Linear Systems and Signals
Power of signals

Anand D. Sarwate

Department of Electrical and Computer Engineering
Rutgers, The State University of New Jersey

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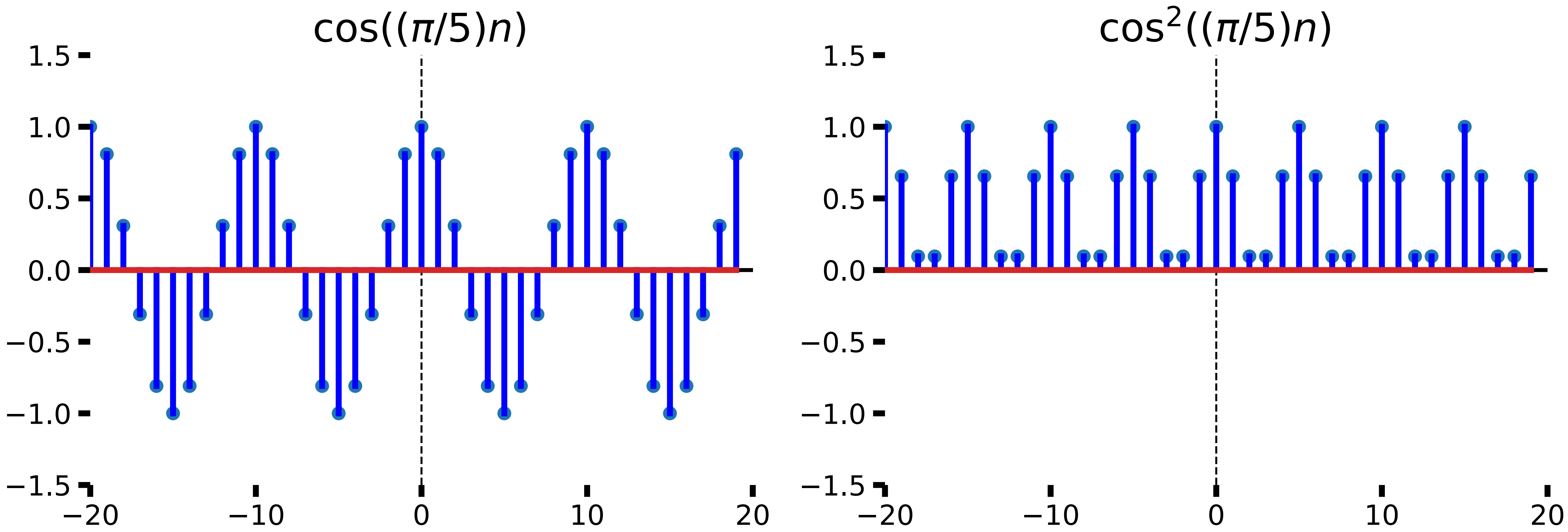
Learning objectives

The learning objectives for this section are:

- compute the power of real and complex signals

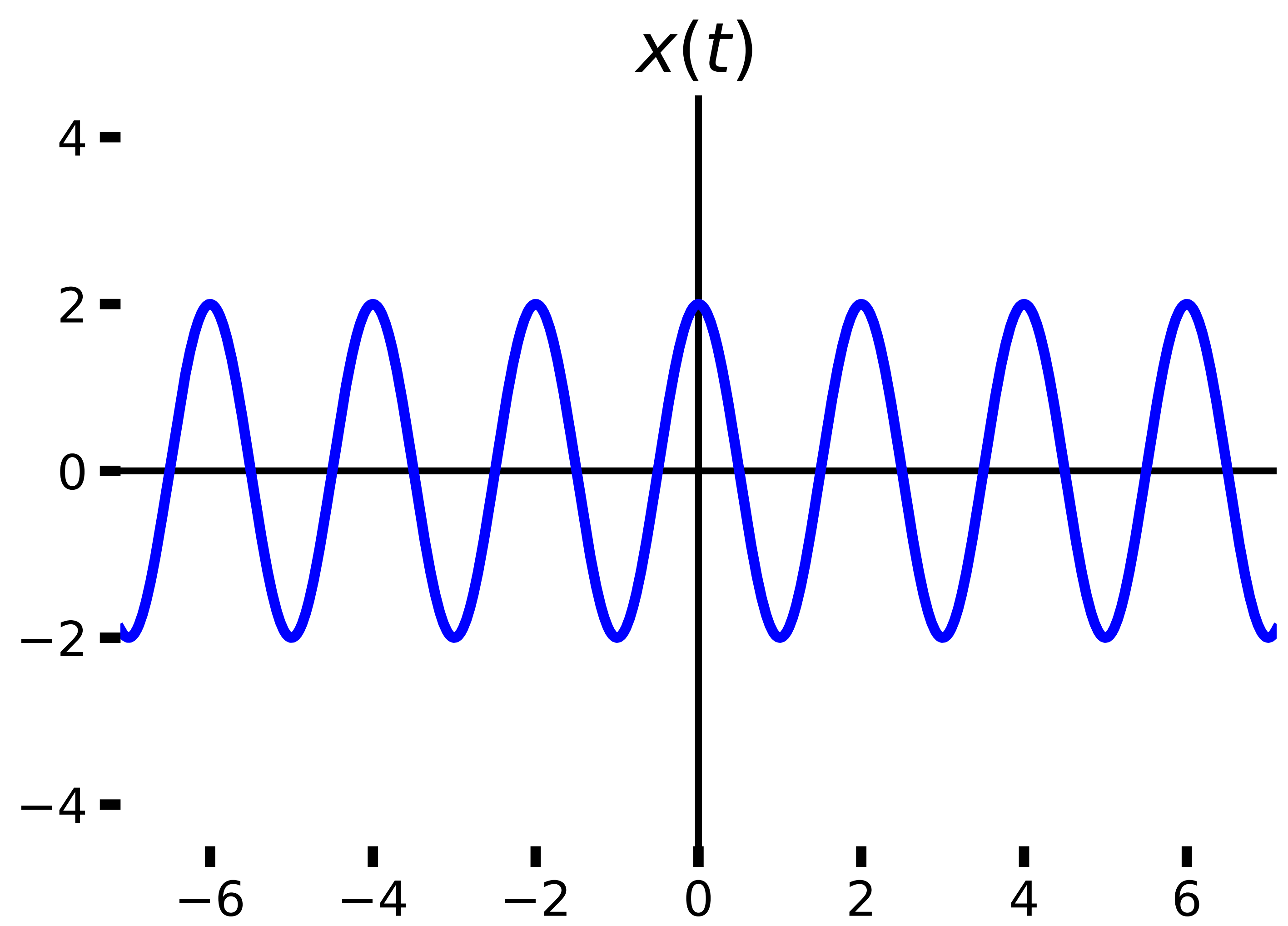
Signals can have infinite energy



We saw before that sinusoids have infinite energy ($\mathcal{E}_x = \infty$ for $x(t) = \cos(10\pi t)$. We see the same phenomenon in DT. If $x[n] = \cos((\pi/5)n)$ then

$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} \cos^2((\pi/5)n) = \infty. \tag{1}$$

Power is energy per unit time



The power of a signal is that *average energy per unit time*:

$$\mathcal{P}_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \tag{2}$$

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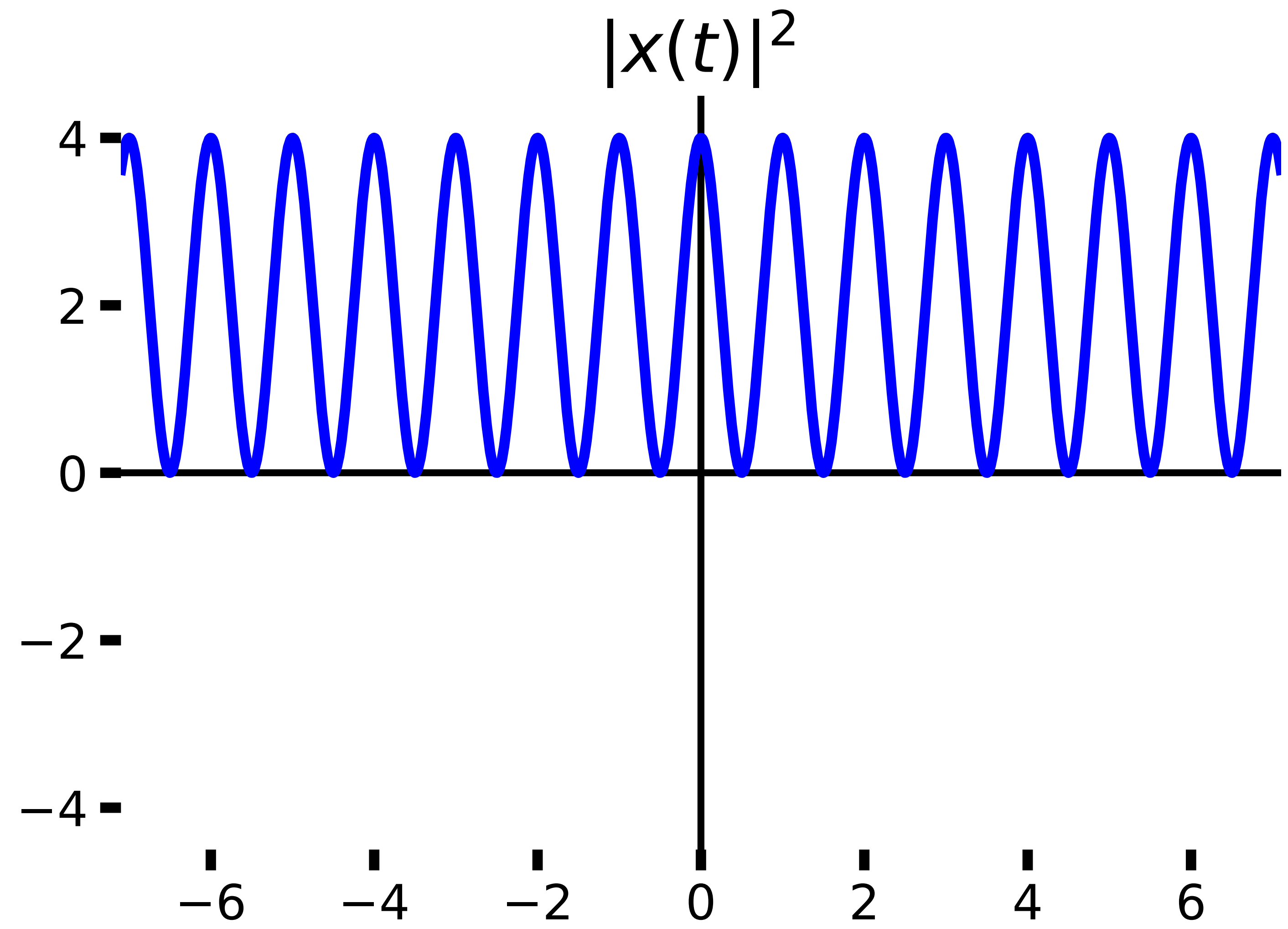

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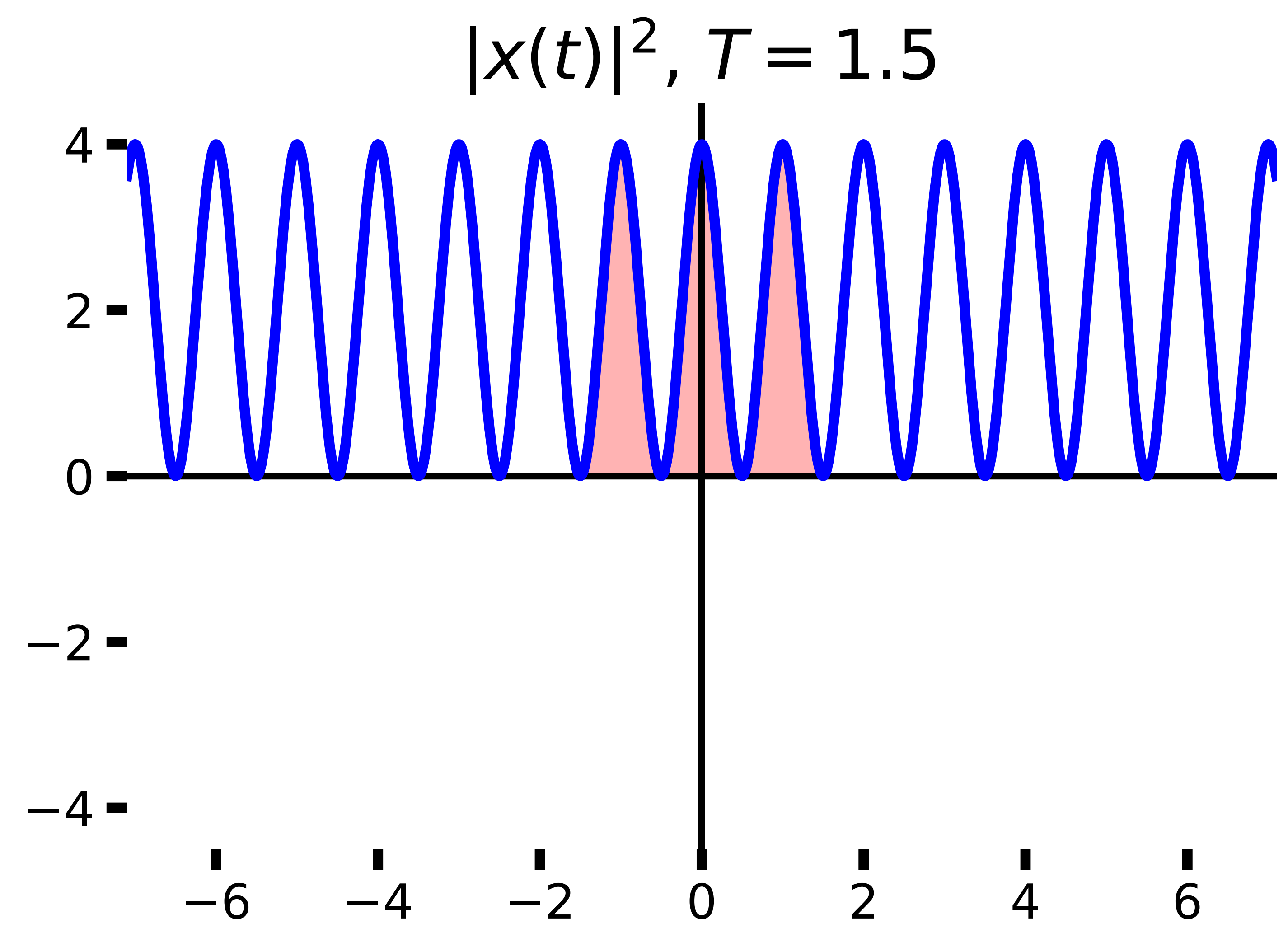

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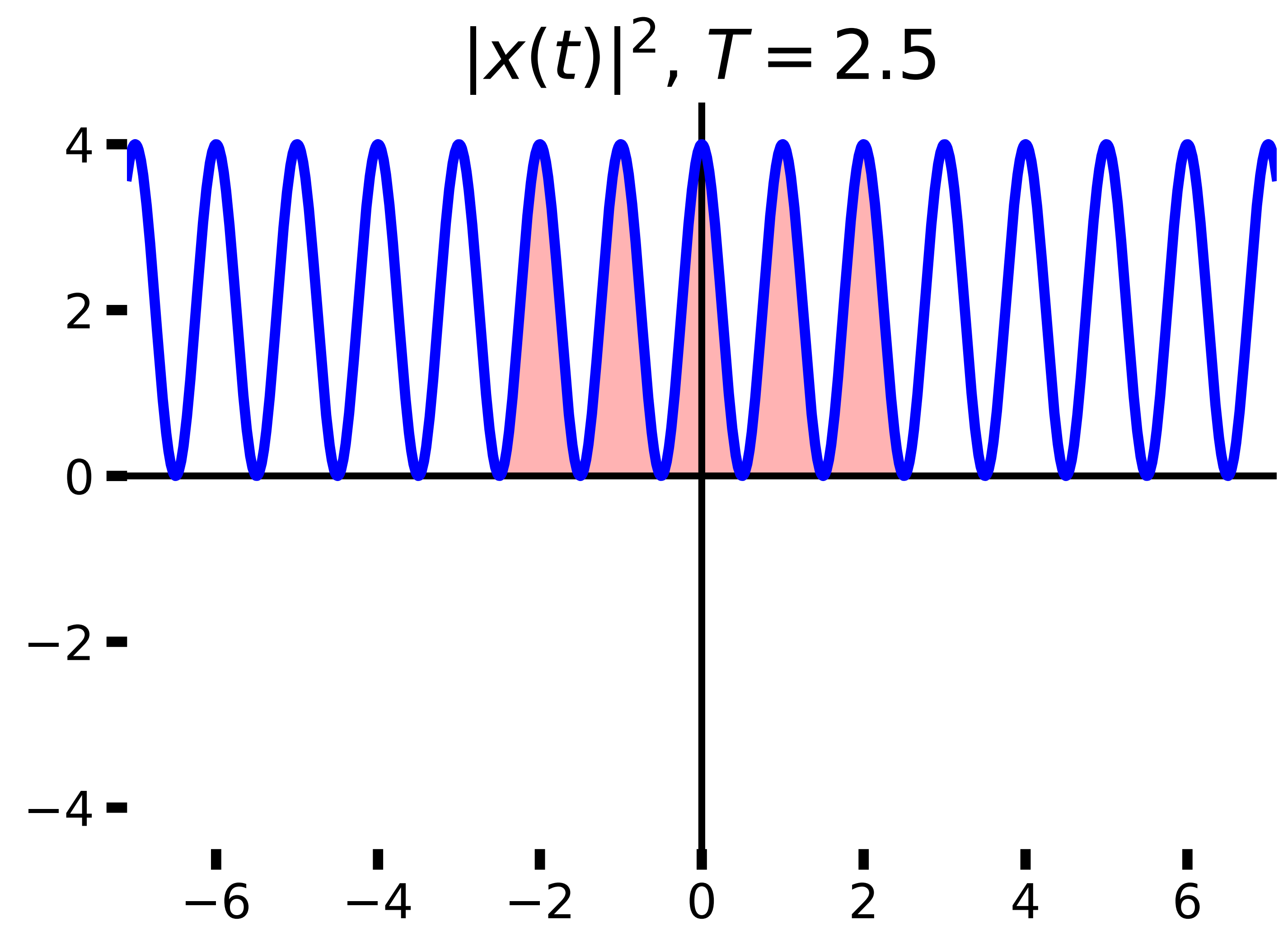

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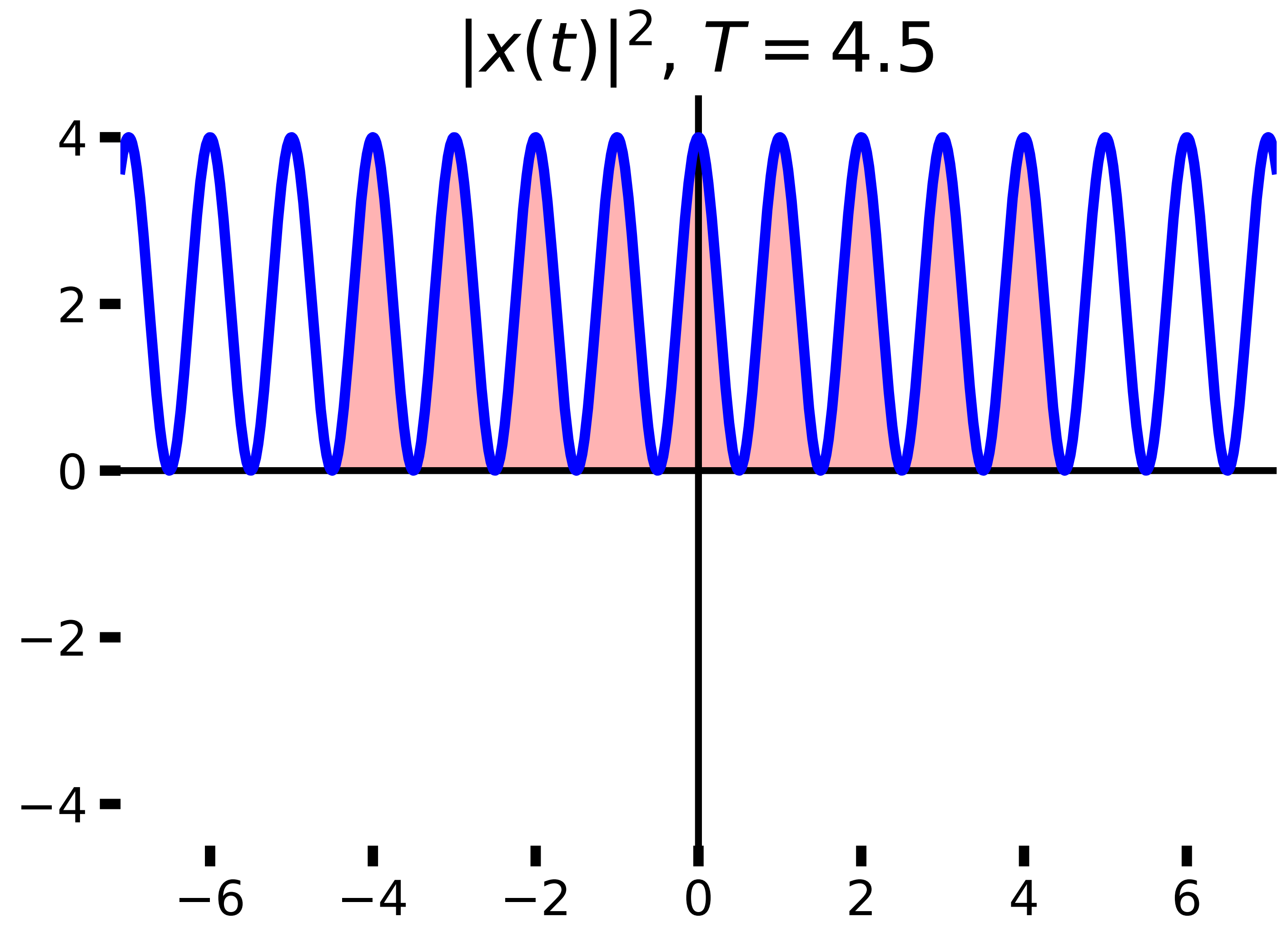

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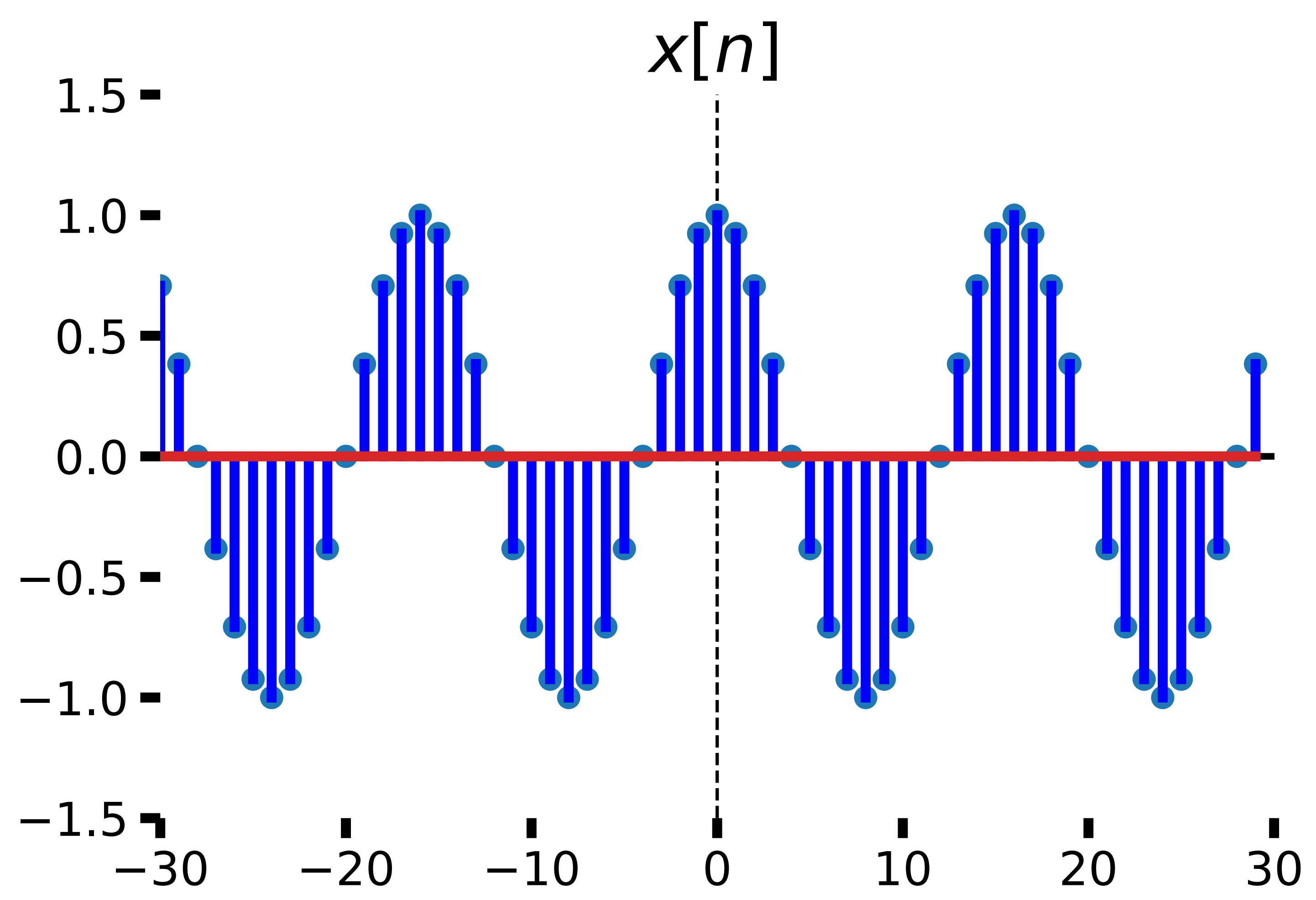
Power of a signal in DT



For DT we have to account for the signal at $n = 0$:

$$\mathcal{P}_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{3}$$

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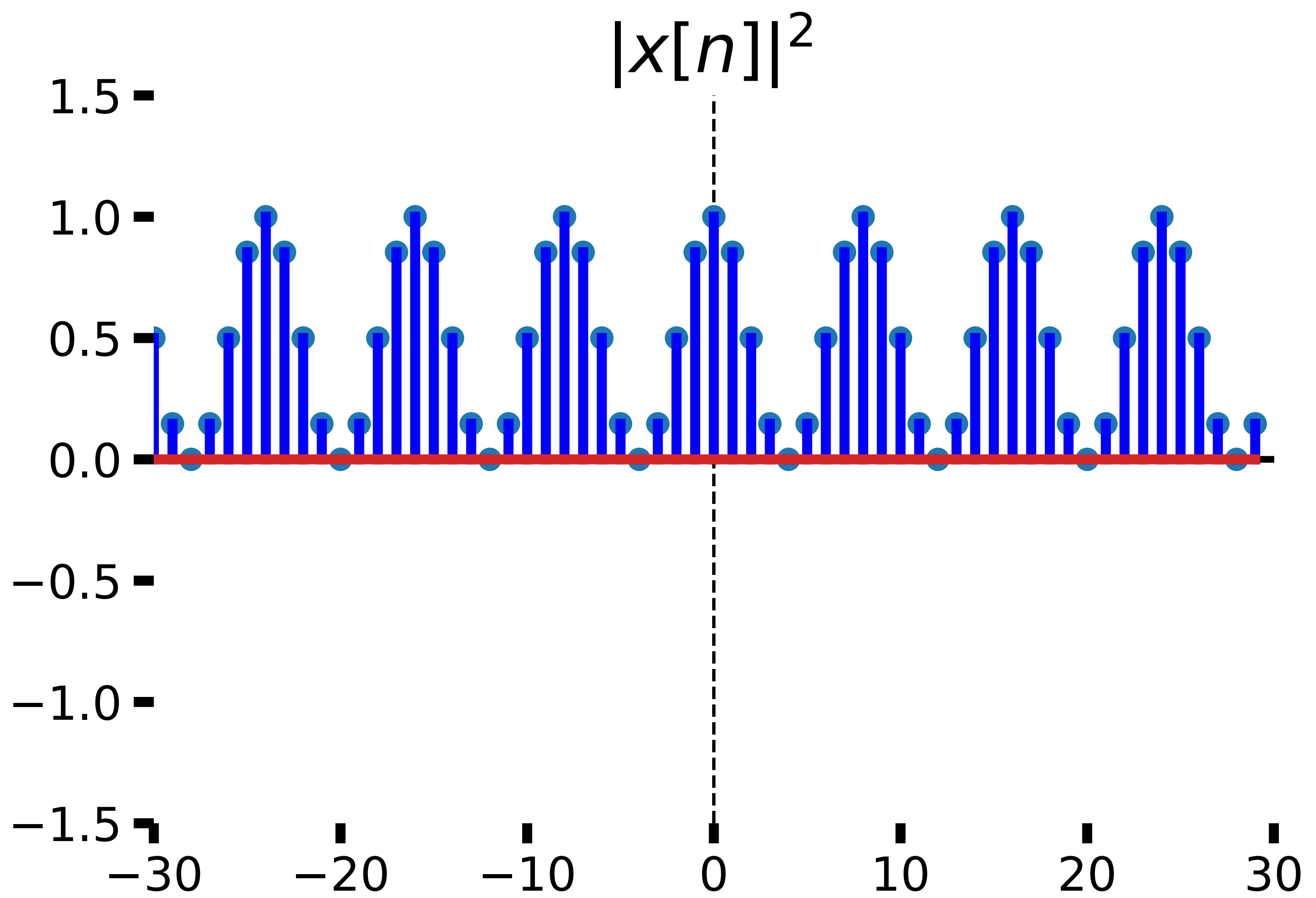

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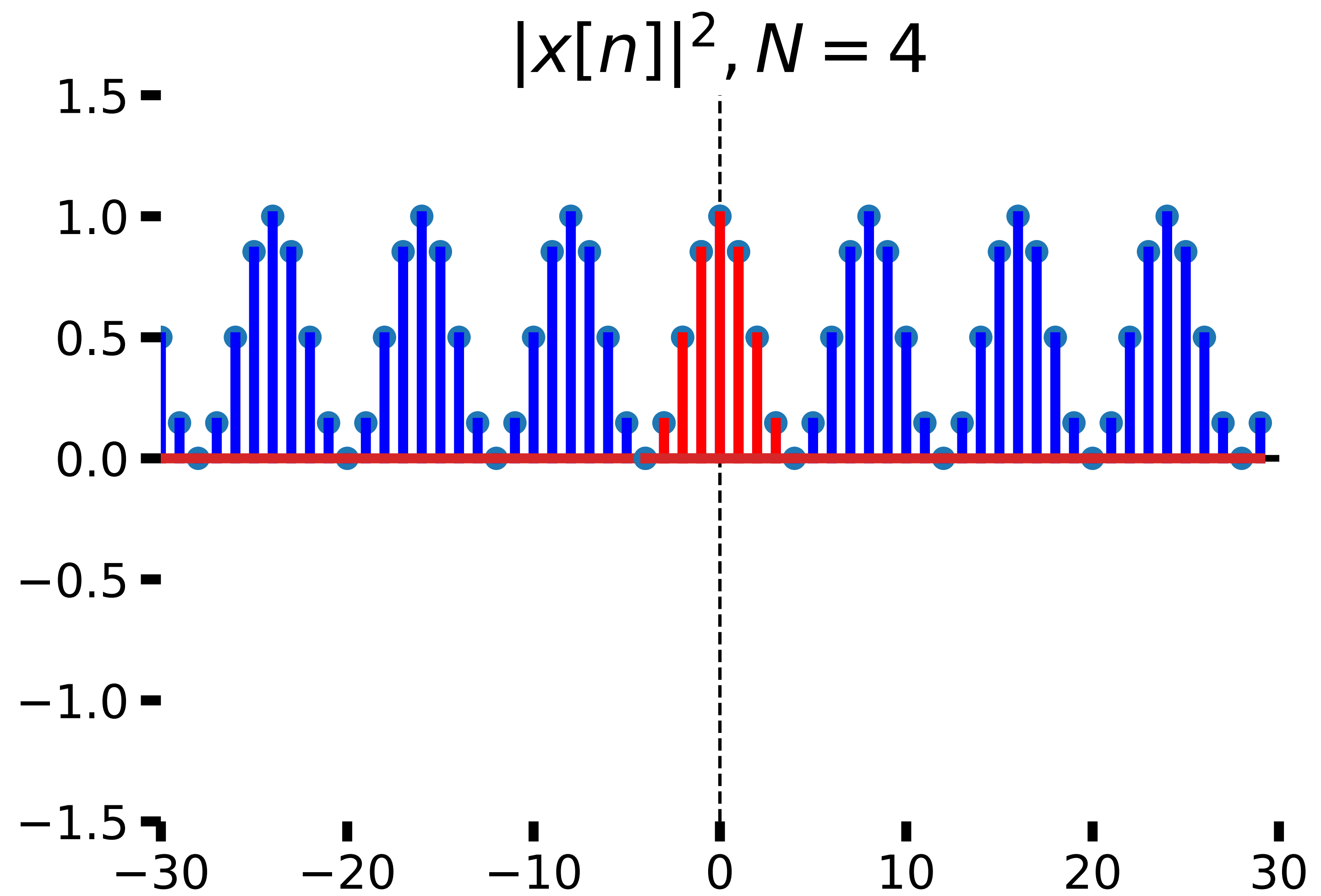

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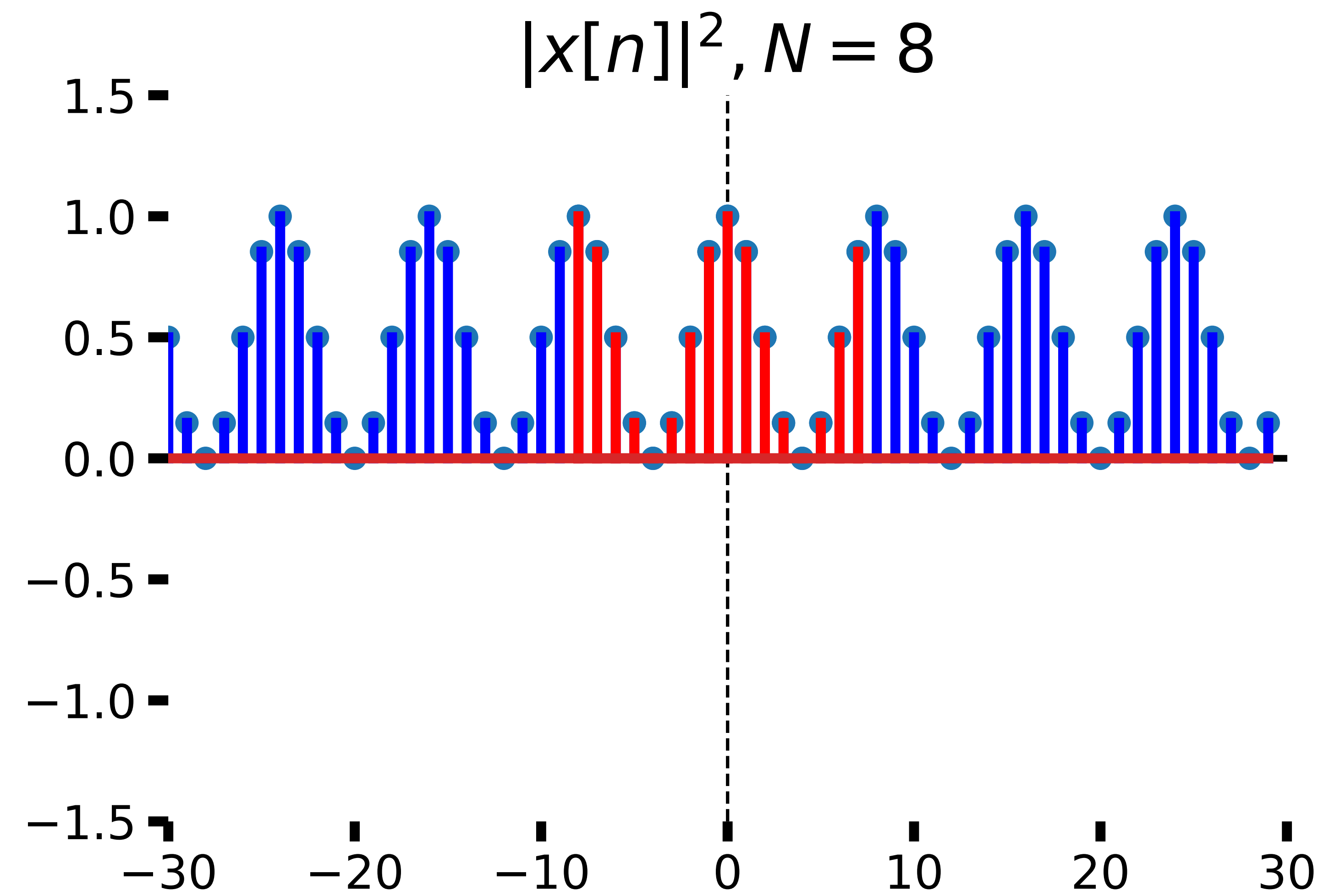

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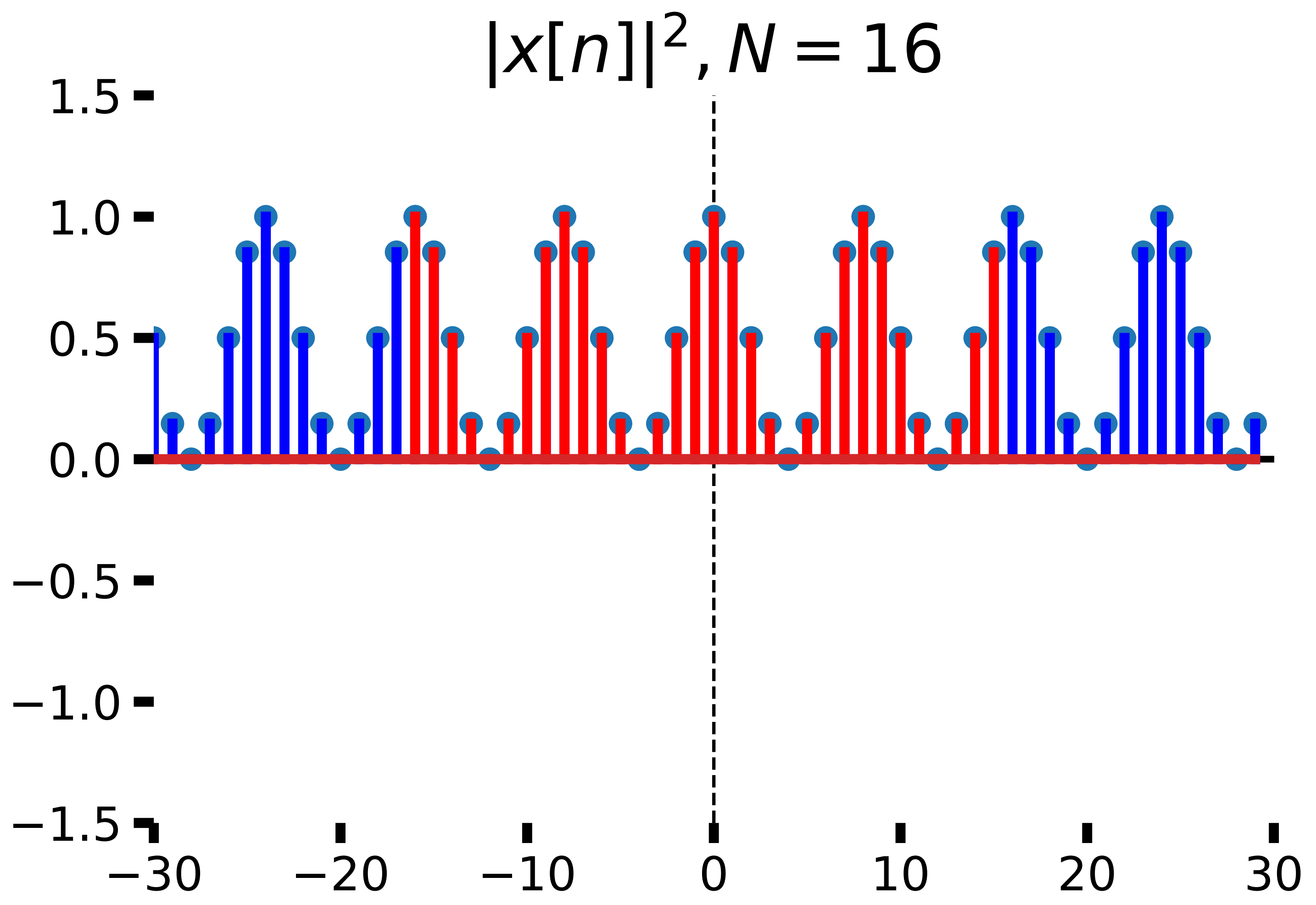

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CT periodic signals

Suppose $x(t)$ is periodic with period T_0. Then we can look at integer multiples of T_0:

$$\begin{aligned}
\mathcal{P}_x &= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt && (4) \\
&= \lim_{K \to \infty} \frac{1}{2KT_0} \int_{-KT_0}^{KT_0} |x(t)|^2 dt && (5) \\
&= \lim_{K \to \infty} \frac{1}{2KT_0} 2K \int_{0}^{T_0} |x(t)|^2 && (6) \\
&= \frac{1}{T_0} \int_{0}^{T_0} |x(t)|^2. && (7)
\end{aligned}$$

The power of a periodic signal is the average instantaneous power over one period.

Example: complex exponentials

Complex periodic exponentials are a good example of power-type signals. The power of $x(t) = e^{j\omega_0 t}$ is the energy in a single period. Since the magnitude $|x(t)| = 1$:

$$\mathcal{P}_x = \int_0^{T_0} \left|e^{j\omega_0 t}\right|^2 dt = \int_0^{T_0} 1 dt = T_0. \tag{8}$$

For a sinusoid $y(t) = \cos(\omega_0 t)$,

$$\mathcal{P}_x = \frac{1}{T_0} \int_0^{2\pi/\omega_0} \cos^2(\omega_0 t) dt \tag{9}$$

$$= \frac{1}{T_0} \int_0^{T_0} \frac{1}{2}(1 + \cos(2\omega_0 t)) dt \tag{10}$$

$$= 1 \tag{11}$$

DT periodic signals

Suppose $x[n]$ is periodic with period N_0. Then we can look at integer multiples of N_0:

$$\mathcal{P}_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{12}$$

$$= \lim_{K\to\infty} \frac{1}{2KN_0+1} \sum_{n=-KN_0}^{KN_0} |x[n]|^2 \tag{13}$$

$$= \frac{1}{2KN_0+1} \left(|x[0]|^2 + 2K \sum_{n=1}^{N_0} |x[n]|^2 \right) \tag{14}$$

$$= \frac{1}{N_0} \sum_{n=1}^{N_0} |x[n]|^2. \tag{15}$$

The power of a periodic signal is the average instantaneous power over one period.