Linear Systems and Signals
The sinc function

Anand D. Sarwate

Department of Electrical and Computer Engineering
Rutgers, The State University of New Jersey

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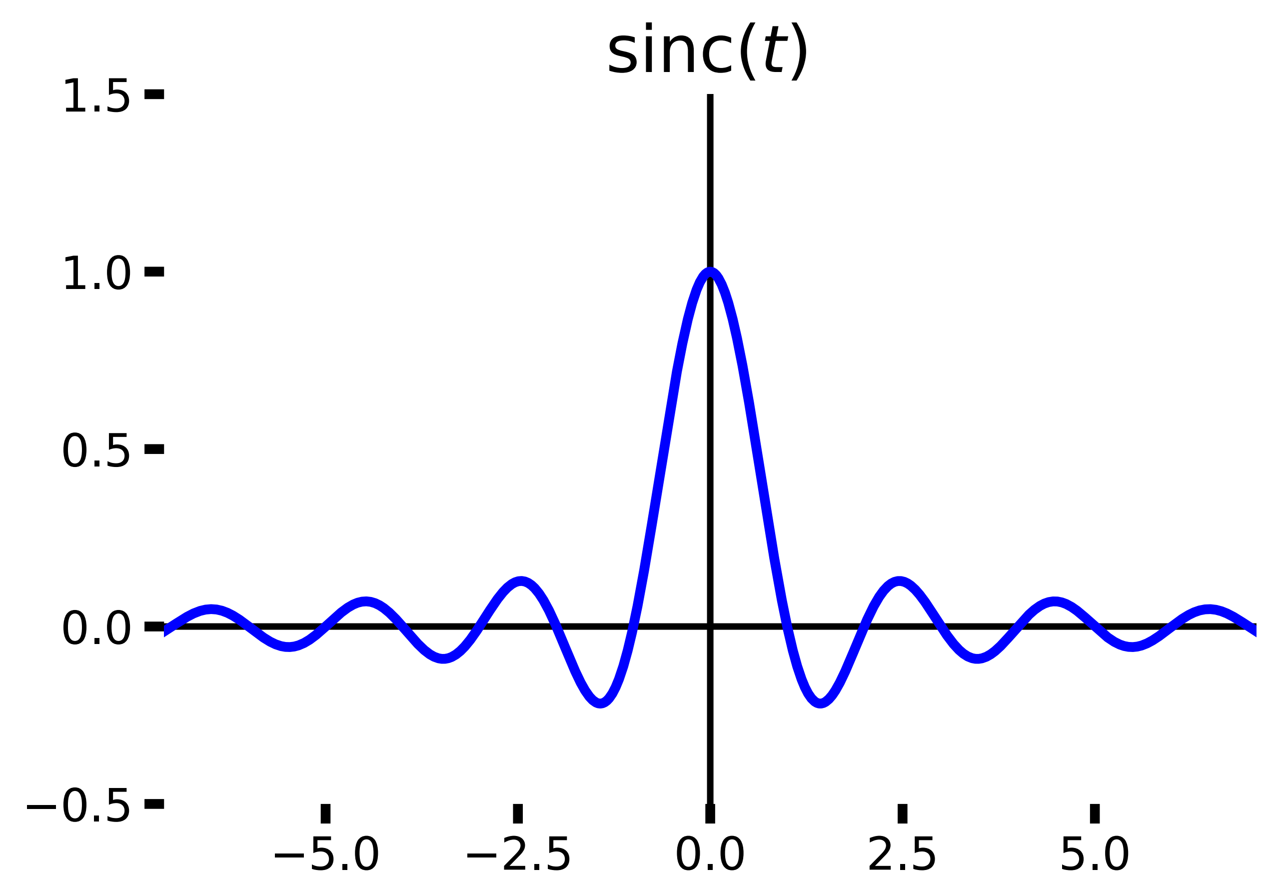
Learning objectives

The learning objectives for this section are:

- use standard calculus tools (e.g. L'Hospital's rule) to understand the shape of functions such as the sinc function
- sketch the sinc function and determine its zero-crossings and maximum value

The CT sinc function

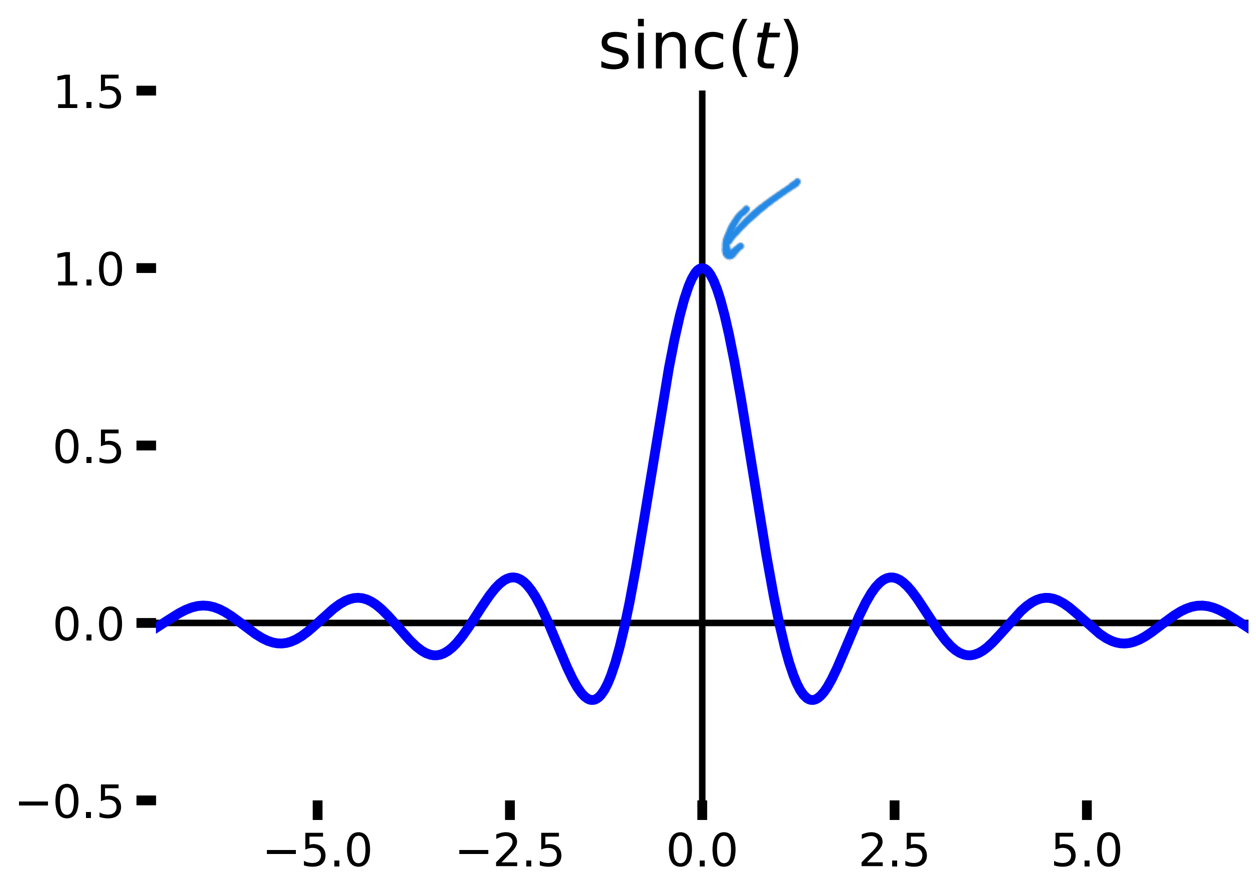


A very important function for understanding signal processing is call the sinc (pronounced the same as "sink") function:

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}. \tag{1}$$

Math folks define it as $\frac{\sin t}{t}$ but it's convenient to put the π in there.

Value at 0

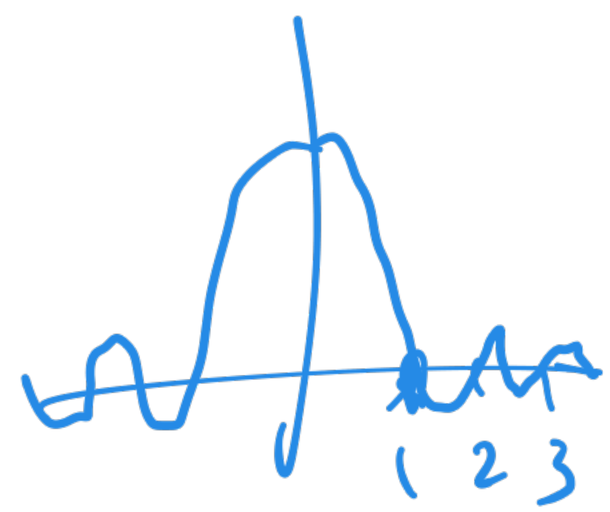


What is the value at $t = 0$? The numerator and denominator are both 0 so we need to use *L'Hôpital's rule* (or L'Hospital):

$$\operatorname{sinc}(0) = \lim_{t \to 0} \frac{\frac{d}{dt} \sin(\pi t)}{\frac{d}{dt}(\pi t)} = \frac{\pi \cos(\pi t)}{\pi} = 1. \tag{2}$$

Zero-crossings at 0

The sinc function is zero at every nonzero integer value of t since $\sin(\pi k) = 0$ for $k \in \mathbb{Z}$. These are the *zero-crossings* of the sinc function.



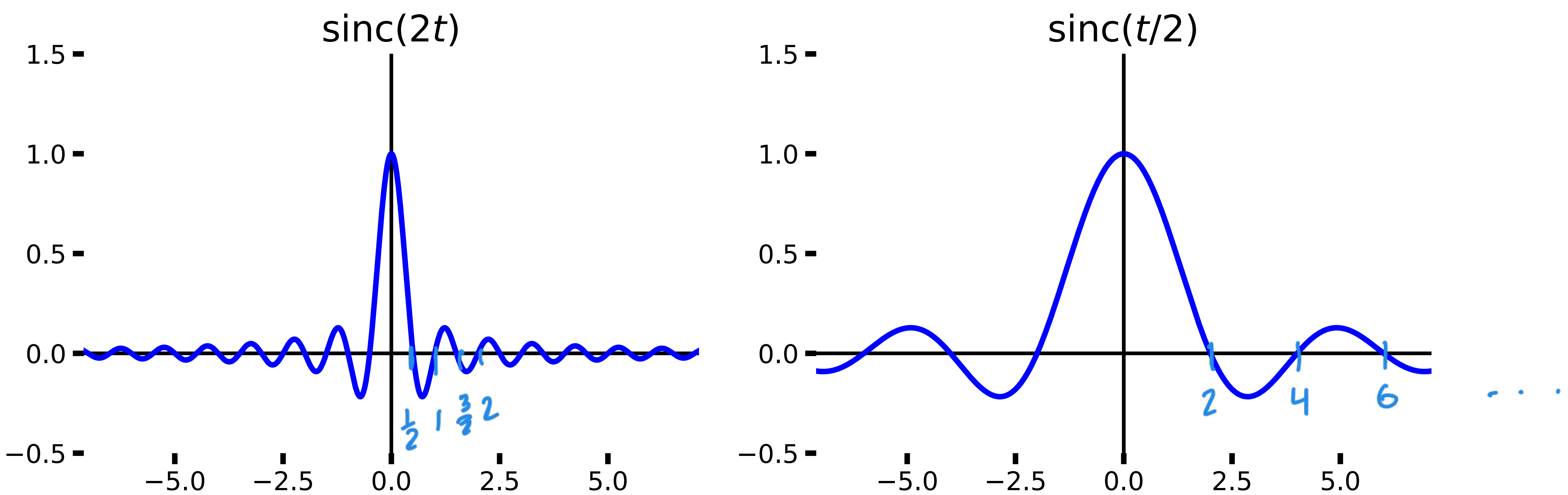
Sinc (0) = 1

What about $\mathrm{sinc}(\alpha t)$?

$$\mathrm{sinc}(\alpha t) = \frac{\sin(\alpha \pi t)}{\alpha \pi t} \tag{3}$$

which is 0 when $\alpha \pi t = \pi k$ or $t = k/\alpha$.

Compression and dilation



If we look at the scaled function $\operatorname{sinc}(\alpha t)$ we keep the same value at 0 but move the zero crossings to $\frac{k}{|\alpha|}$ for $k \in \mathbb{Z}$.

The sinc in DT

The sinc in DT is defined similarly way:

$$\mathrm{sinc}[n] = \frac{\sin(\pi n)}{\pi n} \tag{4}$$

This is just a *sampled* version of the CT sinc function. We will more often write it out instead of using the $\mathrm{sinc}[\cdot]$ shorthand, so you will see

$$h[n] = \frac{\sin((\pi/4)n)}{\pi n} \qquad \frac{\sin(\omega_0 n)}{\pi n} \tag{5}$$

instead. Try to determine the value at $n = 0$ by taking the limit of the CT version.

of $h[0]$