Linear Systems and Signals

Impulse response and causality

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Signal properties of $h(t)$

$\Updownarrow$

System properties of $\mathcal{H}$

Learning objectives

The learning objectives for this section are:

- use the support of the impulse response to determine the dependence of outputs on inputs
- use delays and truncation to approximate non-causal systems with causal systems

System properties from the impulse response

LTI systems are *linear* and *time-invariant*. What about our other system properties like *causality*, *stability*, and *invertibility*? Remember our important fact from CT and DT:

- The impulse response contains everything you need to know about the system.
- We should be able to determine system properties from the impulse response.

Now we'll see how to use the impulse response to figure out whether an LTI system is causal, stable, or invertible.

Causality and convolution

To determine if a system is causal, we can look at the convolution formula:

x(t) delayed by τ

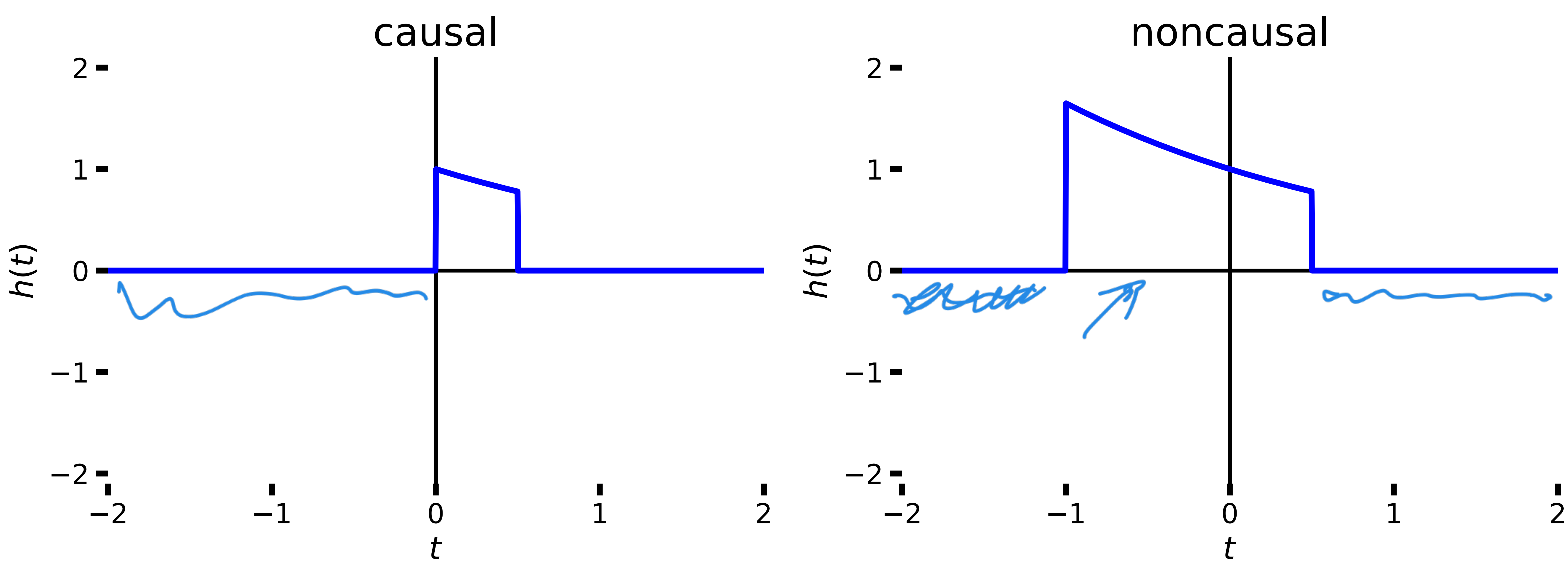
$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau \qquad y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] \qquad (1)$$

If we want $y(t)$ or $y[n]$ to depend only on past or current values, we cannot have any contribution from negative τ or negative k. So we can simplify:

$$y(t) = \int_{0}^{\infty} h(\tau) x(t - \tau) d\tau \qquad y[n] = \sum_{k=0}^{\infty} h[k] x[n-k] \qquad (2)$$

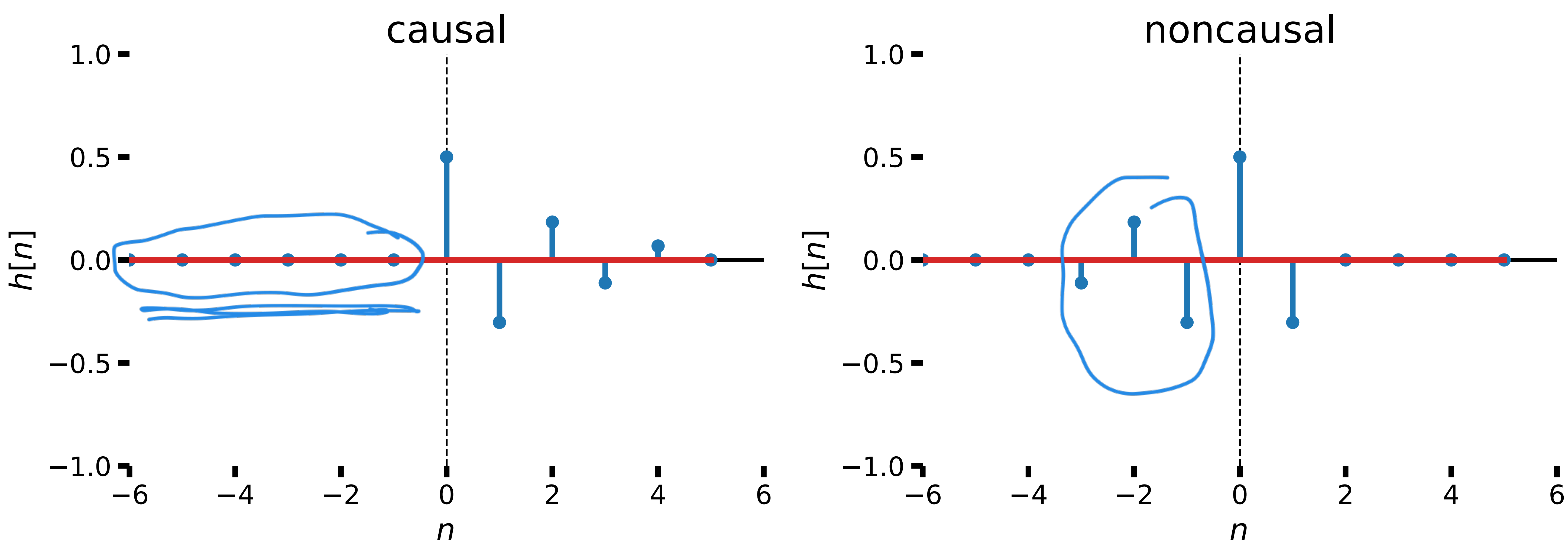
So this means that $h(t) = 0$ for $t < 0$ and $h[n] = 0$ for $n < 0$.

Causal systems



What we have shown is that $h(t) = 0$ for $t < 0$ and $h[n] = 0$ for $n < 0$ implies that the system is causal. It helps to think about this graphically: always draw a picture!

Causal systems



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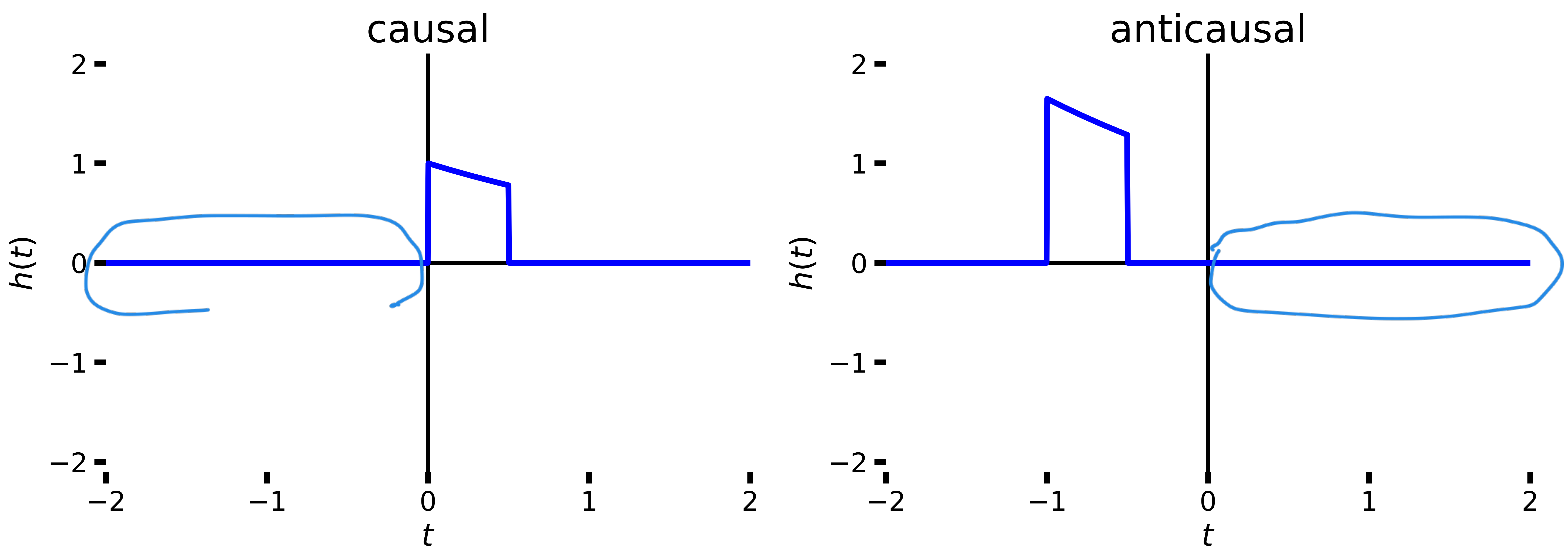
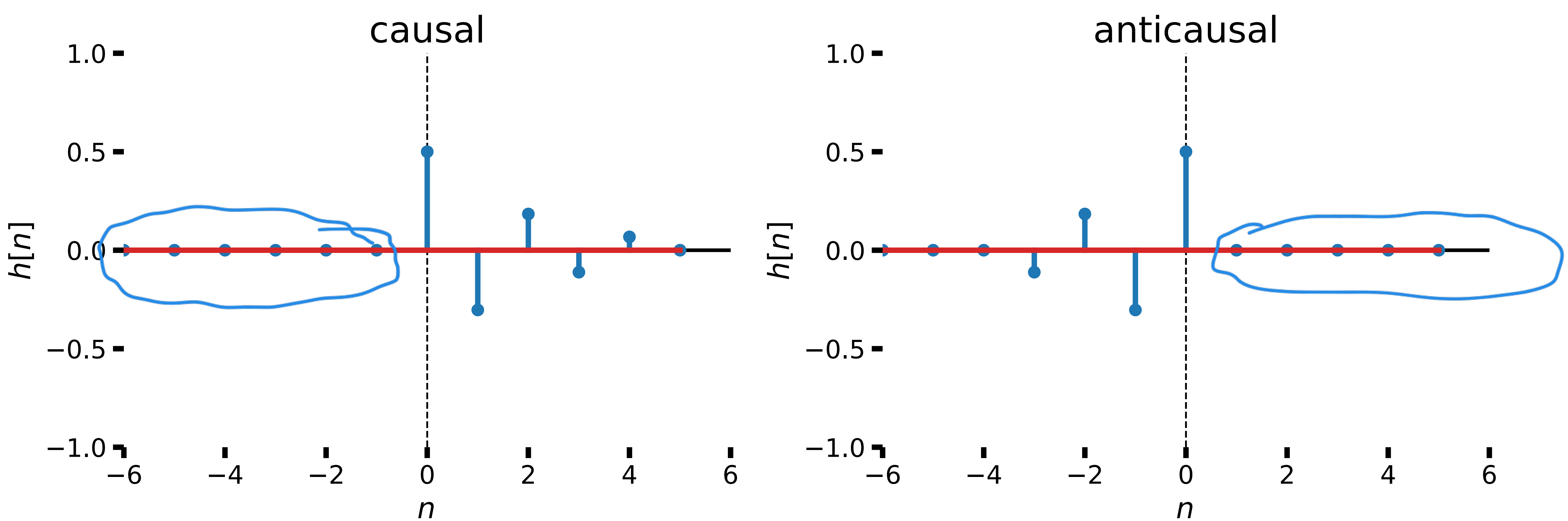
Anticausal systems



Try it yourself: use the same argument to show that $h(t) = 0$ for $t > 0$ and $h[n] = 0$ for $n > 0$ implies that the system is anticausal.

Anticausal systems



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Memorylessness

For a system to be memoryless, the output $y(t)$ has to be a function of $x(t)$ only:

only memoryless system

$$y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)d\tau. \tag{3}$$

This can only happen if $h(\tau)$ is $a\delta(\tau)$ for some a. This system is just a gain of a.

Likewise for DT:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \tag{4}$$

to depend only on $x[n]$ means $h[k]$ must be $a\delta[k]$ for some a, which is also a gain of a.

The uses of causal and noncausal systems

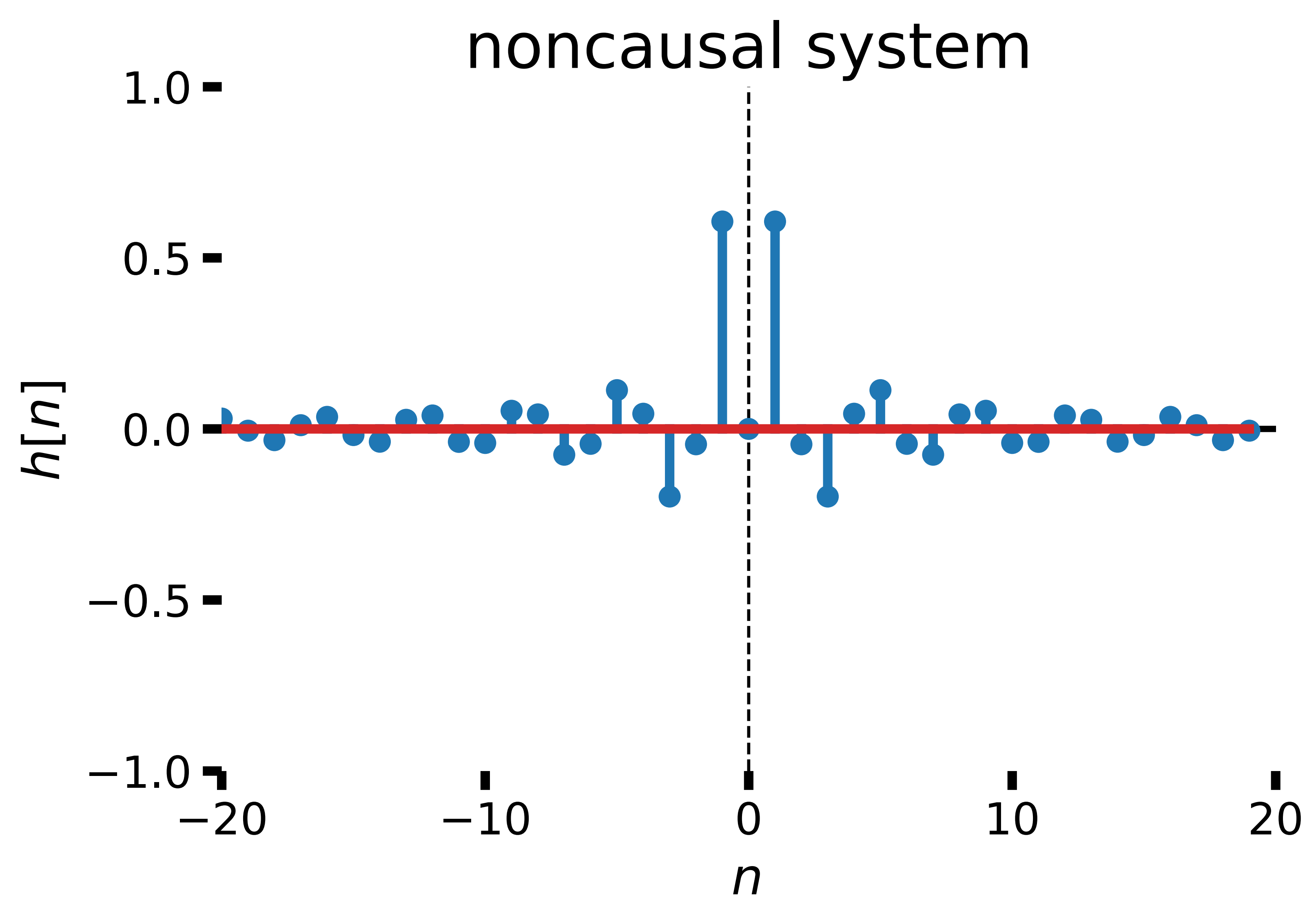
Causal systems can operate in "real-time": to compute the output you only need current and past values of the input signal x.

- Important for online signal processing and filtering.
- Examples: analog radio frequency (RF) filters, live audio effects (vocoding, autotune), servomechanical systems

Noncausal systems can use past and future inputs.

- Important for offline signal processing and filtering.
- Examples: blurring and other image processing effects, offline audio effects, video processing

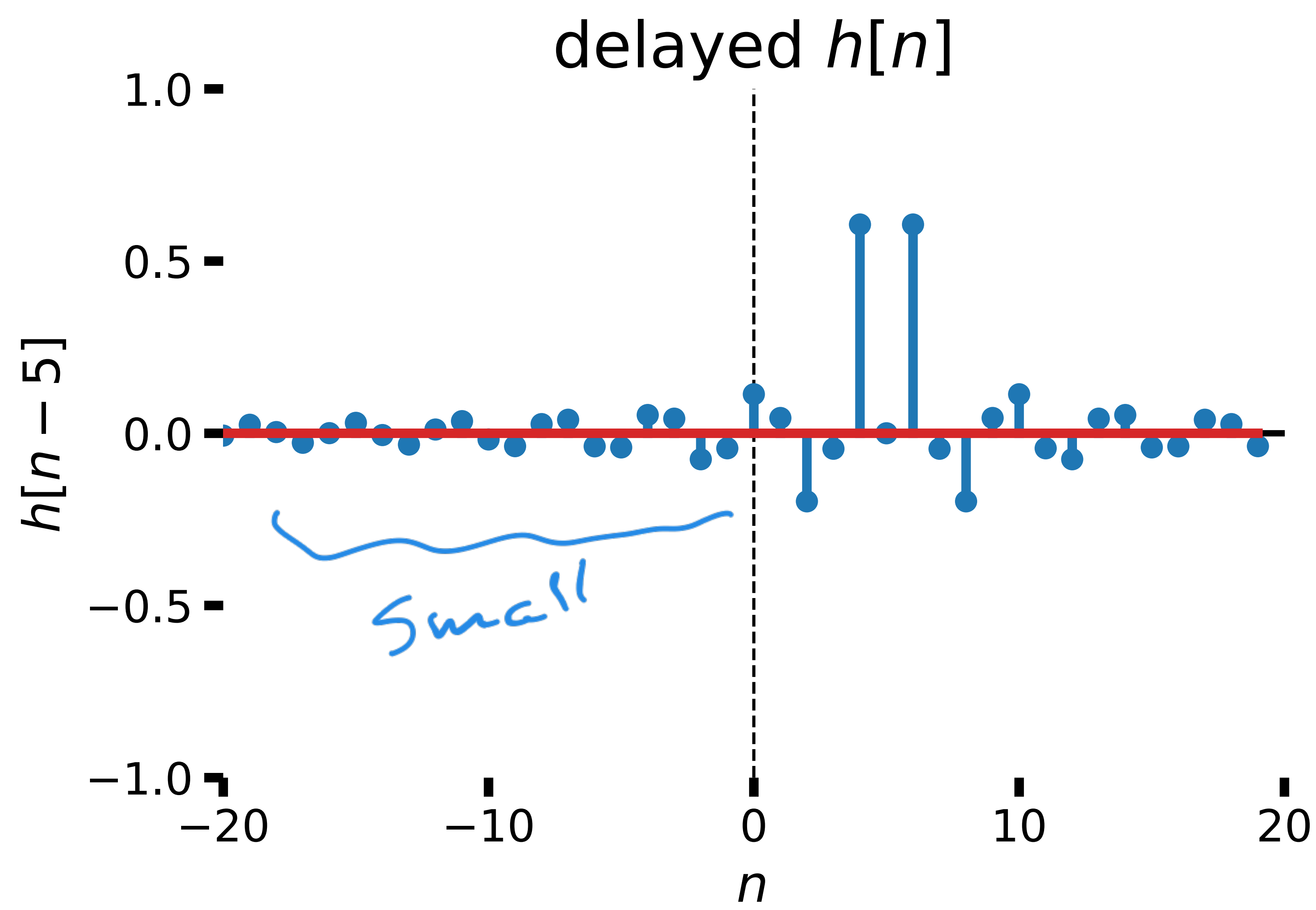
Making systems causal



What if you have a noncausal system that works well but you need to make it causal?

- Introduce a delay to make the impulse response causal if you can
- Truncate the impulse response to make it causal.
- Or both!

Making systems causal



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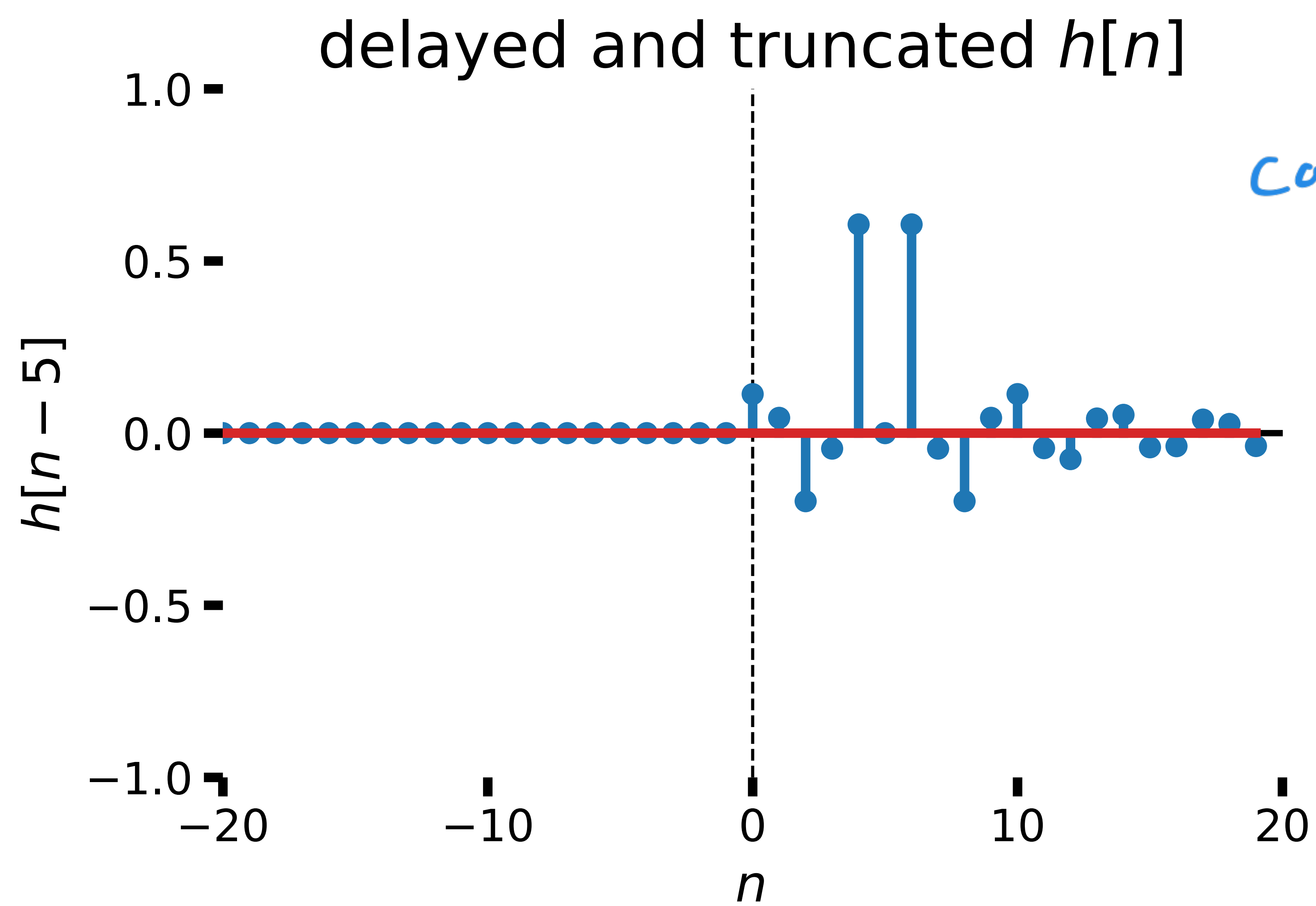

causal approx to h[n]

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Try it yourself!

Problem

For each of the following impulse responses determine if the system is causal, anticausal, or neither causal nor anticausal.

- $h(t) = \mathrm{rect}(t)$.
- $h(t) = e^{-2t}u(t)$
- $h(t) = e^{3t}u(-t)$
- $h[n] = u[n] - u[n-5]$
- $h[n] = 3^n(u[n+5] - u[n])$
- $h[n] = \cos((\pi/3)n)$