Linear Systems and Signals

Calculating signal properties computationally

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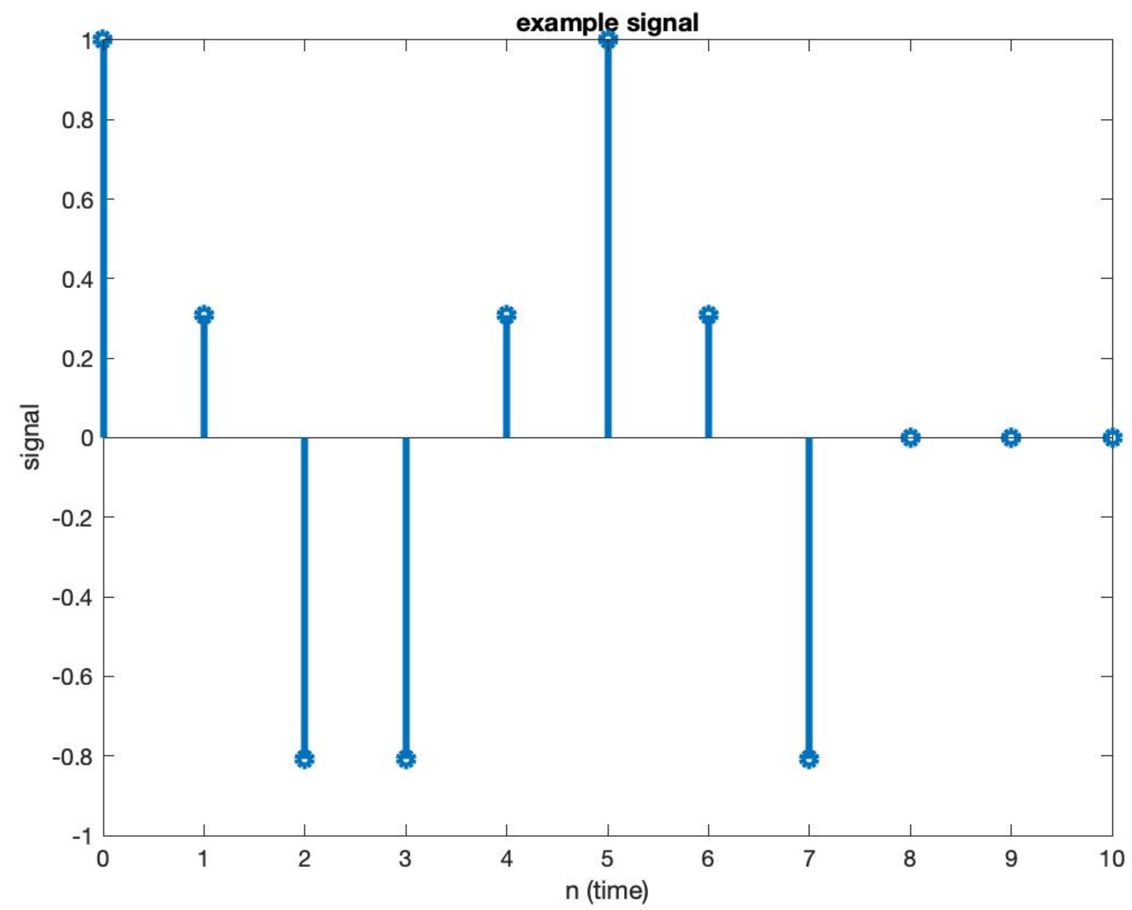
Learning objectives

The learning objectives for this section are:

- find the power and energy for signals computationally
- finding the period of a periodic signal

DT energy of a signal



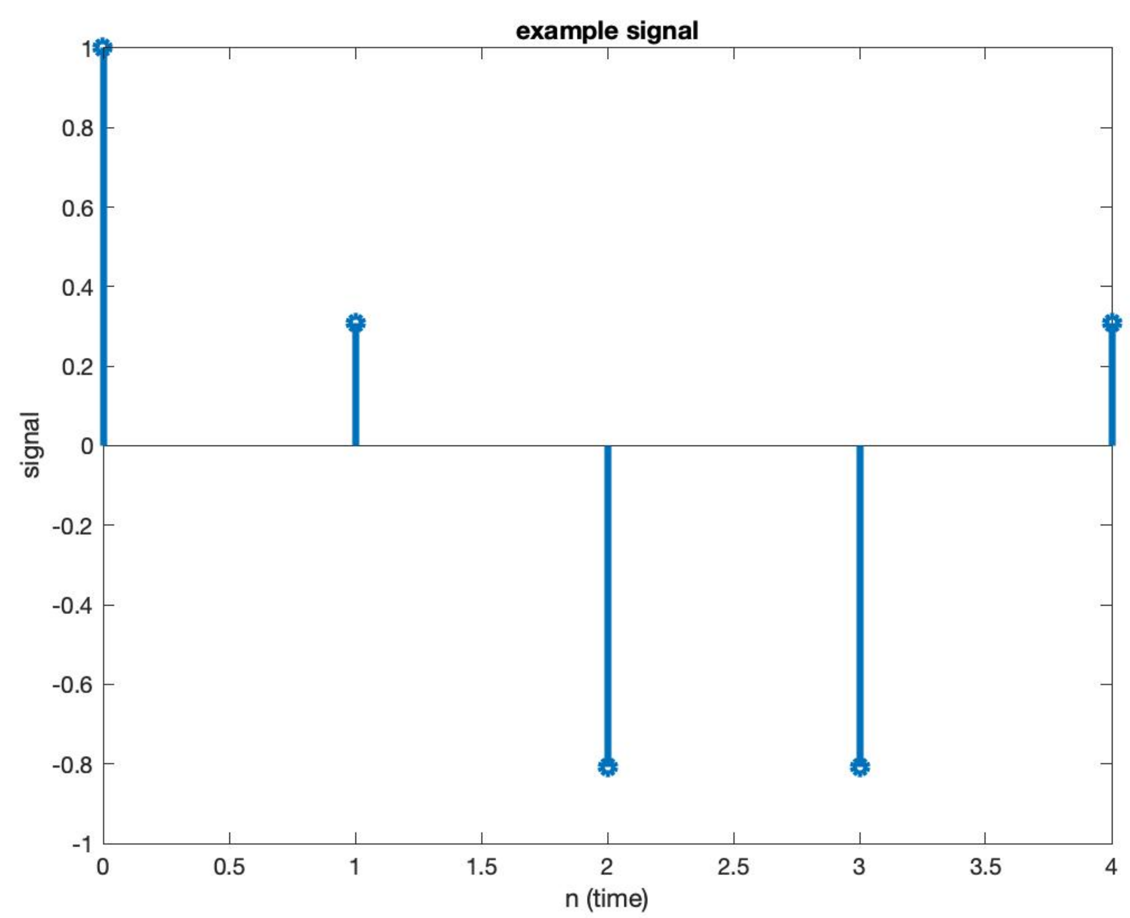
For DT signals, the energy of a signal is easy to calculate:

Code Example 1: DT signal energy

```
xsig = @(n) cos(0.4*pi*n) .* (n >= 0) .* (n < 8);
n = 0:10;
x = xsig(n);
Ex = sum(x.*x);
```

$\sum_n x[n]^2$ abs(x)^2

DT power of a signal



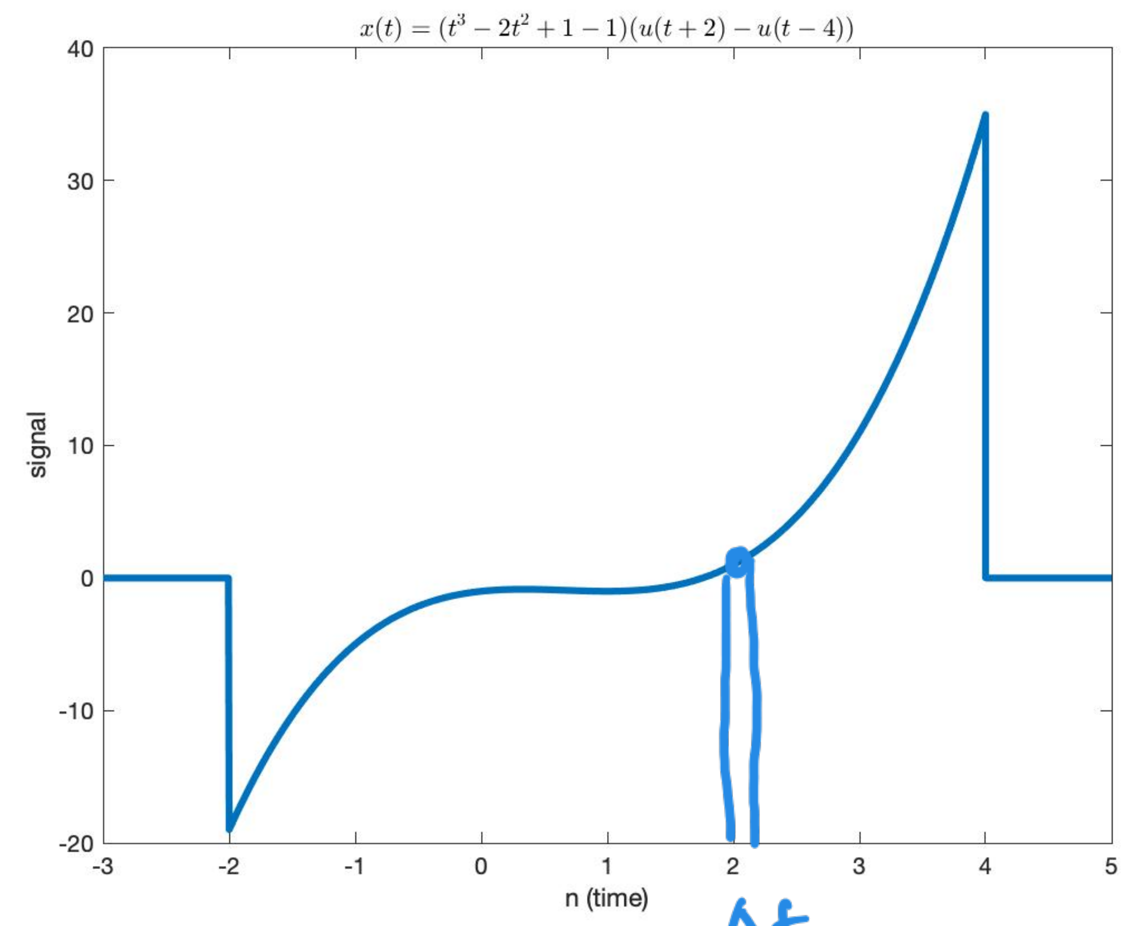
For a periodic signal, we can calculate the power in one period:

Code Example 2: DT signal power

```
xsig = @(n) cos(0.4*pi*n); % period = 5
n = 0:4;
x = xsig(n);
Px = sum(x.*x) / 5;
```

$\frac{1}{N_0} \sum_{n=1}^{N_0} |x[n]|^2$

CT energy of a signal



area under curve is $\Delta t \cdot x(t)$

For CT, you have to remember the sampling interval $1/f_s$:

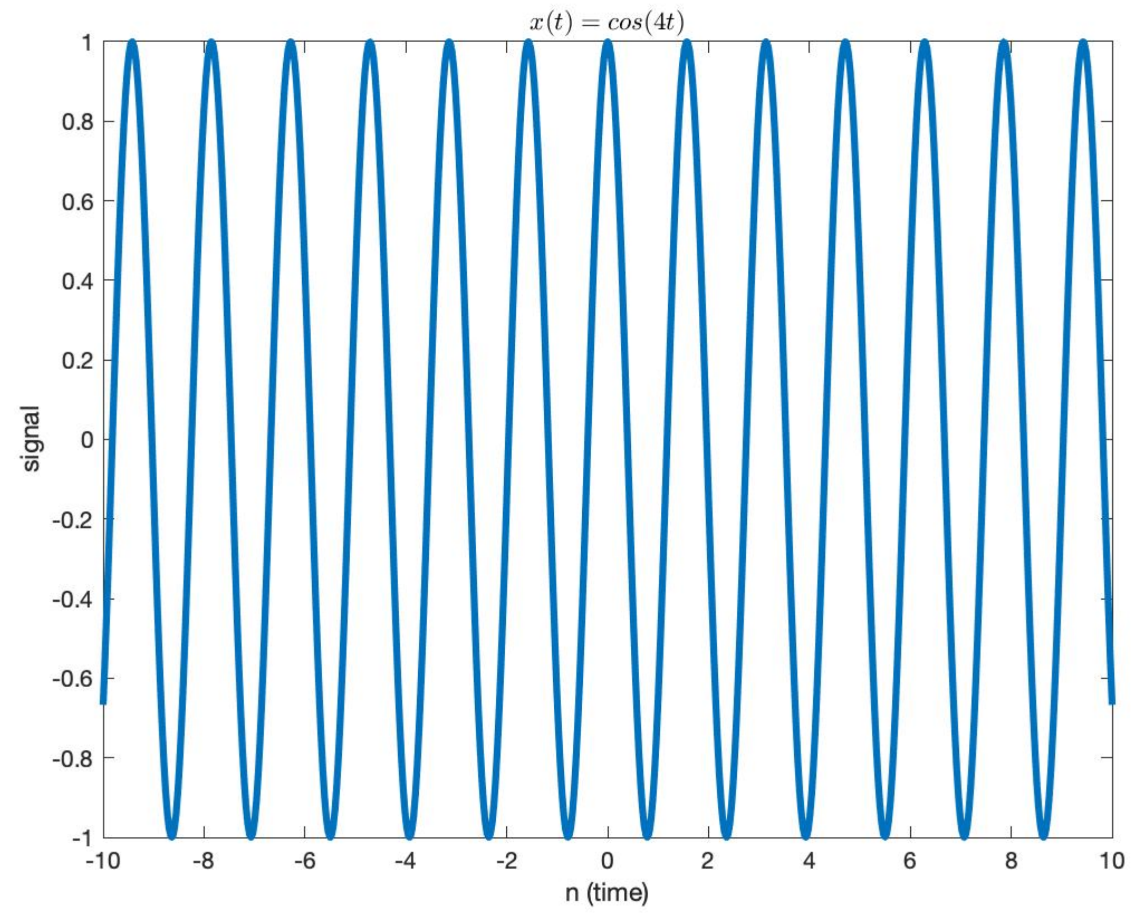
Code Example 3: CT signal energy

$\Delta t = 1/f_s$

```
fs = 1e4;
t = (-3):(1/fs):5;
x = (t.^3 - 2*t.^2 + t - 1) .* (t > -2) .* (t < 4);
Ex = sum(x.^2)/fs;
```

$\sum$ x.^2 ← discrete approx

CT power



For CT signals, we have a bit of a challenge. Even for periodic signals, what if the period is not an integer multiple of $1/f_s$?

1. **Option 1:** empirically calculate the limit $\lim_{T\to\infty} \frac{1}{2T} \int_{t=-T}^{T} |x(t)|^2 dt$.
2. **Option 2:** for periodic signals, estimate the period of the signal and compute $\frac{1}{T_0} \int_0^{T_0} |x(t)|^2 dt$.

Option 1: empirical limit

Code Example 4: numerical limit for CT power

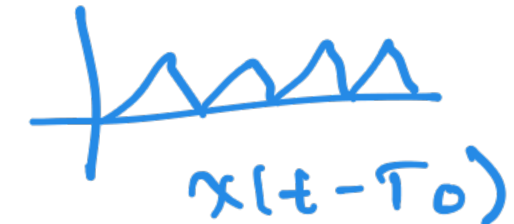
```
fs = 1e6;
xsig = @(t) cos(4*t);
t = -1000:(1/fs):1000;
Ex = (sum(x.*x)/fs) / 2000;
```

energy

time

1. computationally intensive – try it out!
2. how do you know when you reach the limit?

Option 2: estimate the period with autocorrelation



If we delay a periodic signal by an integer multiple of T_0 then we get the same signal. **Idea:** find the delay where the signal "lines up." We use the *autocorrelation function* to do this:

conjugate

$$R_{xx}(\tau) = \frac{1}{2T_0} \int_{t=-T_0}^{T_0} x(t)x^*(t-\tau)dt \tag{1}$$

delayed copy

But we don't know T_0! Just take a large enough segment of time and see where

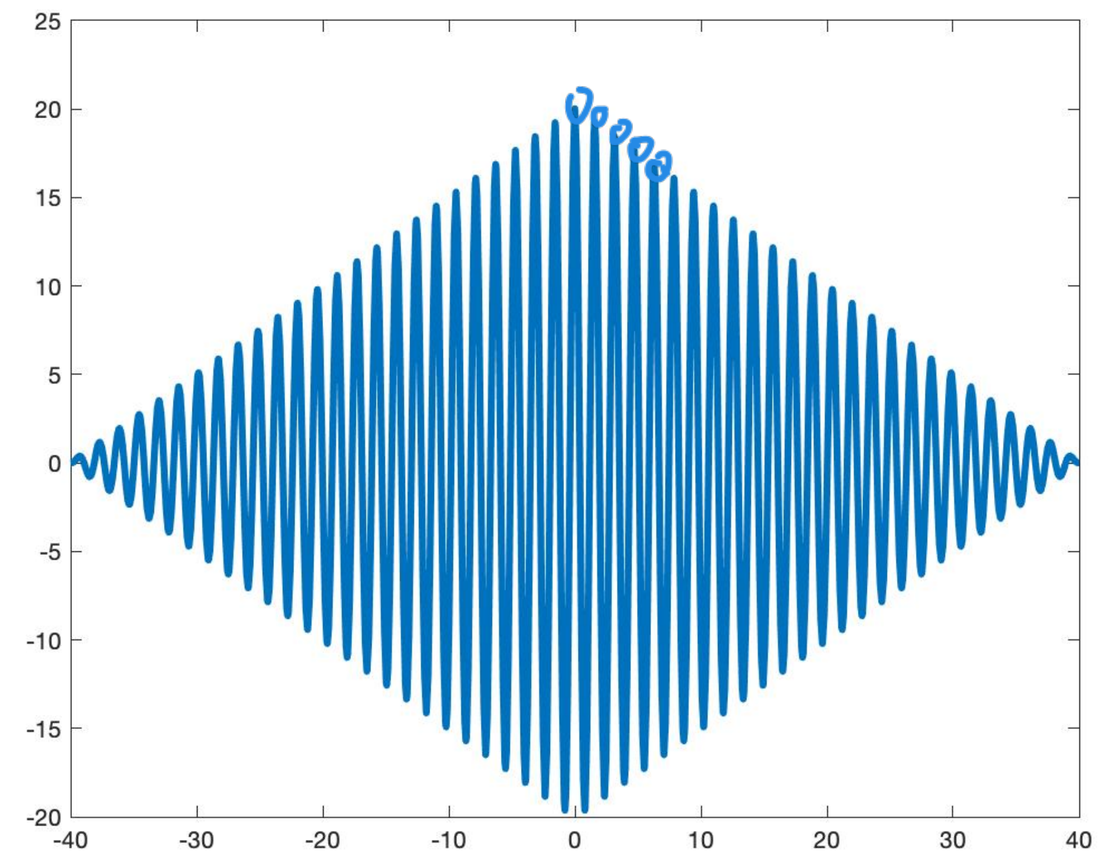
T = big

$$R_{xx}(\tau) = \int_{t=-T}^{T} x(t)x^*(t-\tau)dt \tag{2}$$

delay/lag

is large. The peak is at $\tau = 0$ and the next largest should be $\pm T_0$.

Option 2 in MATLAB



Find 2^{nd} highest peak

$\omega = 4$

$\frac{2\pi}{4} = \frac{\pi}{2}$

not rational

We can use the `xcorr` function in MATLAB to do this:

Code Example 5: using the xcorr function

```
fs = 1e4;
xsig = @(t) cos(4*t);
t = -20:(1/fs):20;
[Rxx,lags] = xcorr(xsig(t));
figure; plot(lags/fs,Rxx/fs,'LineWidth',3);
```

We want the *second-largest peak*.

Finding the second-largest peak

For $x(t) = \cos(4t)$ the period $T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$. Check to see if this is the case!

Code Example 6: finding the second-largest peak

```
[pks,locs] = findpeaks(Rxx);
[srt,idx] = sort(pks,'descend');
second_peak = locs( idx(2) );
second_peak_lag = lags(second_peak)/fs;
T_0 = abs( second_peak_lag );
```

choose 2nd

may choose T_0 as 2nd highest peak

Calculating the power

The last part is to calculate the power, which we can do by calculating the total energy in one period and dividing by T_0.

Code Example 7: finding the second-largest peak

```
t2 = 0:(1/fs):T_0;
x = xsig(t2);
Px = (sum(x.^2)/fs)/T_0;
```

energy

Compare this to what you get by integration.

Try it yourself

Problem

Find the energy and power numerically for signals that we have seen in already. In particular, try out these calculations for complex-valued signals. Look at the documentation for `xcorr` *to see how it works for complex signals.*