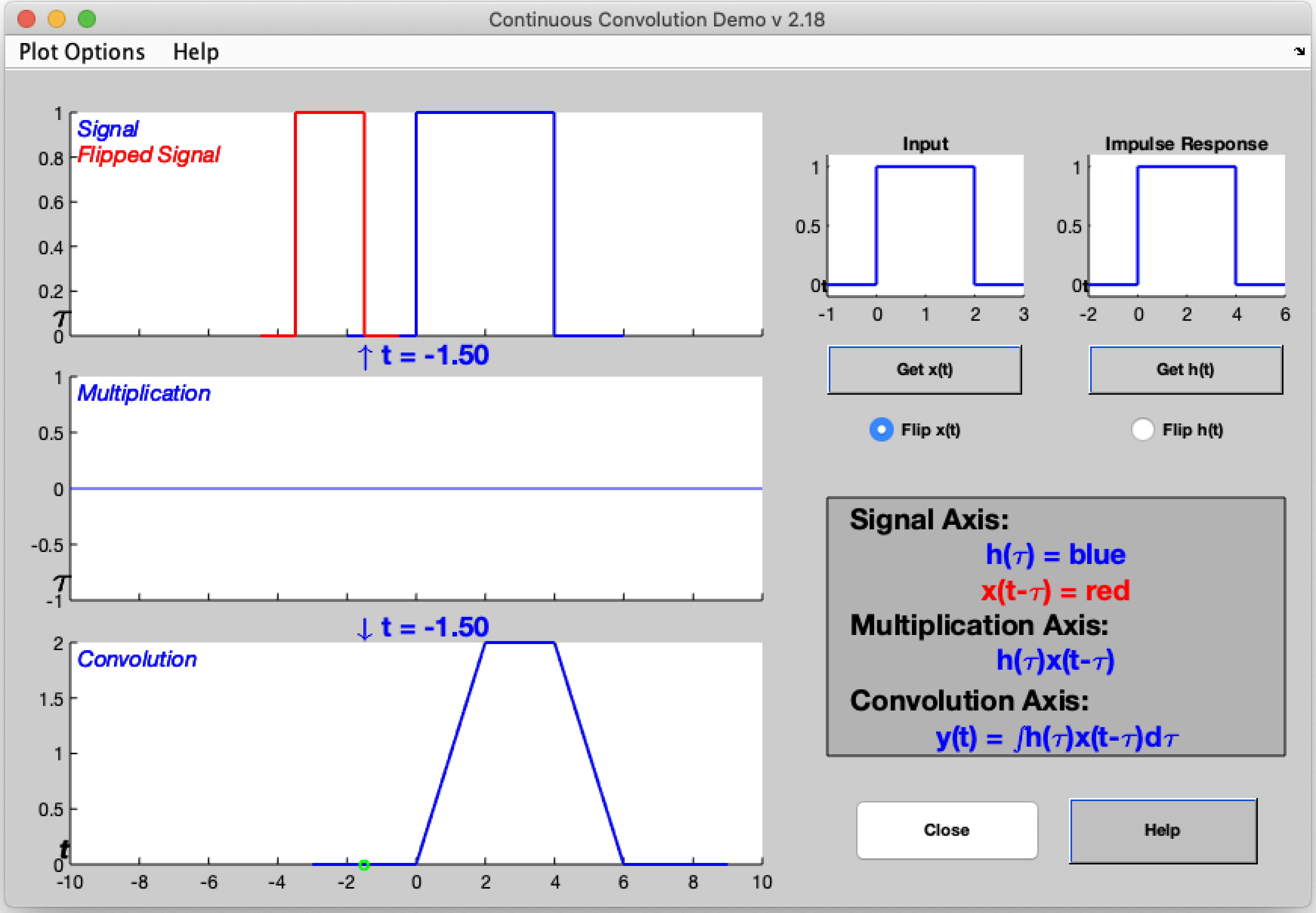

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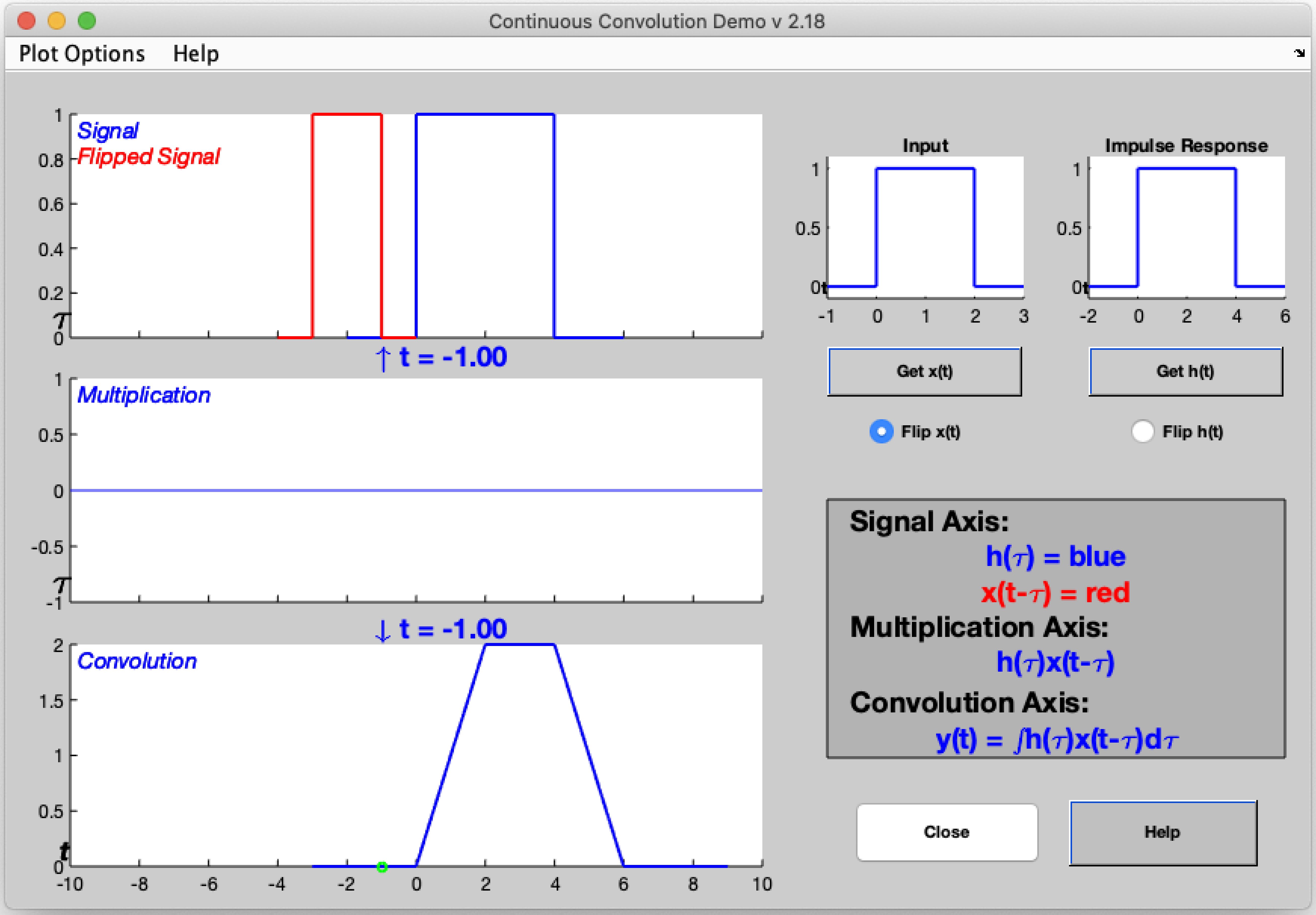

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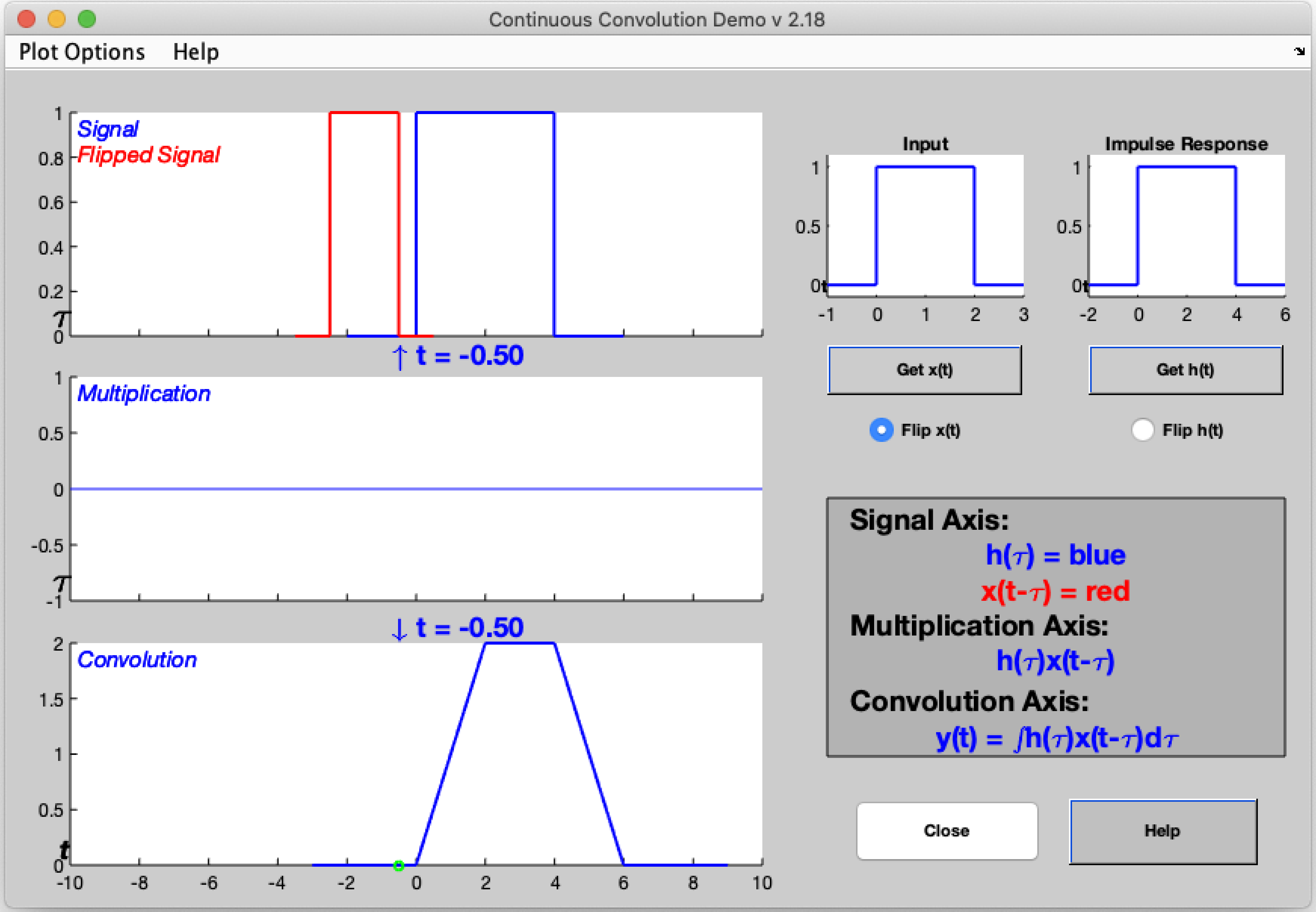

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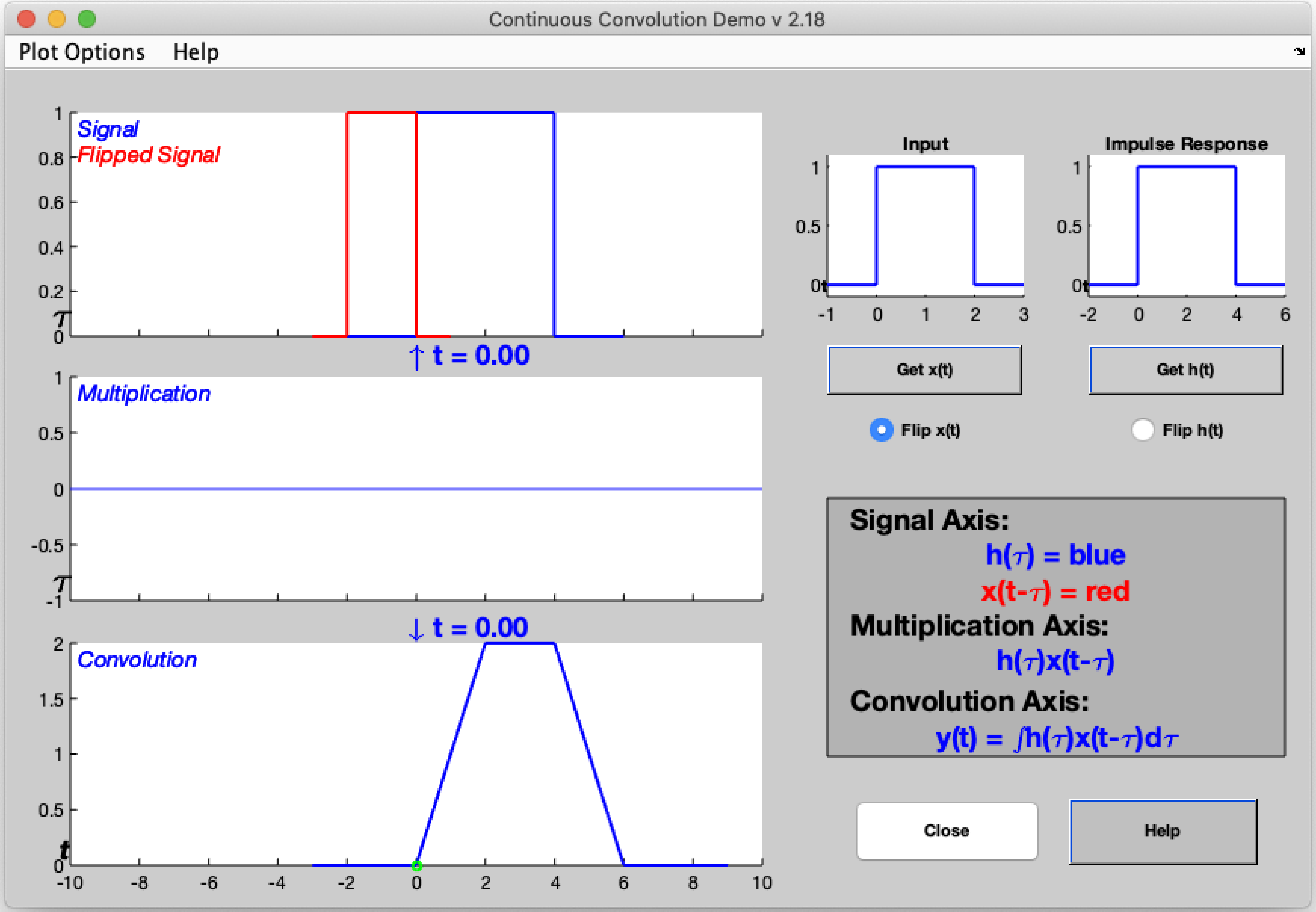

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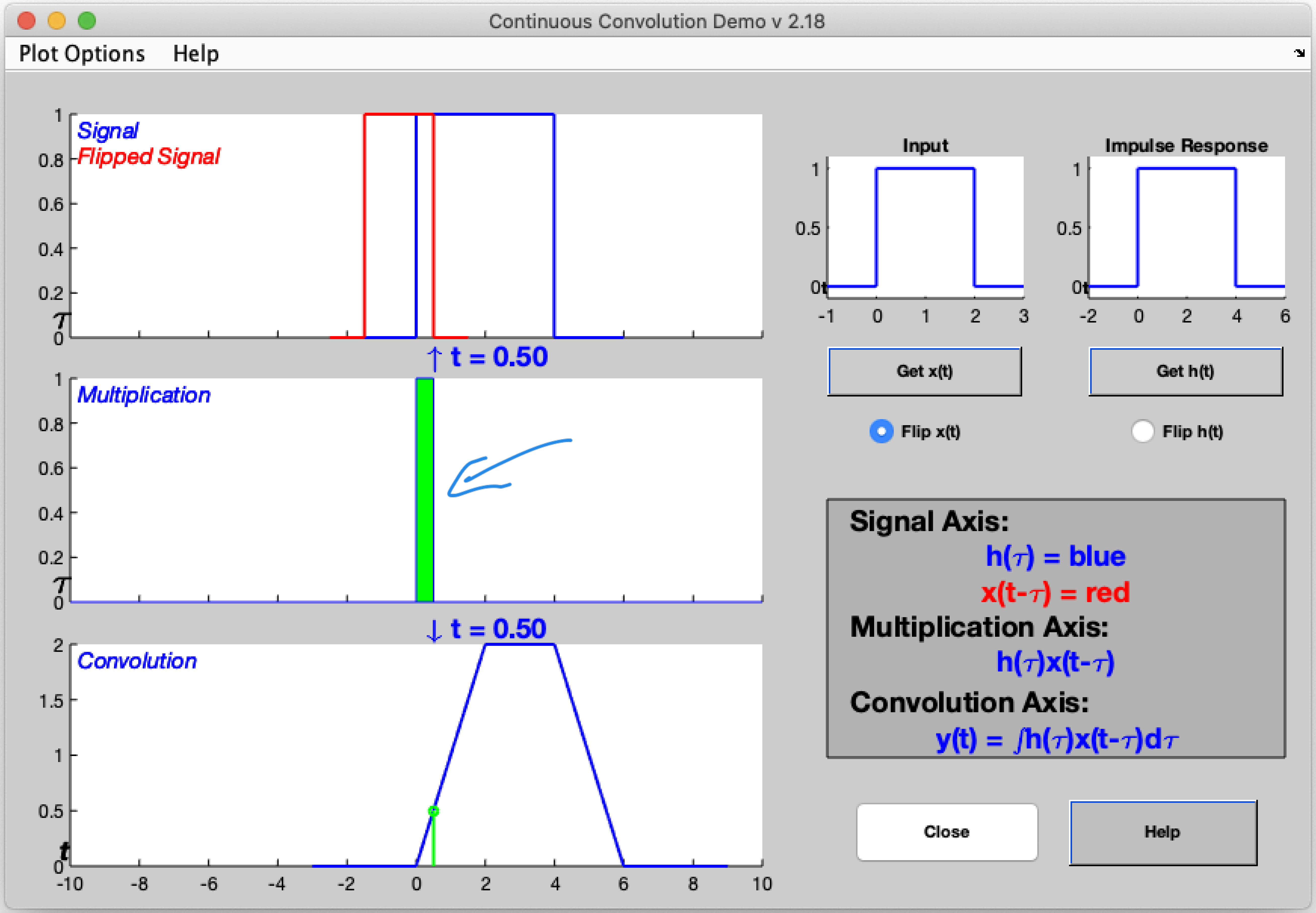

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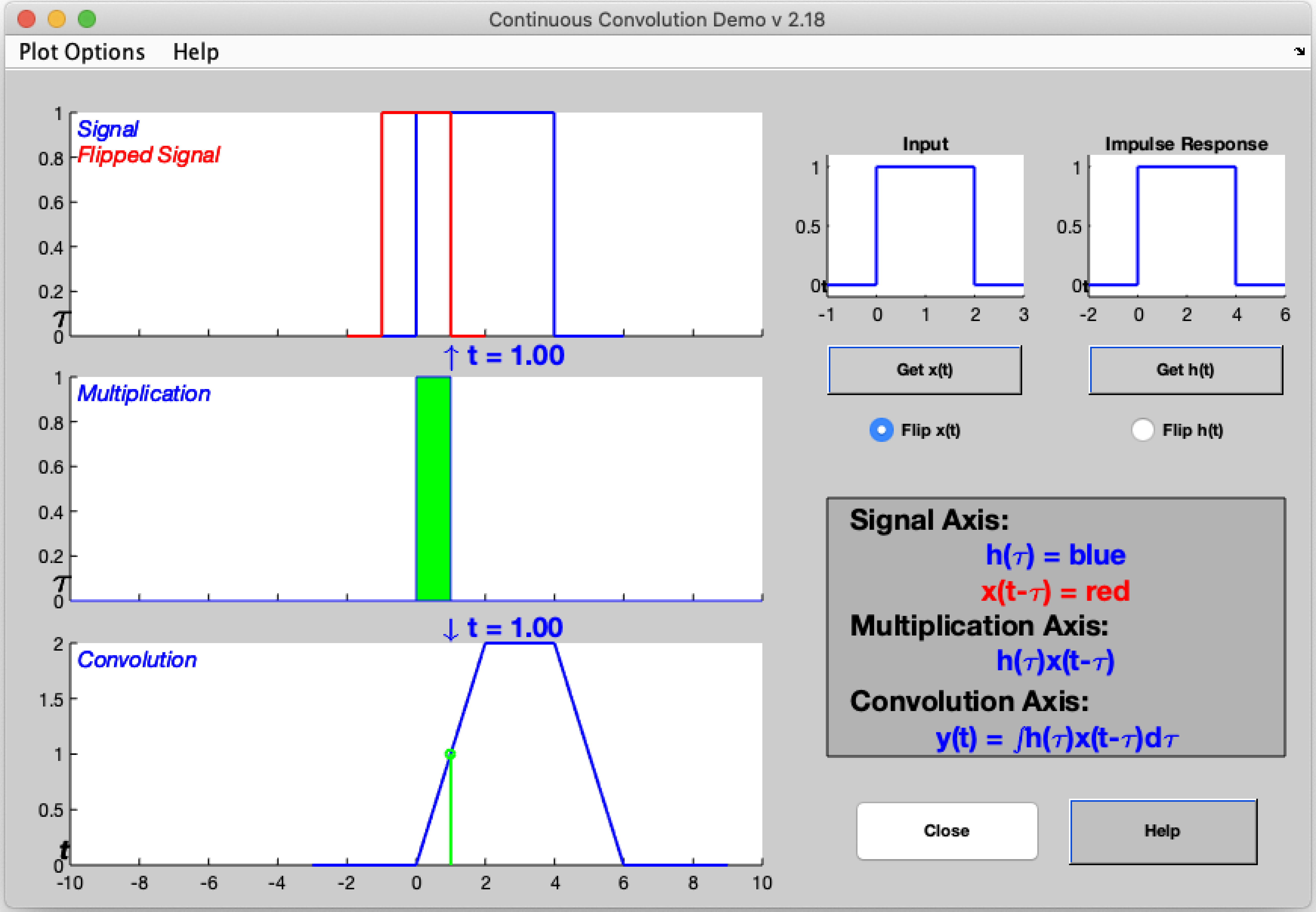

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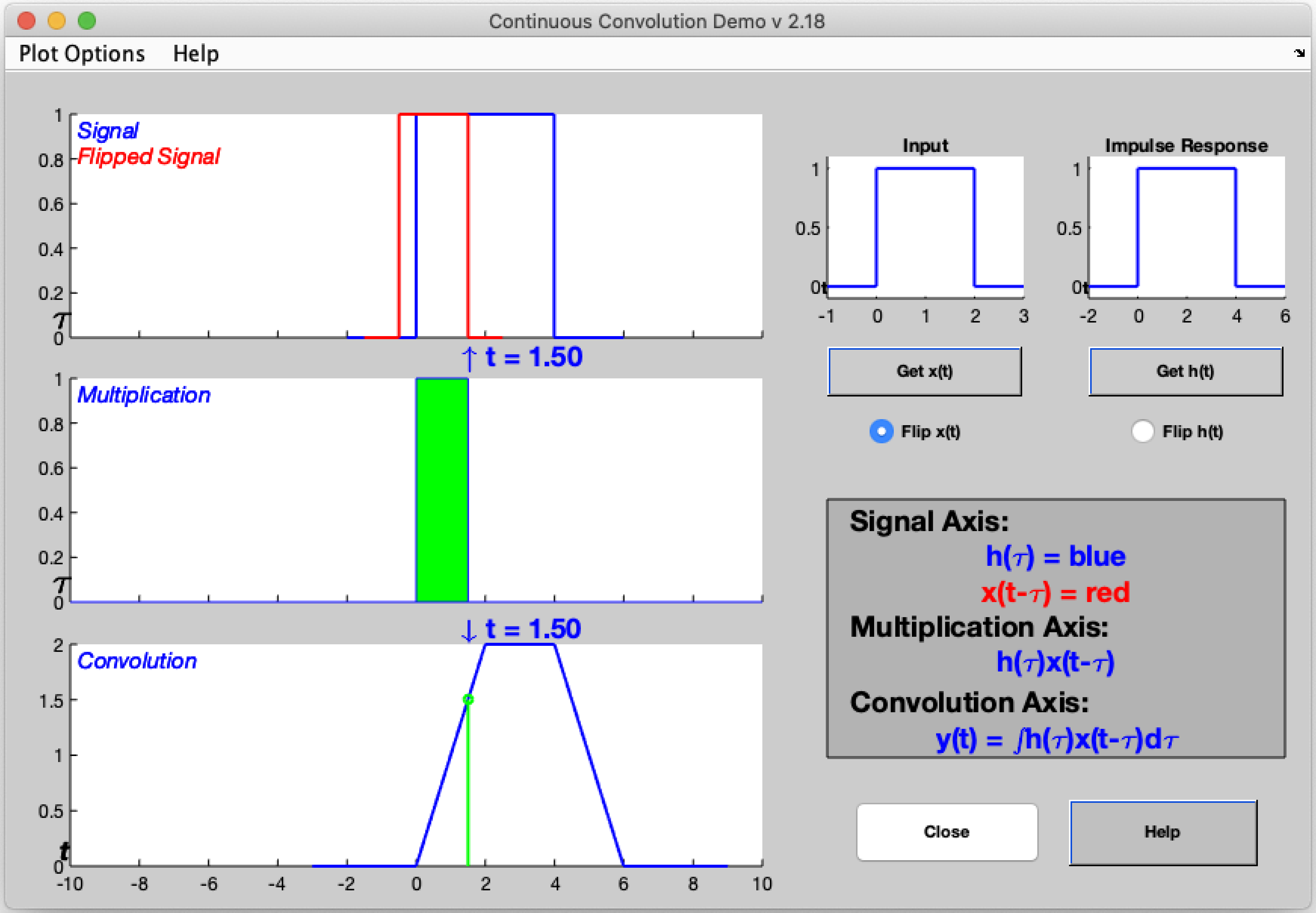

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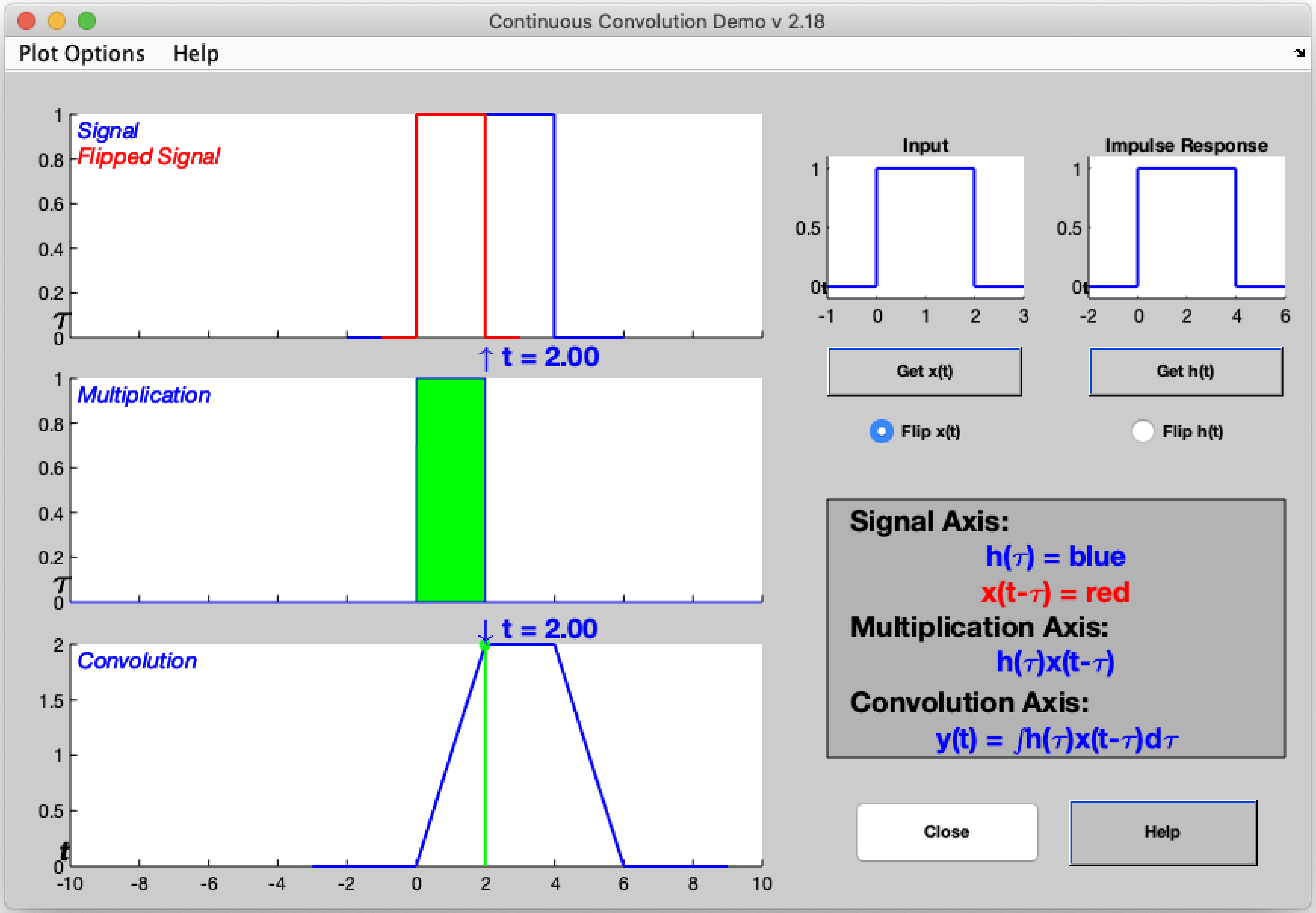

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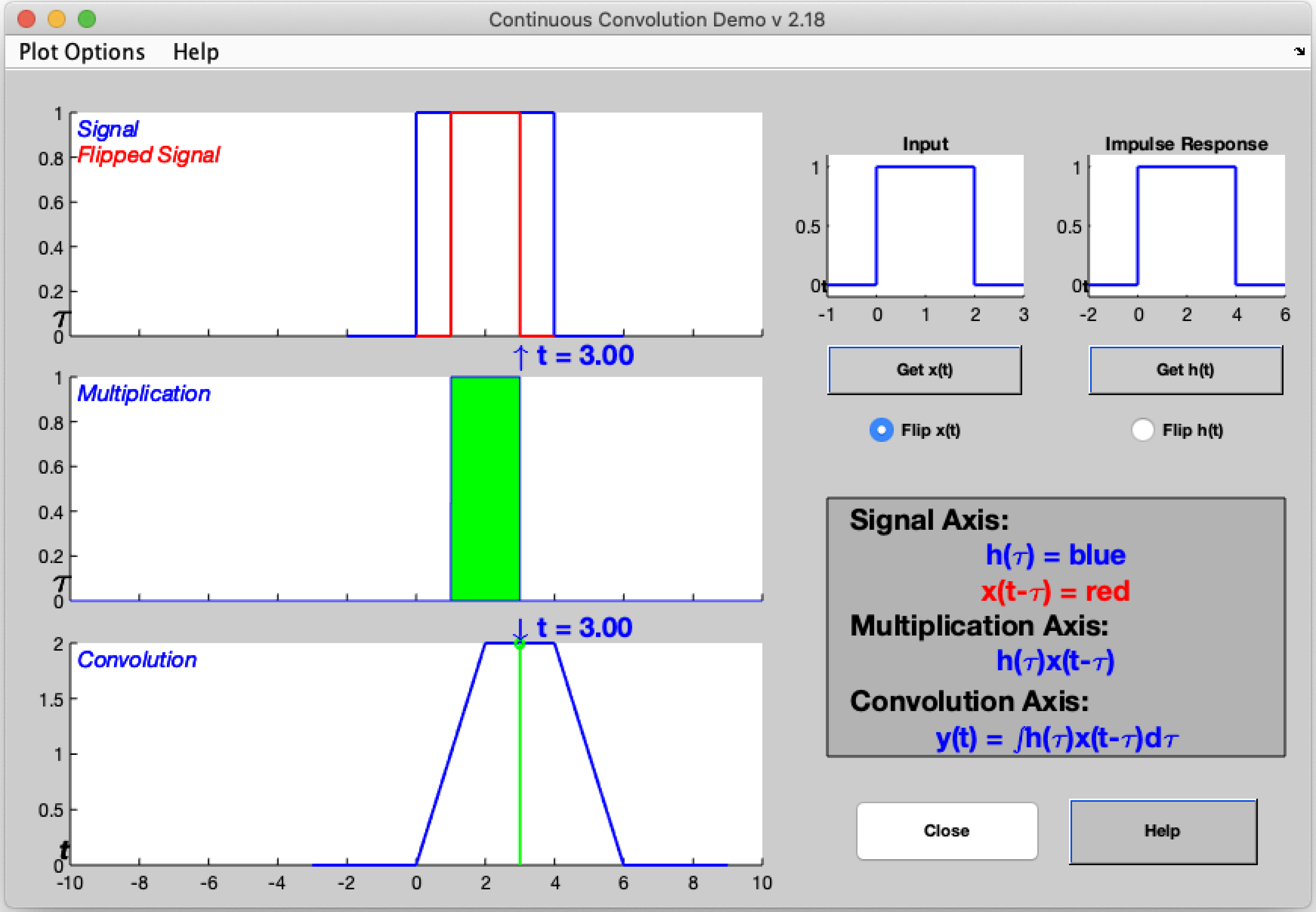

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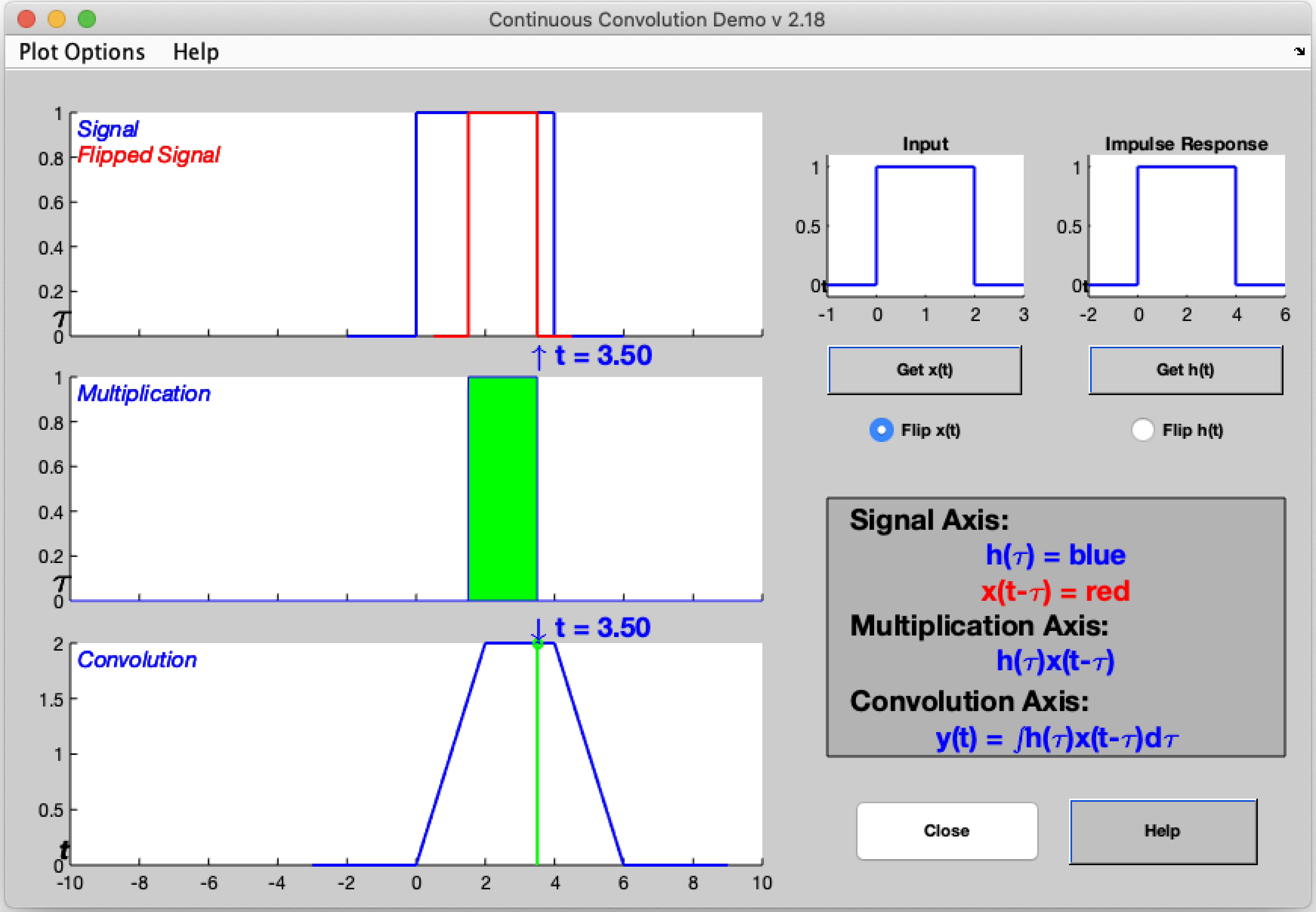

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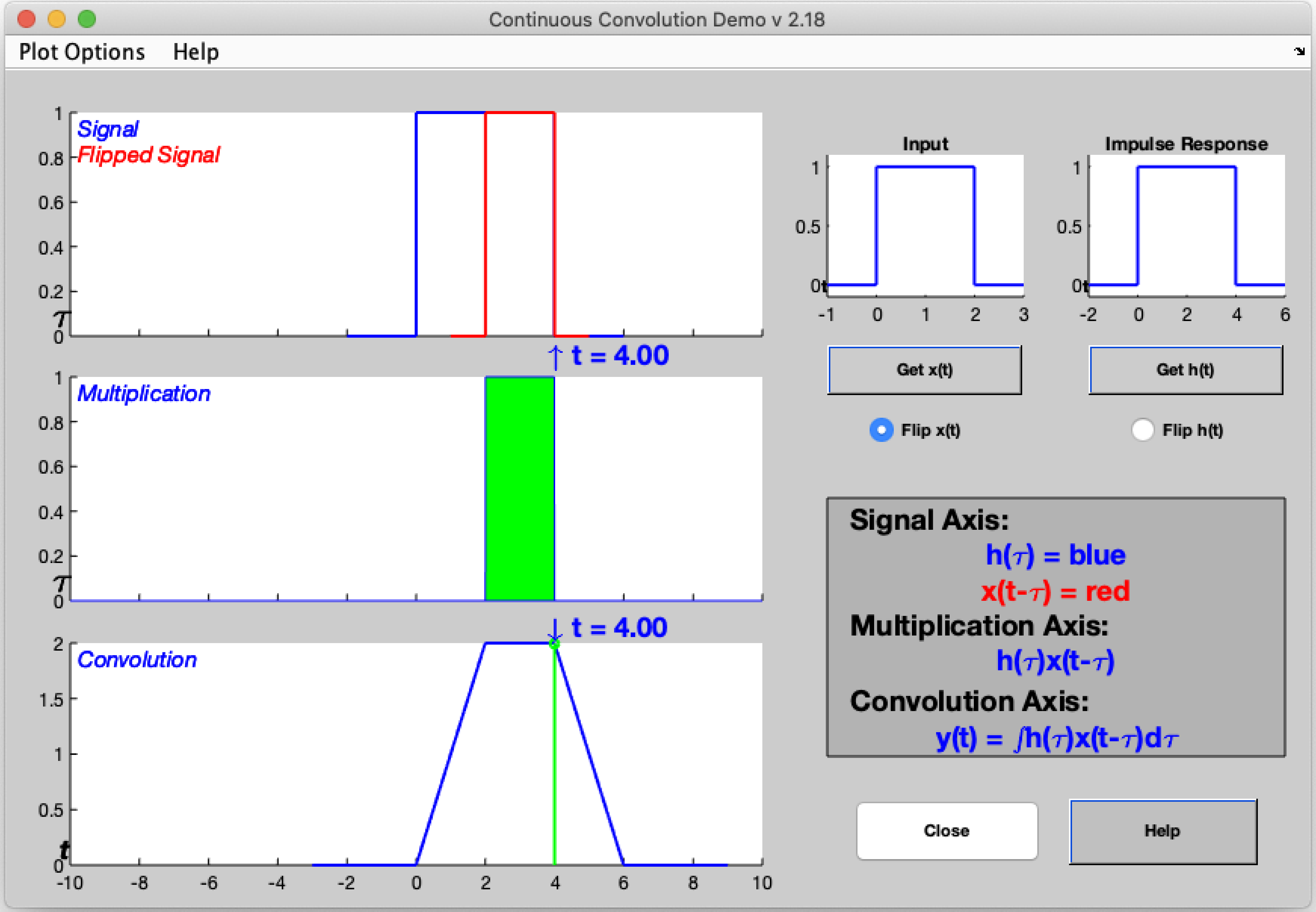

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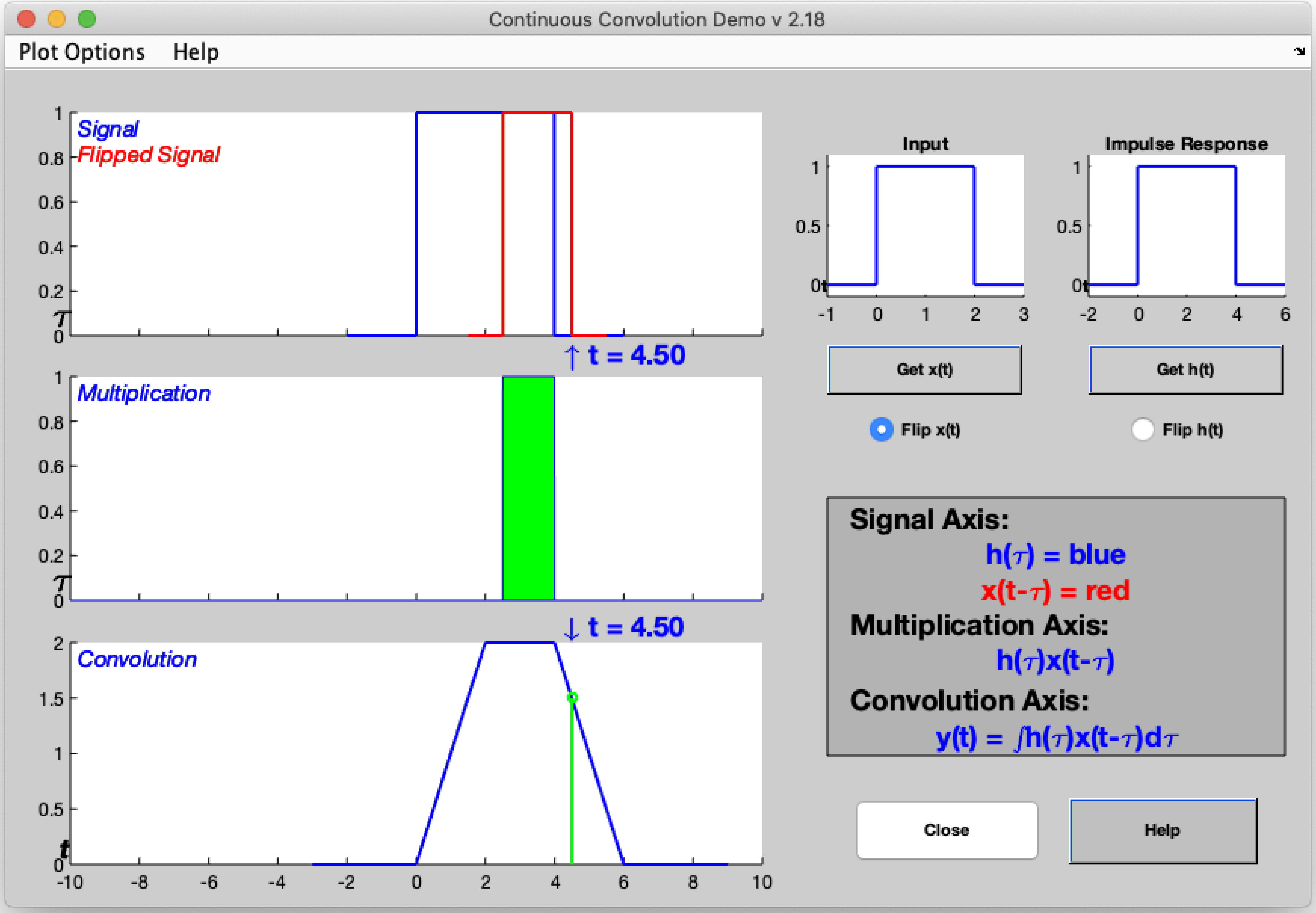

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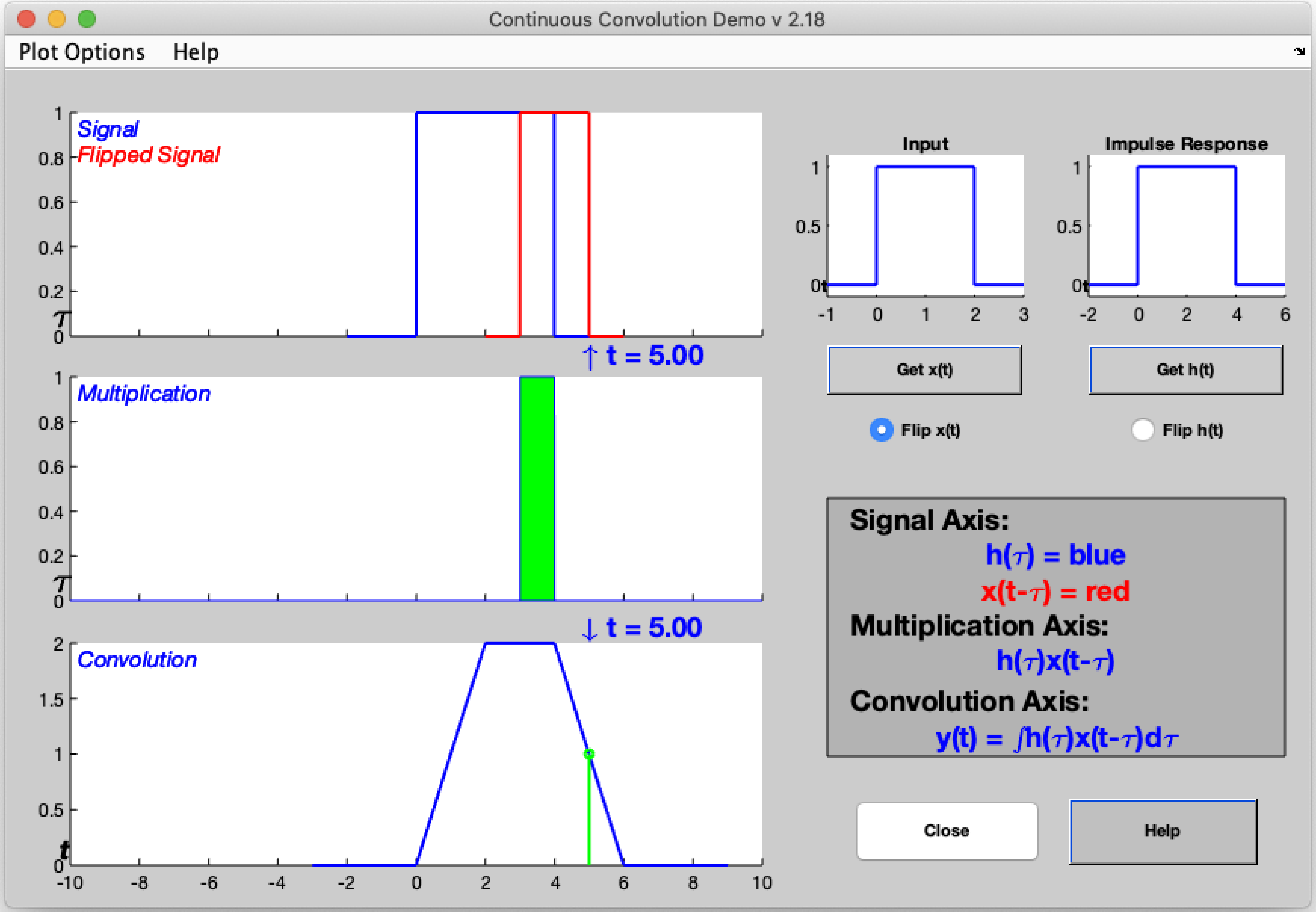

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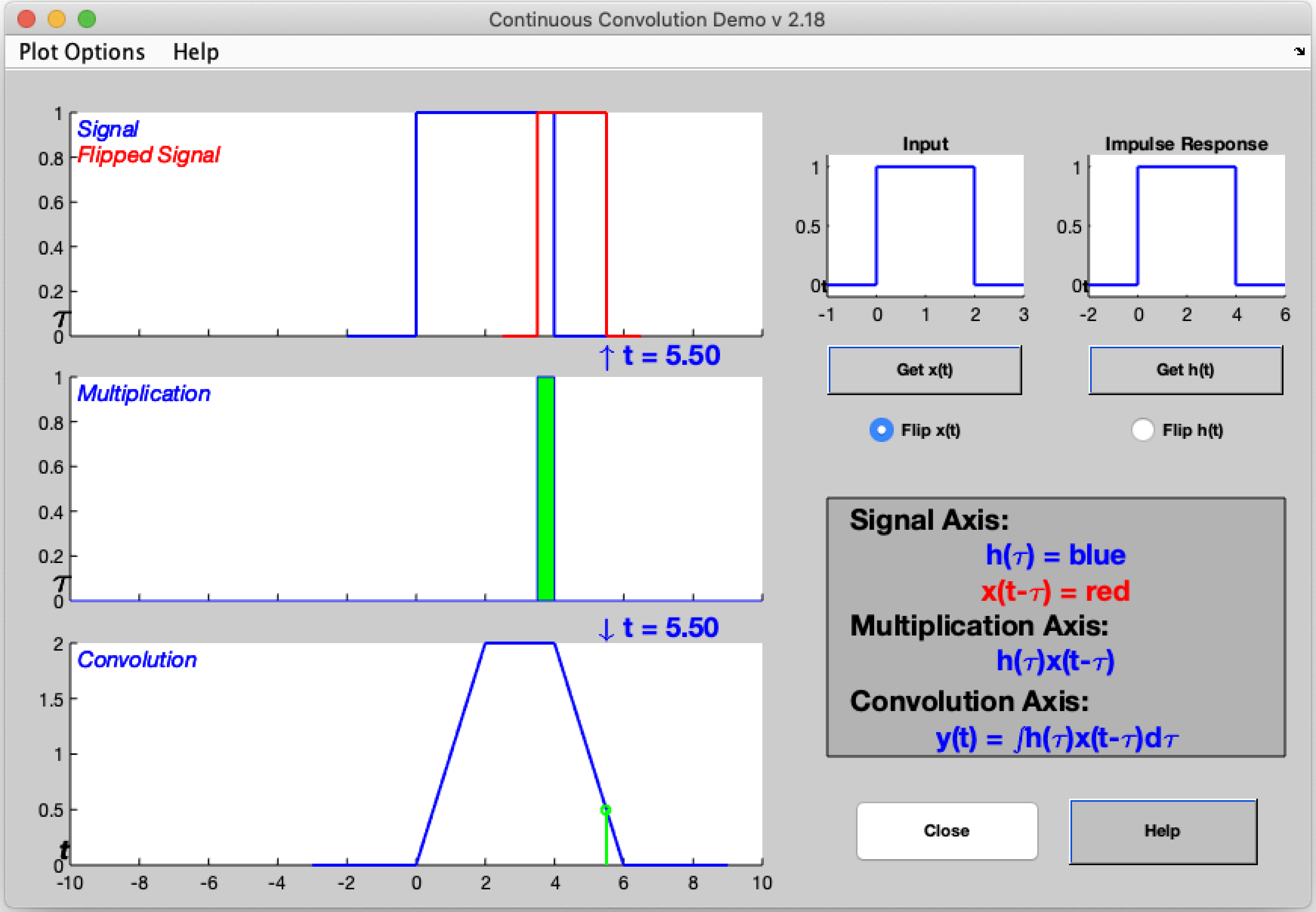

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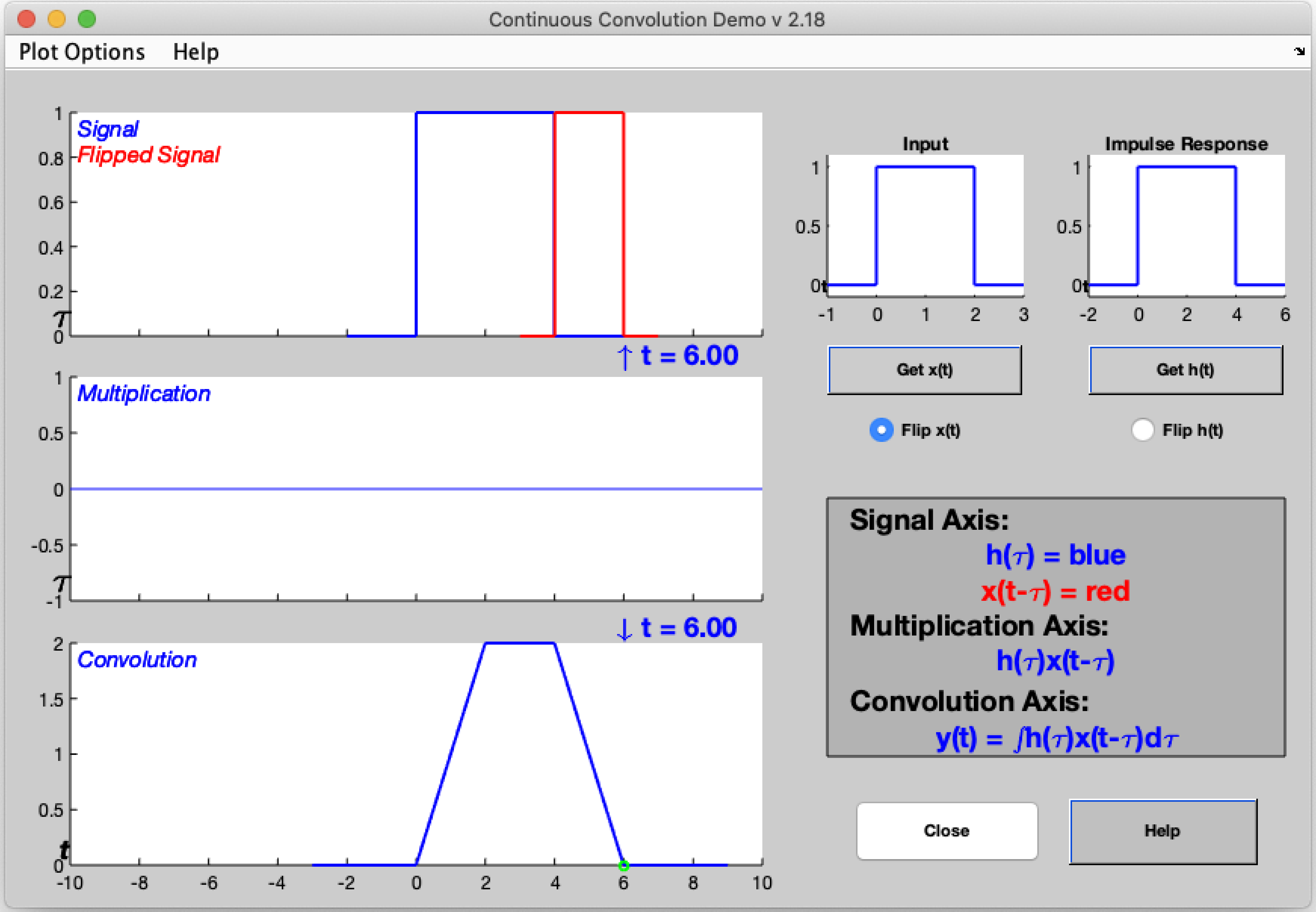
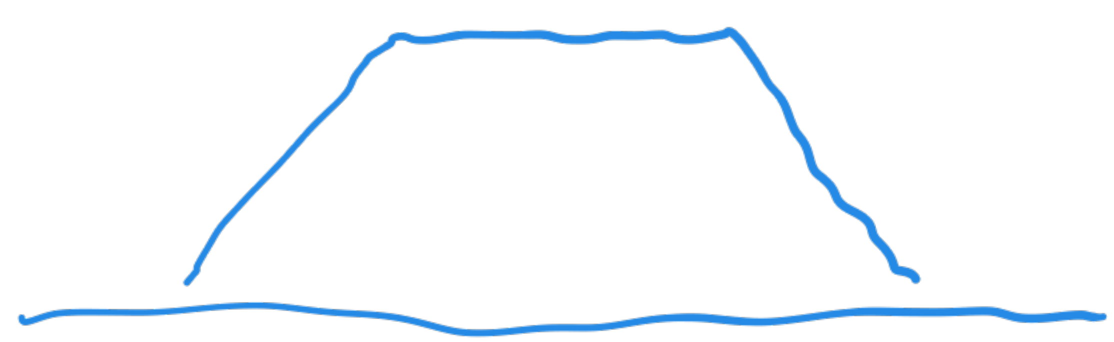


In general

In general, convolving two boxes will give you a trapezoid. Why?

- Phase 1: $x(t-\tau)$ and $h(\tau)$ don't overlap, so $y(t) = 0$.
- Phase 2: $x(t-\tau)$ and $h(\tau)$ partly overlap, so $y(t)$ increases linearly with t.
- Phase 3: $x(t-\tau)$ and $h(\tau)$ totally overlap, so $y(t)$ is constant.
- Phase 4: $x(t-\tau)$ and $h(\tau)$ partially overlap, so $y(t)$ decreases linearly with t
- Phase 5: $x(t-\tau)$ and $h(\tau)$ don't overlap, so $y(t) = 0$.



The 5-phase method

This "5 phase" method works for convolving two signals with finite lengths. If the signals are complicated, the calculations can start getting messier sometimes, so you have to see if graphical or algebraic is the right way to go. Because convolution is commutative, we also have

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad (3)$$

So we could instead flip $h(\tau)$ and slide it over $x(\tau)$. One approach may be easier to think about than ther other. The picture of flip-and-slide is very useful because it lets you "visualize" what is happening.

Try it yourself

Apply LTI system properties

Problem

Using the convolution of two boxes as a template, convolve the following signals:

$$x(t) = (u(t-4) - u(t-7)) * (u(t) - u(t-2)) \tag{4}$$

$$x(t) = (u(t+1) - u(t-6)) * (u(t-2) - u(t-12)) \tag{5}$$

$$x(t) = \mathrm{rect}(3t-6) * (u(t+1) - u(t-1)) \tag{6}$$