Linear Systems and Signals
Time-invariant systems

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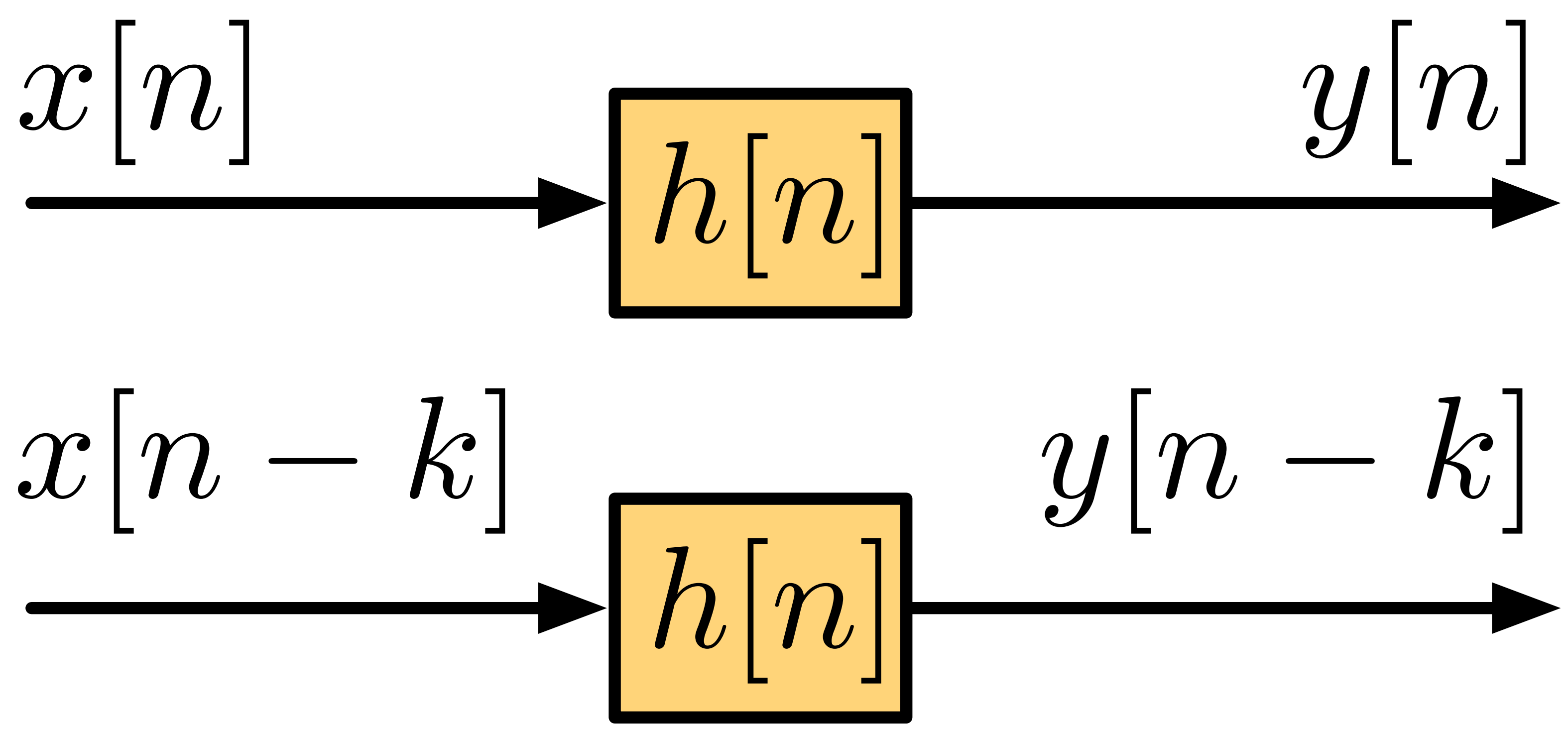
Learning objectives

The learning objective for this section is:

- determine if a system is time-invariant or time-varying

Time-invariance



Definition

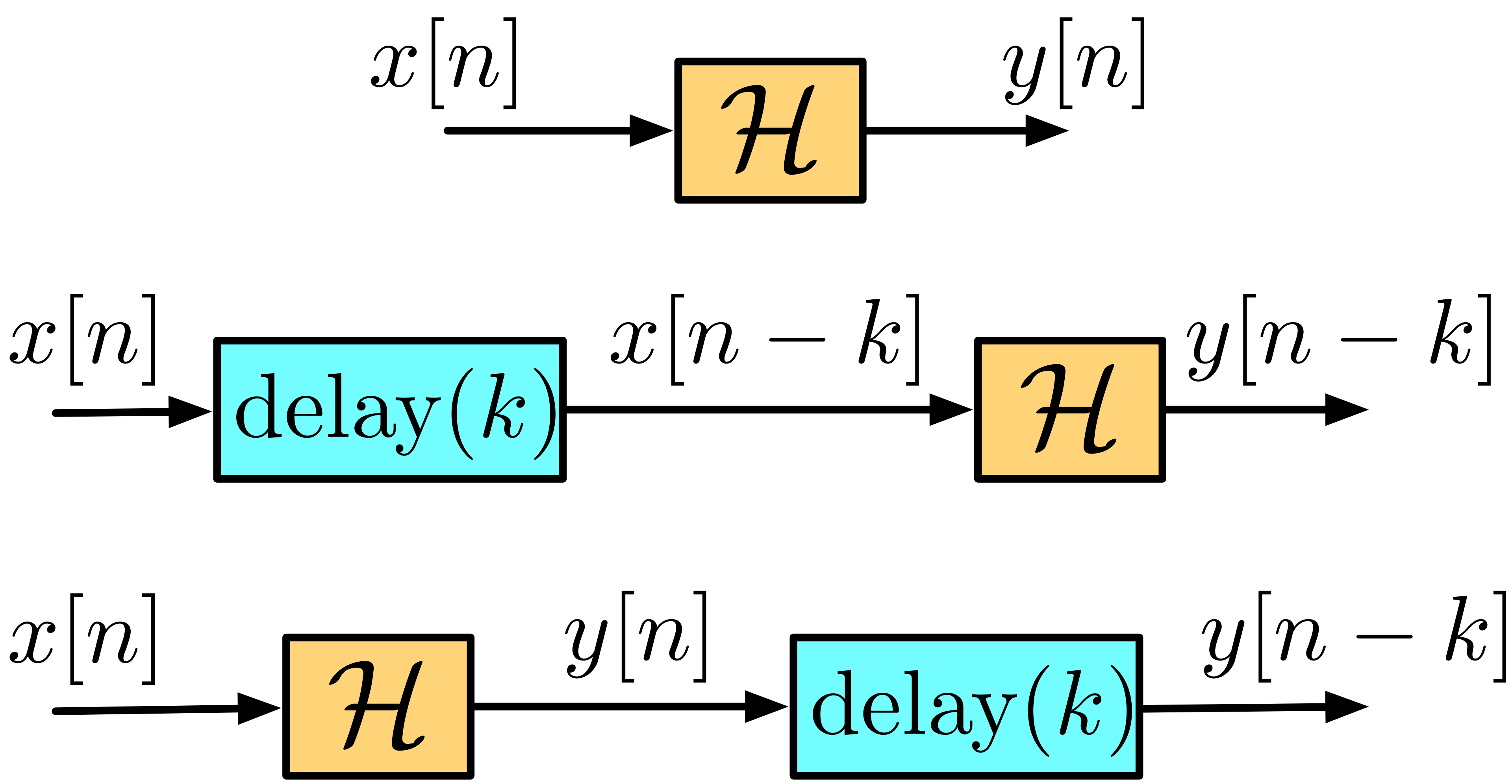
A system $\mathcal{H}$ is *time-invariant* if for all signals $x(t)$ (or $x[n]$) with $y(t) = \mathcal{H}(x(t))$ (or $y[n] = \mathcal{H}(x[n])$) and any time shift $t_0 \in \mathbb{R}$ (or $n_0 \in \mathbb{Z}$), we have

$$\mathcal{H}\left(x(t-t_0)\right) = y(t-t_0) \qquad \mathcal{H}\left(x[n-n_0]\right) = y(n-n_0) \tag{1}$$

A system is *time-varying* if it is not time-invariant.

In another block diagram

$$x[n] \xrightarrow{\mathcal{H}} y[n] \Longrightarrow x[n-k] \xrightarrow{\mathcal{H}} y[n-k]$$



"the system commutes with delay"

Checking for linearity

To avoid confusion, let $z(t)$ (or $z[n]$) be the delayed signal:

- $y(t) = x(t)\cos(6000\pi t)$. Then if $z(t) = x(t - t_0)$, we have

$$\mathcal{H}(z(t)) = z(t)\cos(6000\pi t) \neq x(t - t_0)\cos(6000\pi(t - t_0)) \quad (2)$$

  This is time-varying.

- $y[n] = \cos((3\pi/4)x[n])$. Then if $z(t) = x(t - t_0)$, we have

$$\mathcal{H}(z(t)) = \cos((3\pi/4) z(t)) = y(t - t_0) \quad (3)$$

  This is time-invariant.

Try it yourself

Problem

Determine if each of these systems is time-varying or time-invariant.

- $y[n] = x(t)^2$
- $y[n] = nx[n]$
- $y(t) = x(t-2) + x(2-t)$
- $y(t) = x(3t)$
- $y[n] = x_e[n]$