Linear Systems and Signals

Periodic signals in DT

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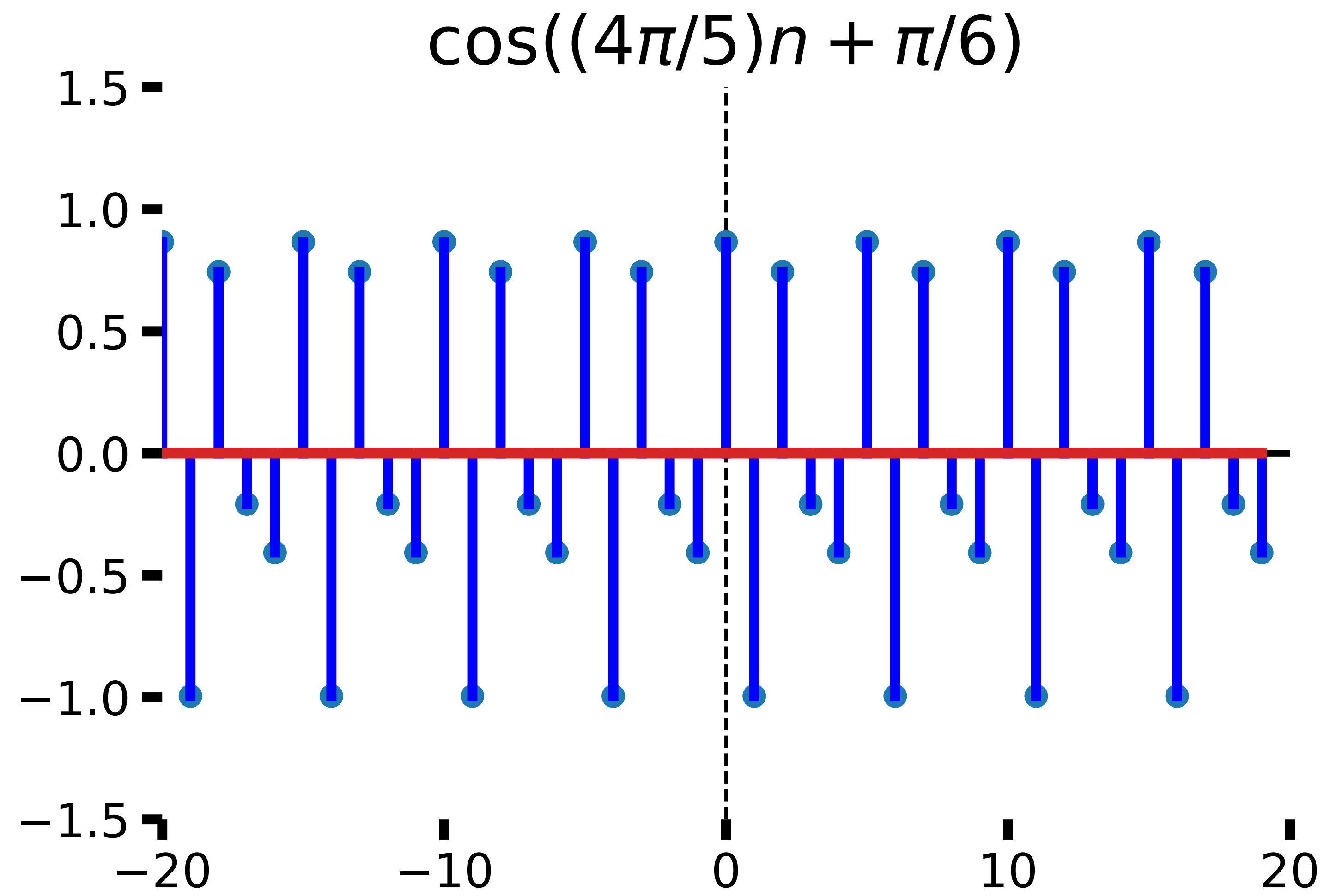
Learning objectives

The learning objectives for this section are:

- determine if a DT signal is periodic or not
- find the period and fundamental angular frequency of periodic DT signals

Definition



A DT signal $x[n]$ is periodic if there is an integer time shift N_0 such that

$$x[n + N_0] = x[n] \qquad \text{for all } n \tag{1}$$

The *fundamental period* of $x[n]$ is the smallest such N_0. The *fundamental angular frequency* is $\frac{2\pi}{N_0}$.

DT sinusoids

DT signals don't have the same periodicity properties as CT signals. Let's take two different sinusoidal examples:

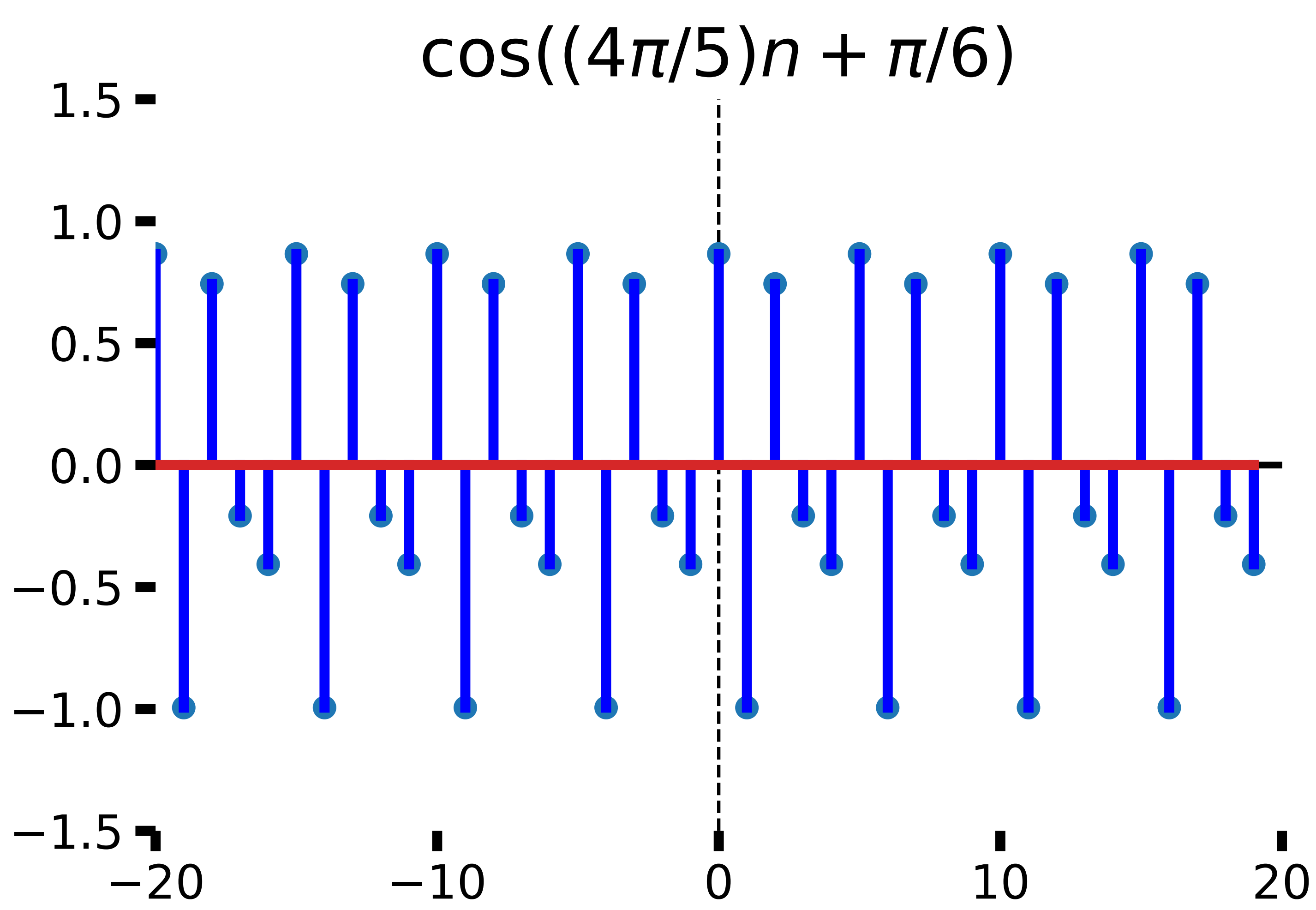
$$x_1[n] = \cos((4/5)\pi n + \pi/6) \tag{2}$$
$$x_2[n] = \sin((3/4)n + \pi/7) \tag{3}$$

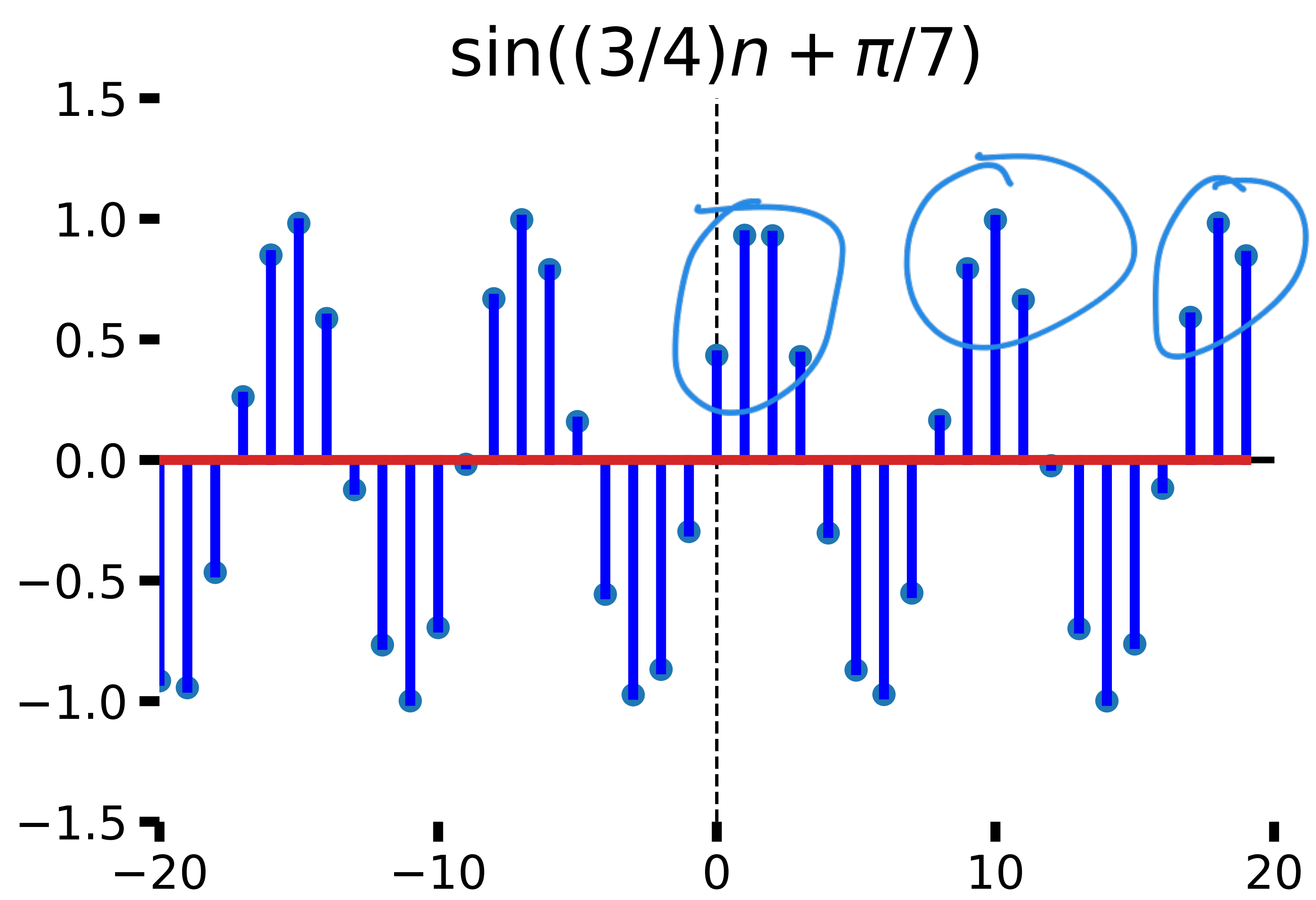
The first signal is periodic. We need to look to see for what time n the term $\frac{4}{5}\pi n$ is equal to a multiple of 2π: for that value of n, $x[n] = \cos((4/5)\pi n + \pi/6) = x_1[0]$. So $N_1 = 5$.

The second signal is not periodic. If we look at the sequence $\frac{3}{4}n$ we cannot find an integer n such that $\frac{3}{4}n = 2\pi$.

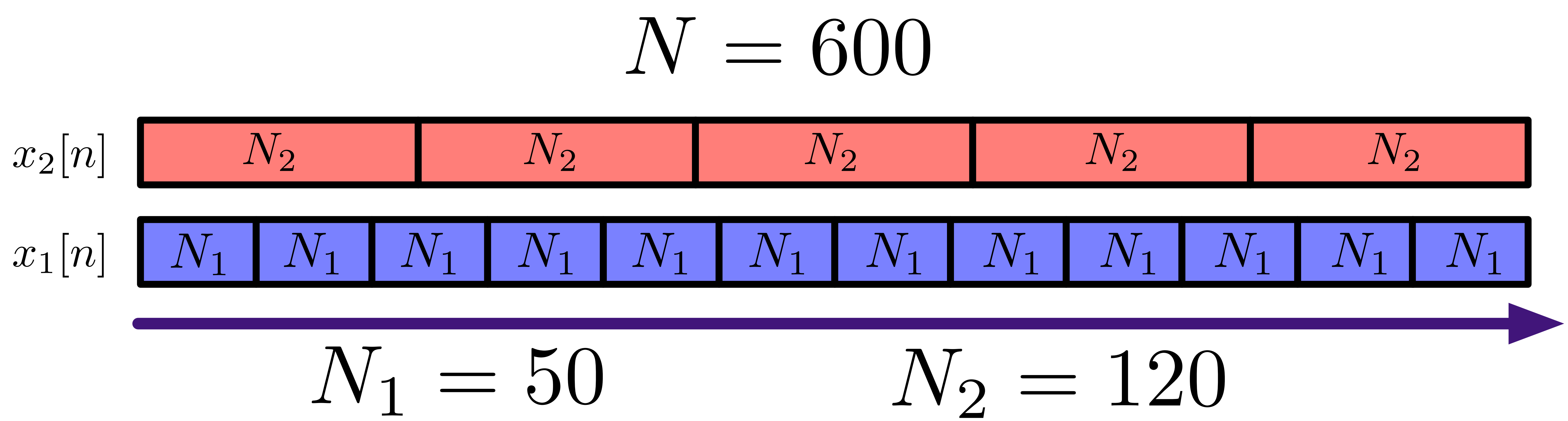
For a sinusoid of form $\cos(\omega n + \phi)$ where ω doesn't have a π in it, you should immediately suspect its not periodic.

Sinusoids in pictures





Sums of DT periodic signals



Suppose $x_1[n]$ is periodic with fundamental period N_1 and $x_2[n]$ is periodic with fundamental period N_2. What can we say about $x[n] = x_1[n] + x_2[n]$?

Find integers M_1 and M_2 to make $N_1 M_1 = N_2 M_2$. Do you see a connection to the least common multiple?

Examples

Problem

Is $x_1[n] = \cos((\pi/3)n + 2) + 2\sin((\pi/8)n + 5)$ *periodic? If so, what is its period?*

We have $\omega_1 = \frac{\pi}{3}$, $\omega_2 = \frac{\pi}{8}$ so $N_1 = 6$, $N_2 = 16$ so the least common multiple is 48. So it's periodic with period $N_0 = 48$.

Problem

Is $x_1[n] = \cos(n/4 + 2) + 2\sin((\pi/8)n + 5)$ *periodic? If so, what is its period?*

We have $\omega_1 = \frac{1}{4}$ so the first signal is itself not periodic.

One more example

Problem

Is $x_1[n] = \cos((2\pi/5)n + 2) + 2\sin((2\pi/7)n + 5)$ *periodic? If so, what is its period?*

We have $\omega_1 = \frac{2\pi}{5}$, $\omega_2 = \frac{2\pi}{7}$ so $N_1 = 5$, $N_2 = 7$ so the least common multiple is 35. So it's periodic with period $N_0 = 35$.

Try it out

Problem

Determine whether each of these signals is periodic. If it is, find the fundamental period and angular frequency.

$$x_1[n] = |\cos((\pi/7)n)| \tag{4}$$

$$x_2[n] = e^{-j(\pi/5)n} + e^{j(3\pi/10)n} \tag{5}$$

$$x_3[n] = e^{j(1/3)n} + e^{j(\pi/5)n} \tag{6}$$

Make up some of your own!