Linear Systems and Signals
Invertible and non-invertible systems

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Learning objectives

The learning objective for this section is:

- determine if a system is invertible or not

Stability

Definition

A system $\mathcal{H}$ is *invertible* if there is a system $\mathcal{H}^{-1}$ such that for any all signals $x(t)$ (or $x[n]$) with

$$y(t) = \mathcal{H}(x(t)) \qquad y[n] = \mathcal{H}(x[n]), \tag{1}$$

we have

$$x(t) = \mathcal{H}^{-1}(y(t)) \qquad x[n] = \mathcal{H}^{-1}(y[n]), \tag{2}$$

The system $\mathcal{H}^{-1}$ is called an inverse system.

Finding an inverse system

- To show a system is invertible you have to show that different input signals produce different output signals or construct an inverse system.
  Suppose $y[n] = x[n]^3$, Then if $x_1[n] \neq x_2[n]$, there is a time n such that $x_1[n]^3 \neq x_2[n]^3$. Alternatively, you can write the inverse system $x[n] = y[n]^{1/3}$.
- To show a system is not invertible you need to find two different inputs x_1 and x_2 which lead to the same output y.
  Suppose $y(t) = x(t)^2$. Then choose $x(t)$ and $-x(t)$. These both lead to the same output signal but because the square we cannot know the sign.

Inverse systems and invertibility

We should make a distinction between finding an inverse system $\mathcal{H}^{-1}$ and invertibility.

- Some people call a system invertible only when the inverse system is *realizable* which often means causal and stable. So when they say is the system invertible they mean "with a causal stable inverse."
- We will be looking at inverse systems which are not causal later on, so causality is perhaps less necessary.
- Stability might be more important – building an unstable system can be bad for many reasons.

Try it yourself

Problem

Determine if each of these systems is invertible or not invertible.

- $\mathcal{H}(x(t)) = \frac{d}{dt}x(t)$
- $\mathcal{H}(x[n]) = x[1-n]$
- $\mathcal{H}(x(t)) = \cos(x(t))$
- $\mathcal{H}(x[n]) = x[n]x[n-1]$