Linear Systems and Signals
Introduction to LTI systems

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Learning objectives

The learning objectives for this section is:

- describe how linearity and time invariance go together

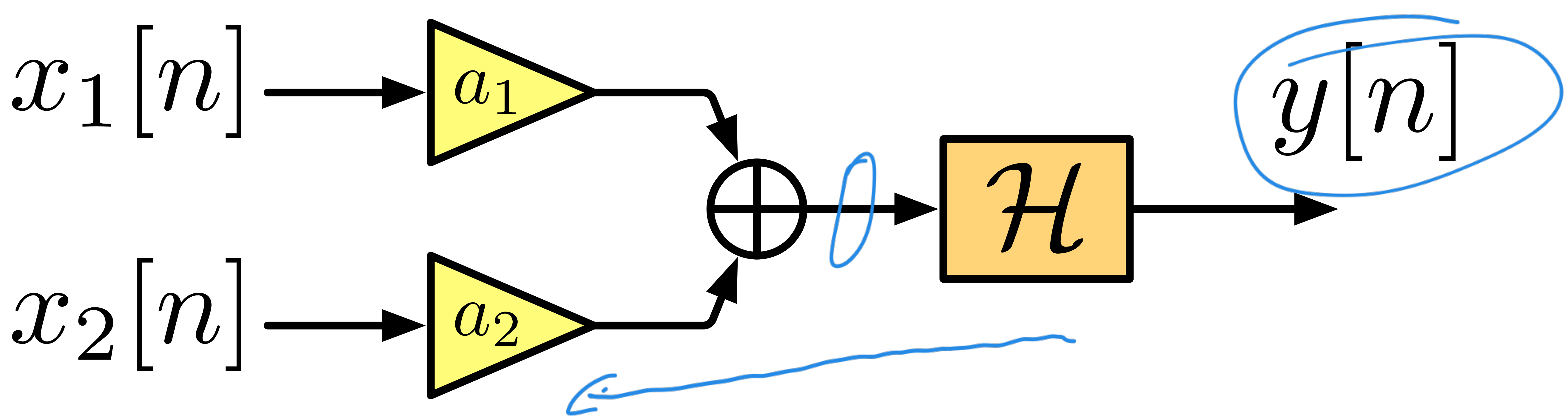
Linearity and time-invariance: two great flavors

The combination of linearity and time-invariance turns out to be really useful in modeling and designing systems:

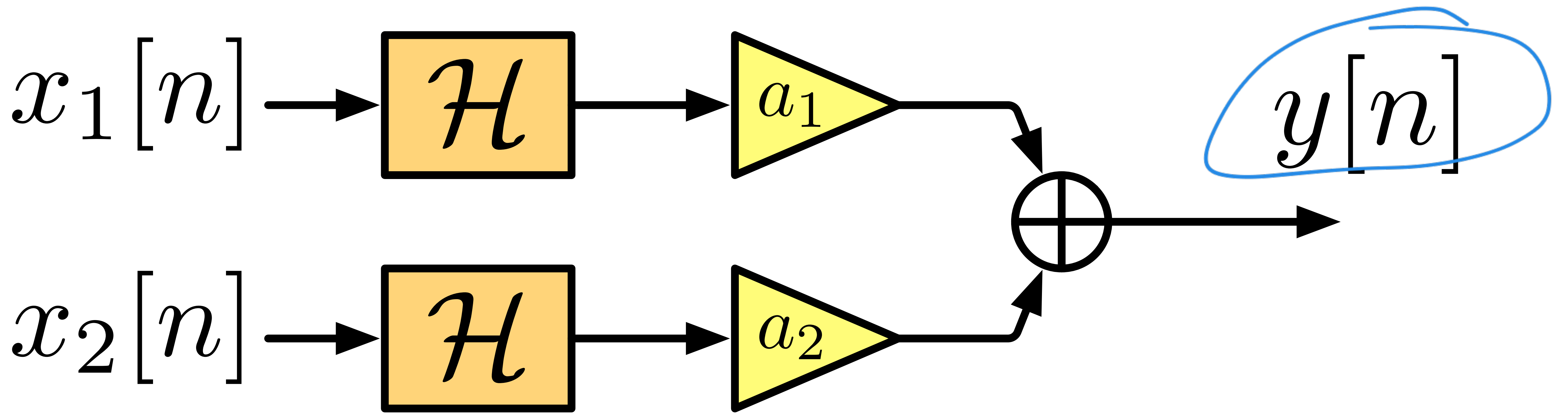
- linearity means we can interchange linear combinations and applying the system
- time-invariance means we can interchange delays and the system

For most of the rest of this class, we will be studying *linear time-invariant (LTI) systems*. We will use the term *filter* to describe LTI systems.

In pictures

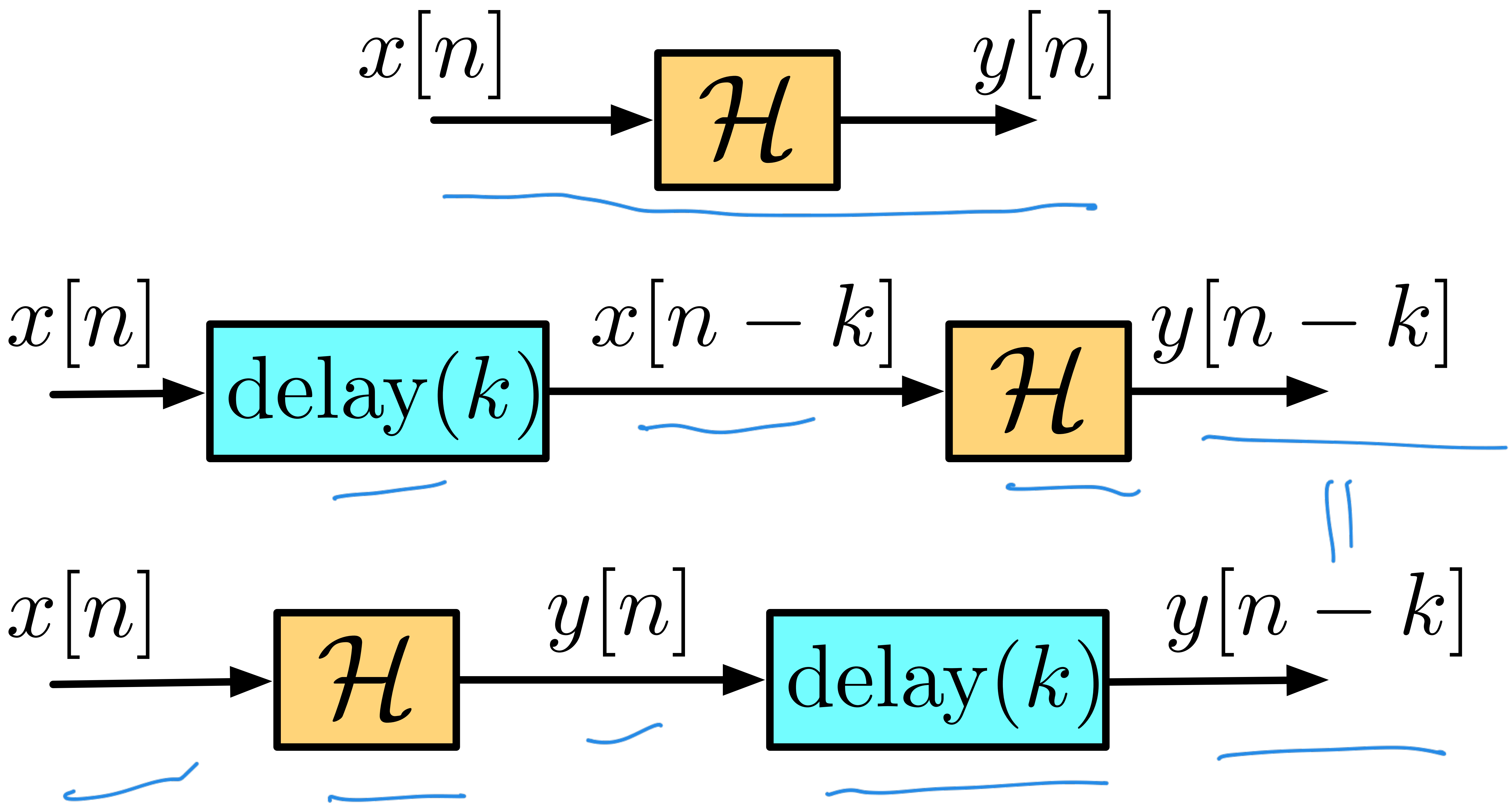


$$\mathcal{H}(a_1 x_1[n] + a_2 x_2[n]) = a_1 \mathcal{H}(x_1[n]) + a_2 \mathcal{H}(x_2[n])$$



In pictures

$$\boxed{x[n] \xrightarrow{\mathcal{H}} y[n] \Longrightarrow x[n-k] \xrightarrow{\mathcal{H}} y[n-k]}$$



Representing DT signals

The key ingredient is the *representation of DT signals in terms of impulses*. If we have a DT signal

$$\ldots, x[-2], x[-1], x[0], x[1], x[2], \ldots \tag{1}$$

we can also write it as

δ@ n=-2 δ@ n=-1

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] \tag{2}$$

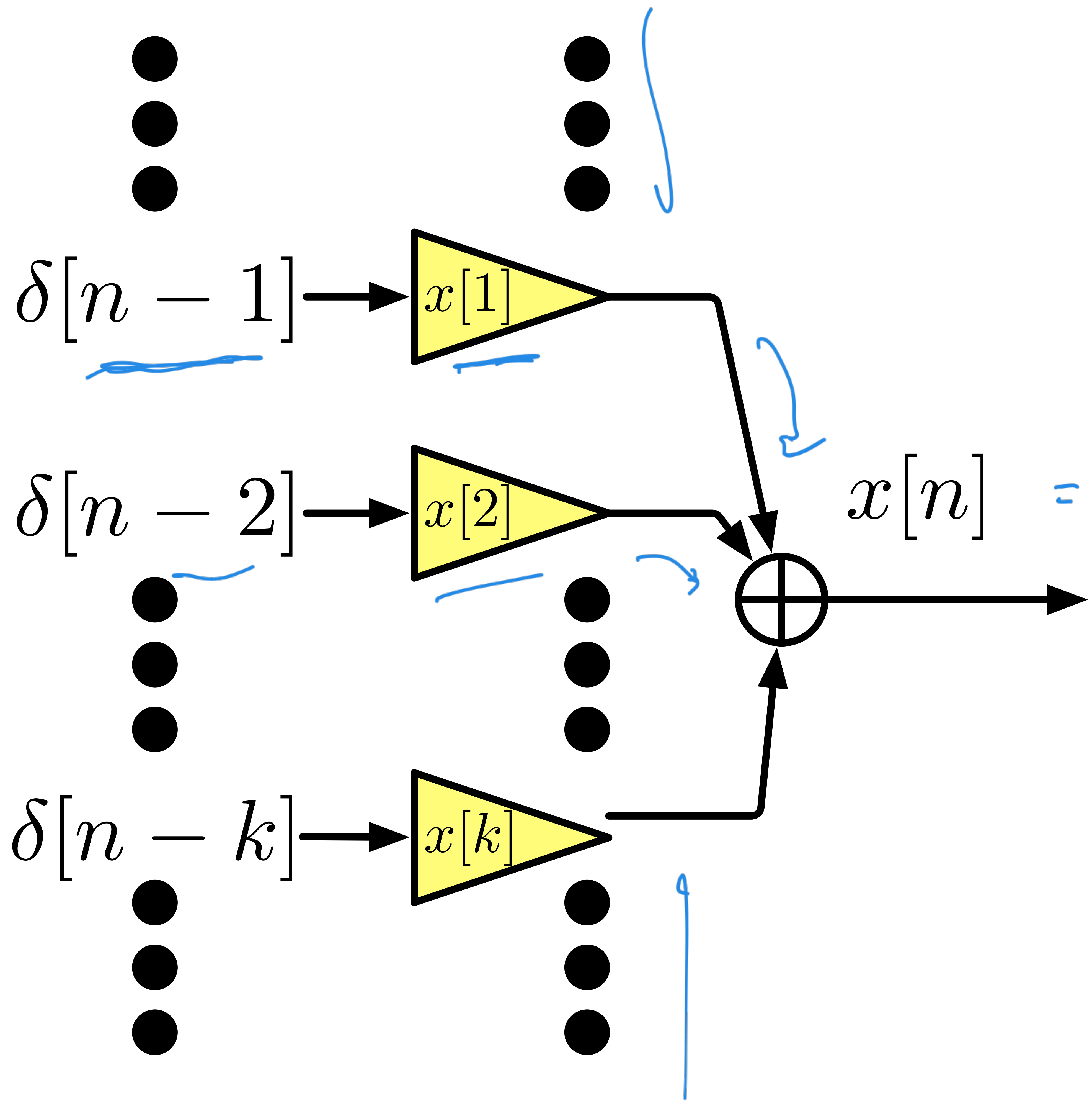
$$+ x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots , \tag{3}$$

δ @ n=1

or

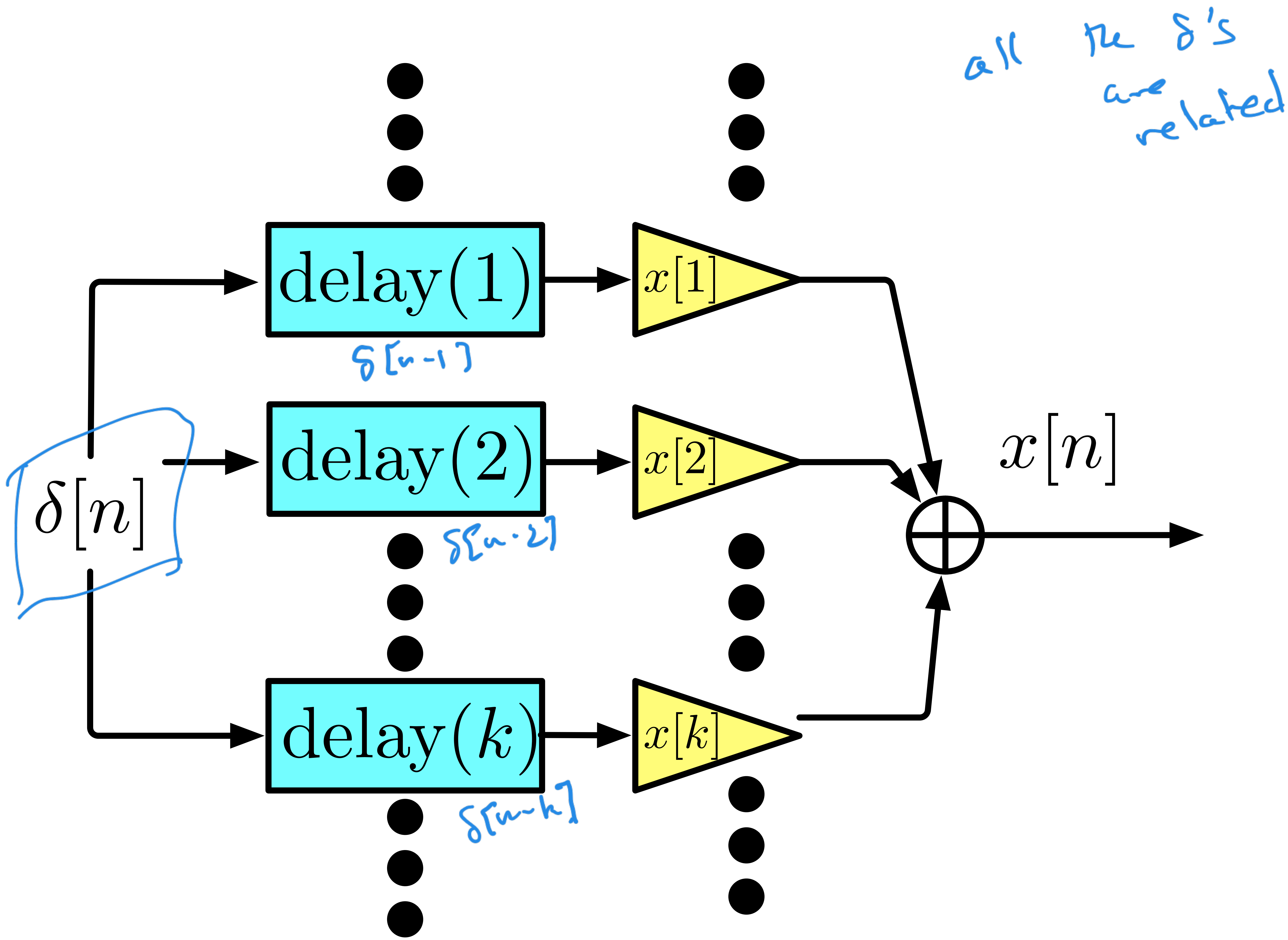
$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].} \tag{4}$$

DT signals in block diagrams



$$= \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

DT signals in block diagrams



So now the question...