Linear Systems and Signals

Energy of signals

Anand D. Sarwate

Department of Electrical and Computer Engineering
Rutgers, The State University of New Jersey

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Learning objectives

The learning objectives for this section are:

- compute the energy of real and complex signals

Energy of a signal: what does it mean

We can model all sorts of phenomena as signals:

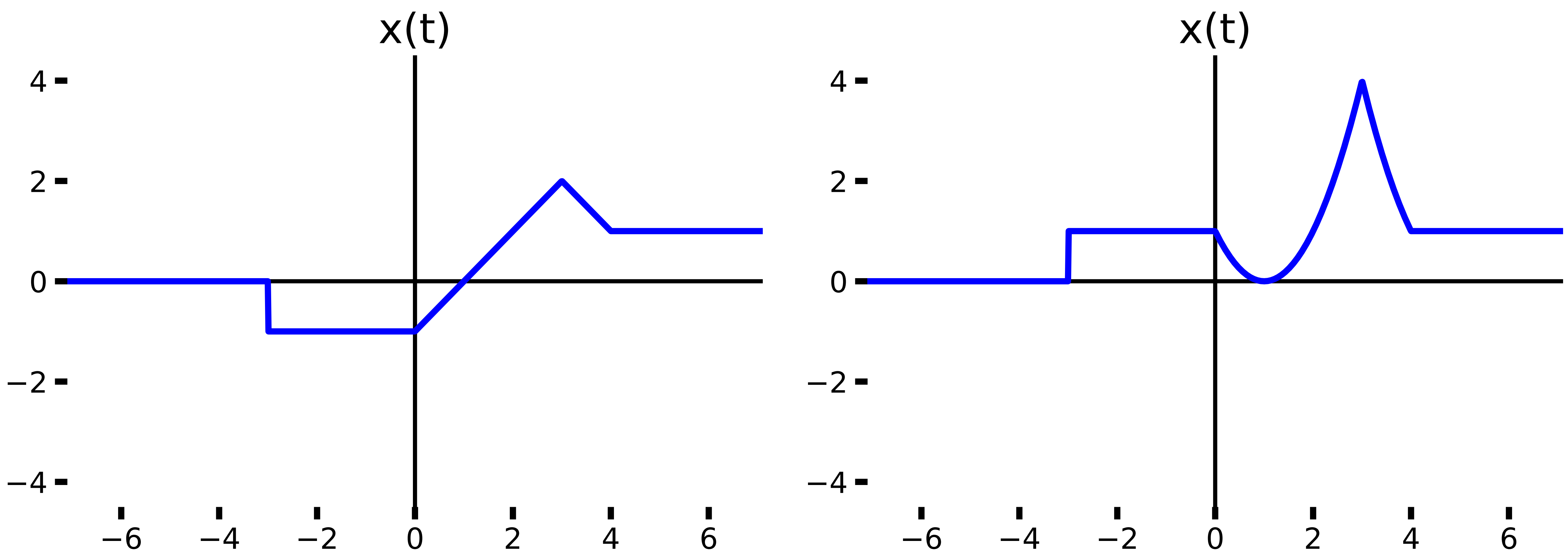
- What do we mean by the energy of a signal?
- Is it measured in Joules?
- Are there some sort of conservation laws?

We will define the energy of a signal *as if* the signal were a voltage going over a 1Ω resistor:

$$\mathcal{E}_v = \int_{-\infty}^{\infty} \frac{V(t)^2}{R} dt = \int_{-\infty}^{\infty} V(t)^2 dt \tag{1}$$

We need to generalize this to complex signals too.

Instantaneous power



The *instantaneous power* of a signal $x(t)$ (or $x[n]$) is

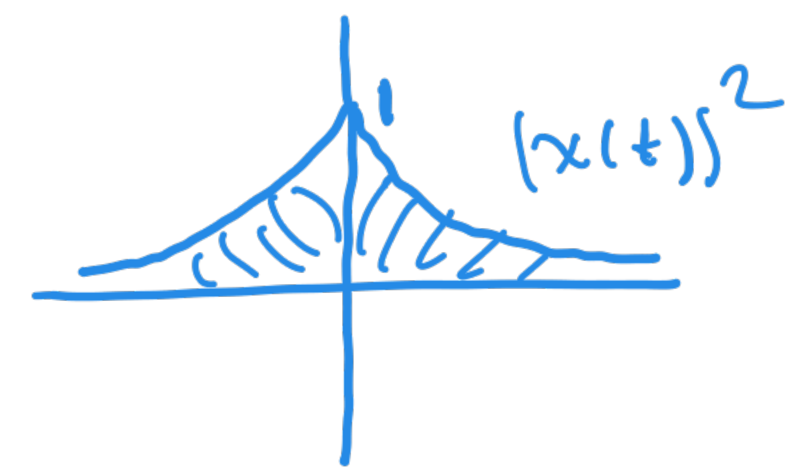
$$|x(t)|^2 \qquad |x[n]|^2. \tag{2}$$

The energy is the power integrated (summed) over time:

$$\mathcal{E}_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad \sum_{n=-\infty}^{\infty} |x[n]|^2. \tag{3}$$

Example 1: finite energy

Take the signal $x(t) = e^{-|t|}$. The instantaneous power is $e^{-2|t|}$. Then energy of this signal is



$$
\begin{aligned}
\mathcal{E}_x &= \int_{-\infty}^{\infty} |x(t)|^2 dt \\
&= \int_{-\infty}^{\infty} e^{-2|t|} dt \\
&= 1
\end{aligned}
$$

Example 2: complex signal

Find the energy of the signal

$$x(t) = e^{-|t|}\cos(\omega_0 t) + je^{-|t|}\sin(\omega_0 t)$$

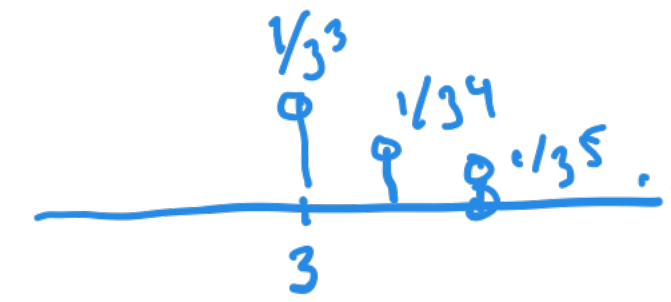
mag 1

Eulerizing, we get $x(t) = e^{-|t|}e^{j\omega_0 t}$. the magnitude squared (instantaneous power) is $e^{-2|t|}$ so

$$\begin{aligned}\mathcal{E}_x &= \int_{-\infty}^{\infty} e^{-2|t|}\,dt \\ &= 1\end{aligned}$$

Example 3: real DT signal

Find the energy of the signal



$$x[n] = \left(\frac{1}{3}\right)^n u[n-3]. \tag{4}$$

We start by applying the definition and then using the *geometric series*.

$$\mathcal{E}_x = \sum_{k=3}^{\infty} \left(\left(\frac{1}{3}\right)^k \right)^2 \tag{5}$$

$$= \sum_{k=3}^{\infty} \left(\frac{1}{9}\right)^k \tag{6}$$

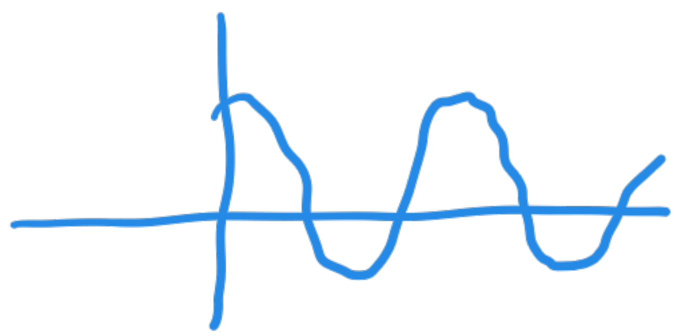
$$= \frac{1}{9^3} \sum_{k=0}^{\infty} \left(\frac{1}{9}\right)^k \tag{7}$$

$$= \frac{1/9^3}{1 - \frac{1}{9}}. \tag{8}$$

Example 4: a sinusoid

Find the energy of the signal

$$x(t) = \cos(10\pi t)u(t). \tag{9}$$



Let's try it:

$$\mathcal{E}_x = \int_0^{\infty} \cos^2(10\pi t)dt \tag{10}$$

$$= \infty. \tag{11}$$

This signal has infinite energy! $\Rightarrow$ what do we do?

Try it yourself

Problem

Take any of the signals that we saw earlier in class. Can you compute their energies? Can you tell whether the energy is finite or infinite?