Linear Systems and Signals

Finding the impulse response

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Learning objectives

The learning objective for this section is:

- find the impulse response of a DT system from an input-output formula

Impulse response and convolution

We have the convolution formula:

a system w/ impulse response

$x*h$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \quad (1)$$

$\delta[n-k]$ is a system that delays its input by k.

$h*x$

$$= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \quad (2)$$

How do we use a given input/output formula to write down $h[n]$?

Easy case: if $y[n]$ is already a linear combination of $x[n-k]$'s...

$$y[n] = 3x[n] - 2x[n-1] - x[n-3] \quad (3)$$

$$h[n] = 3\delta[n] - 2\delta[n-1] - \delta[n-3] \quad (4)$$

replace x with δ

A general approach

Goal: try to rewrite $y[n]$ as a linear combination of delayed versions of $x[n]$. Then $h[k]$ is the coefficient of $x[n-k]$. Let's try an example with *recursion*:

$\alpha\, y[n-1] = \alpha^2 y[n-2] + \alpha x[n-1]$

substitute into

$y[n-2] = \alpha y[n-3] + x[n-2]$

$$
\begin{aligned}
y[n] - \alpha y[n-1] &= x[n] && (5)\\
y[n] &= \alpha y[n-1] + x[n] && (6)\\
&= \alpha^2 y[n-2] + \alpha x[n-1] + x[n] && (7)\\
&= \alpha^3 y[n-3] + \alpha^2 x[n-2] + \alpha x[n-1] + x[n] && (8)\\
&= \sum_{k=0}^{\infty} \alpha^k x[n-k] && (9)
\end{aligned}
$$

$h[k] = \alpha^k$ for $k \geq 0$

So $h[n] = \alpha^n$ for $n \geq 0$ and 0 otherwise. Another way to write this is $h[n] = \alpha^n u[n]$.

Try some yourself

Problem

Find the impulse response from the following input-output relations:

$$y[n] + \frac{1}{2}y[n-1] \cancel{- \frac{1}{4}y[n-2]} = x[n]$$

$$y[n] = 4x[n+1] - 2x[n-2] + 4x[n-4]$$

$$y[n] = \frac{1}{3}y[n-2] + x[n-1]$$

$$y[n] = \sum_{k=0}^{\infty} x[n-k]$$