Linear Systems and Signals
Periodic signals in CT

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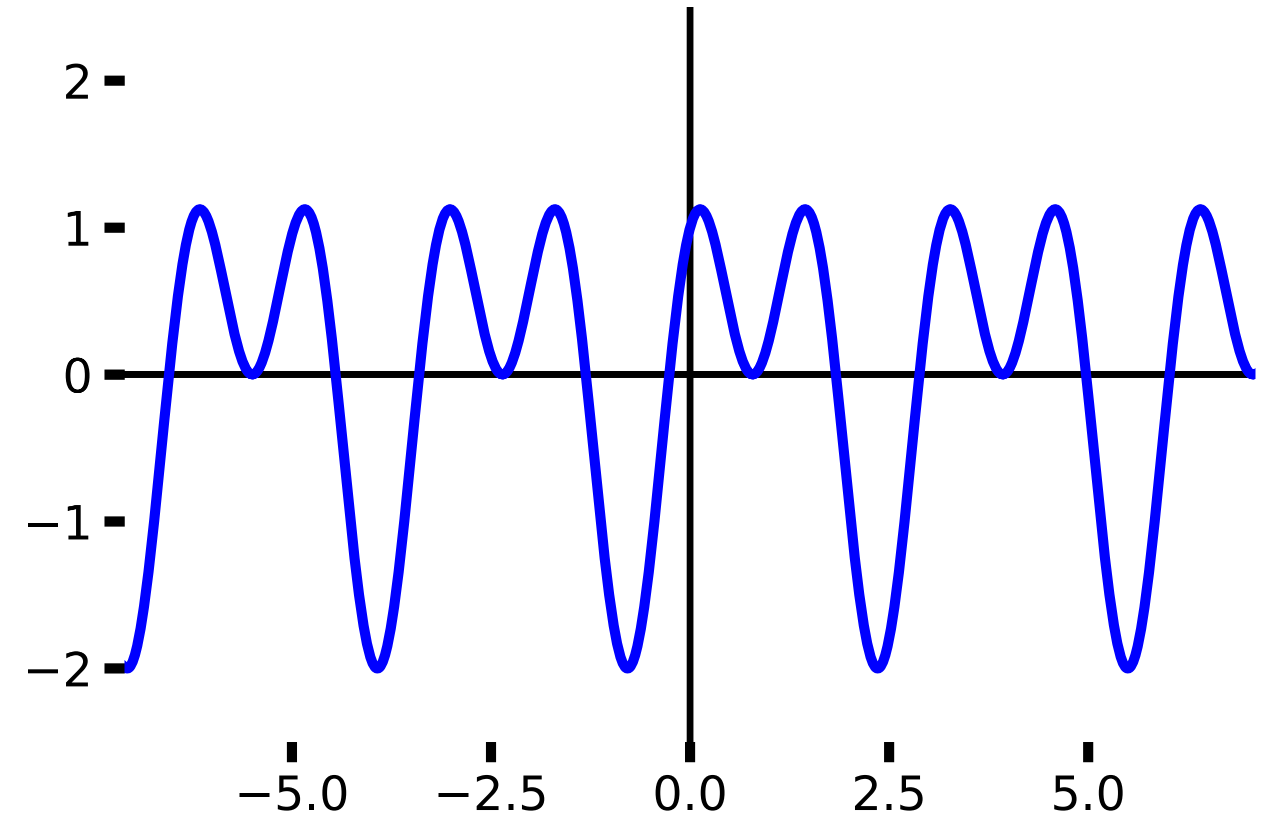
Learning objectives

The learning objectives for this section are:

- determine if a CT signal is periodic or not
- find the period and fundamental angular frequency of periodic CT signals

Definition



A CT signal $x(t)$ is *periodic* if there is a time shift T_0 such that

$$x(t + T_0) = x(t) \qquad \text{for all } t \tag{1}$$

The *fundamental period* of $x(t)$ is the smallest such T_0. The *fundamental angular frequency* is $\frac{2\pi}{T_0}$.
A signal which is not periodic is *aperiodic*

CT sinusoids

For sinusoidal functions this is more straightforward:

$$x(t) = A\cos(\omega t + \phi) \tag{2}$$

To figure out the fundamental period, focus on the term multiplying t: the phase shift ϕ and amplitude A don't affect the periodicity. Note that $x(0) = A\cos(\phi)$. When $\omega t = 2\pi$ we get $A\cos(\phi)$ again,

$$T_0 = \frac{2\pi}{\omega}. \tag{3}$$

Example

Problem

Find the fundamental period of $x(t) = 3\cos(120\pi t)$.

Applying the formula:

$$T_0 = \frac{2\pi}{120\pi} = \frac{1}{60}\mathrm{sec} \tag{4}$$

Problem

Find the fundamental period of $x(t) = \sin(0.14\pi t + 2\pi/5)$

Again,

$$T_0 = \frac{2\pi}{0.14\pi} = \frac{100}{7}\mathrm{sec} \tag{5}$$

Sums of CT sinusoids

$T_0 = 3T_1 = 8T_2$

What if we have 2 (or more) sinusoids?

$$x(t) = \cos(\omega_1 t) + \cos(\omega_2 t) \tag{6}$$

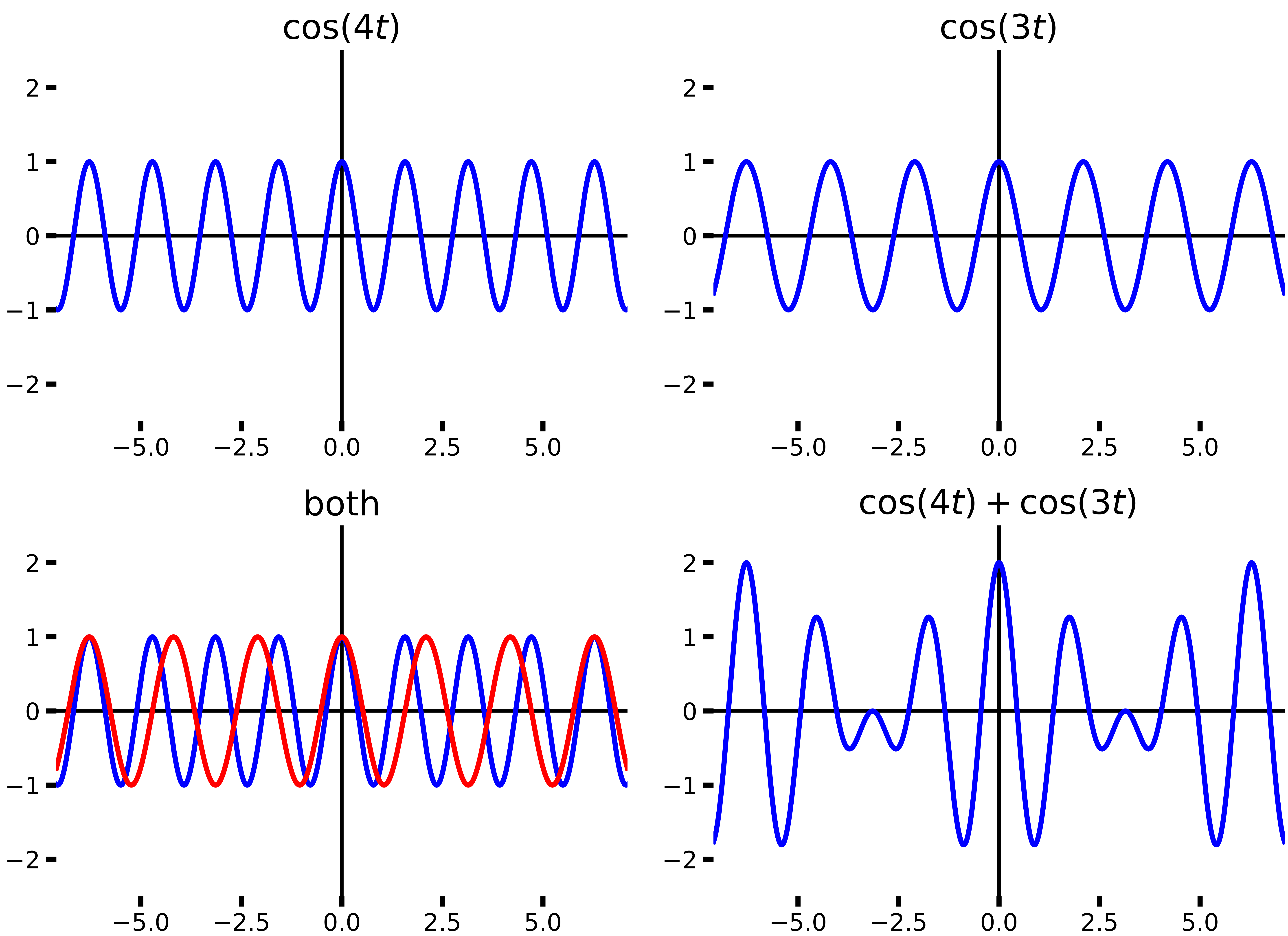
First signal repeats every $T_1 = \frac{2\pi}{\omega_1}$ seconds, second repeats every $T_2 = \frac{2\pi}{\omega_2}$ seconds. What can happen?

For $x(t)$ to be periodic there needs to be some (smallest) T_0 such that

$$T_0 = m_1 T_1 = m_2 T_2 \qquad m_1, m_2 \in \{1, 2, \ldots, \} \tag{7}$$

That is, we need positive integers so that in time T_0 the first signal goes through m_1 cycles and the second goes through m_2 cycles. If there are no such integers then the signal is aperiodic, otherwise the period is T_0.

In pictures



More general signals

1. Try drawing a picture (or use MATLAB) to see if it “looks” periodic.
2. Make an educated guess about what the period might be.
3. Verify your guess mathematically

Examples

- Is $x_1(t) = \cos(3\pi t + 2) + 2\sin(18\pi t + 5)$ periodic? If so, what is its period?
  We have $\omega_1 = 3\pi$, $\omega_2 = 18\pi$ and $T_1 = \frac{2}{3}$, $T_2 = \frac{1}{9}$. Note that $\frac{2}{3} = 6\frac{1}{9}$ so it is periodic with period $\frac{2}{3}$.

Examples

- Is $x_1(t) = \cos(3\pi t + 2) + 2\sin(18\pi t + 5)$ periodic? If so, what is its period?
  We have $\omega_1 = 3\pi$, $\omega_2 = 18\pi$ and $T_1 = \frac{2}{3}$, $T_2 = \frac{1}{9}$. Note that $\frac{2}{3} = 6\frac{1}{9}$ so it is periodic with period $\frac{2}{3}$.
- Is $x_2(t) = \cos(3\pi t + 2) + 2\sin(17\pi t + 5)$ periodic? If so, what is its period?
  We have $\omega_1 = 3\pi$, $\omega_2 = \overset{17}{\cancel{18}}\pi$ and $T_1 = \frac{2}{3}$, $T_2 = \frac{2}{17}$. Note that $3\frac{2}{3} = 17\frac{2}{17}$ so it is periodic with period 2.

Examples

- Is $x_1(t) = \cos(3\pi t + 2) + 2\sin(18\pi t + 5)$ periodic? If so, what is its period?
  We have $\omega_1 = 3\pi$, $\omega_2 = 18\pi$ and $T_1 = \frac{2}{3}$, $T_2 = \frac{1}{9}$. Note that $\frac{2}{3} = 6\frac{1}{9}$ so it is periodic with period $\frac{2}{3}$.
- Is $x_2(t) = \cos(3\pi t + 2) + 2\sin(17\pi t + 5)$ periodic? If so, what is its period?
  We have $\omega_1 = 3\pi$, $\omega_2 = 18\pi$ and $T_1 = \frac{2}{3}$, $T_2 = \frac{2}{17}$. Note that $3\frac{2}{3} = 17\frac{2}{17}$ so it is periodic with period 2.
- Is $x_3(t) = \cos(3\pi t + 2) + 2\sin(12t + 5)$ periodic? If so, what is its period?
  We have $\omega_1 = 3\pi$, $\omega_2 = 12$ and $T_1 = \frac{2}{3}$, $T_2 = \frac{\pi}{6}$. There are no integers such that $\frac{2}{3}m_1 = \frac{\pi}{6}m_2$. So this is not periodic.

Try it out

Problem

Determine whether each of these signals is periodic. If it is, find the fundamental period and angular frequency.

$$x_1(t) = |\cos(7\pi t)| \tag{8}$$

$$x_2(t) = e^{-j20\pi t} + e^{j14\pi t} \tag{9}$$

$$x_3(t) = e^{j3t} + e^{j3\pi t} \tag{10}$$

Make up some of your own!