Linear Systems and Signals

Linear and nonlinear systems

Anand D. Sarwate

Department of Electrical and Computer Engineering
Rutgers, The State University of New Jersey

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Learning objectives

The learning objective for this section is:

- determine if a system is linear or nonlinear

Linear systems

Definition

A CT system $\mathcal{H}$ is *linear* if

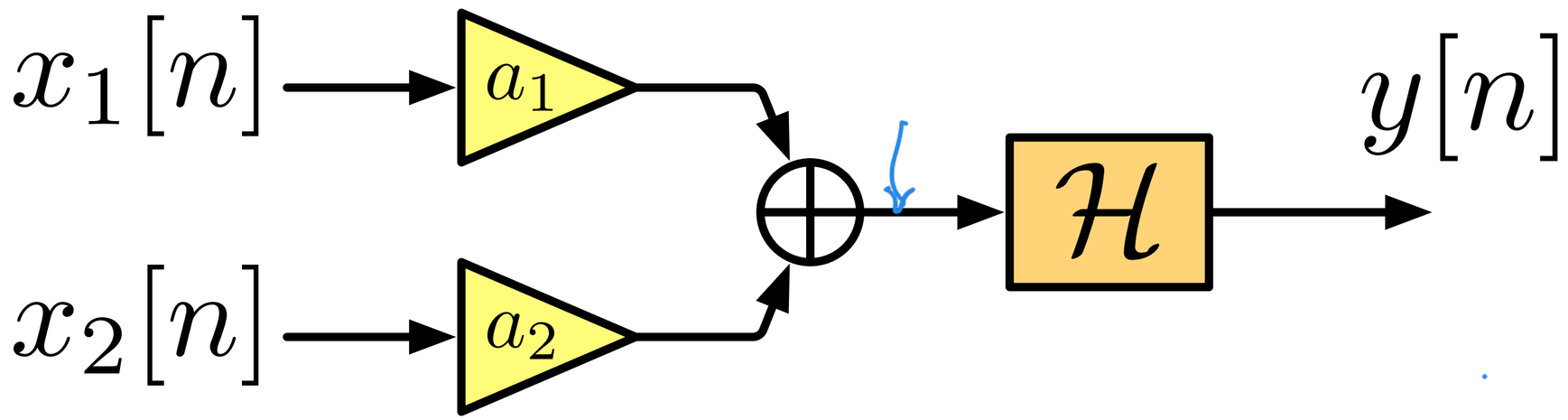
$$\mathcal{H}\left(a_1 x_1(t) + a_2 x_2(t)\right) = a_1 \mathcal{H}(x_1(t)) + a_2 \mathcal{H}(x_2(t)). \tag{1}$$

A DT system $\mathcal{H}$ is *linear* if

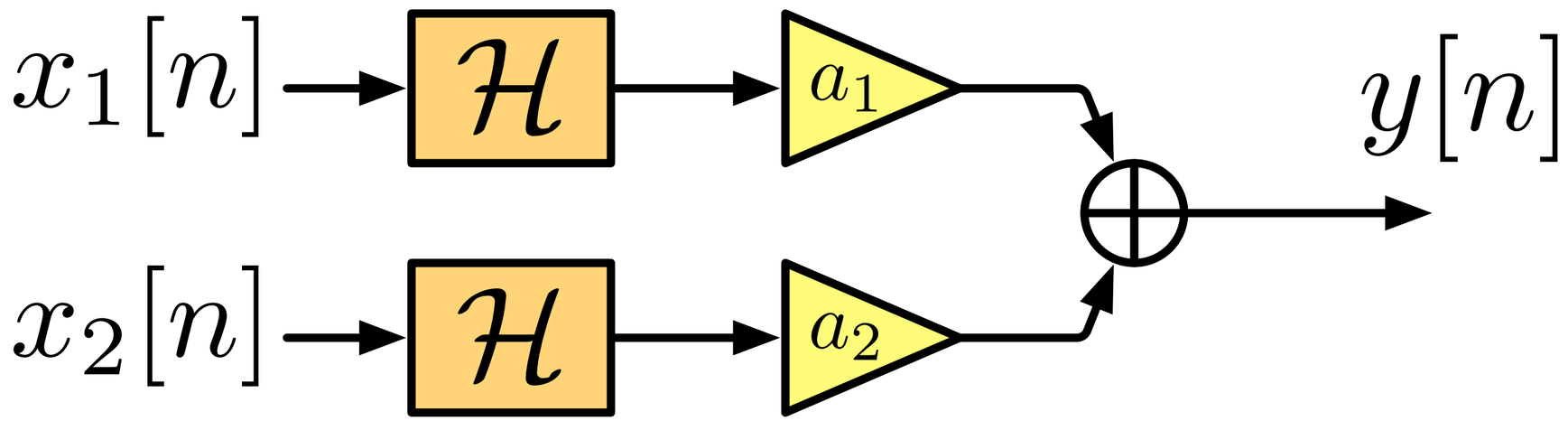
$$\mathcal{H}\left(a_1 x_1[n] + a_2 x_2[n]\right) = a_1 \mathcal{H}(x_1[n]) + a_2 \mathcal{H}(x_2[n]). \tag{2}$$

So the output of a linear combination of signals is a linear combination of the outputs of the individuals with the same coefficients.

In block diagrams

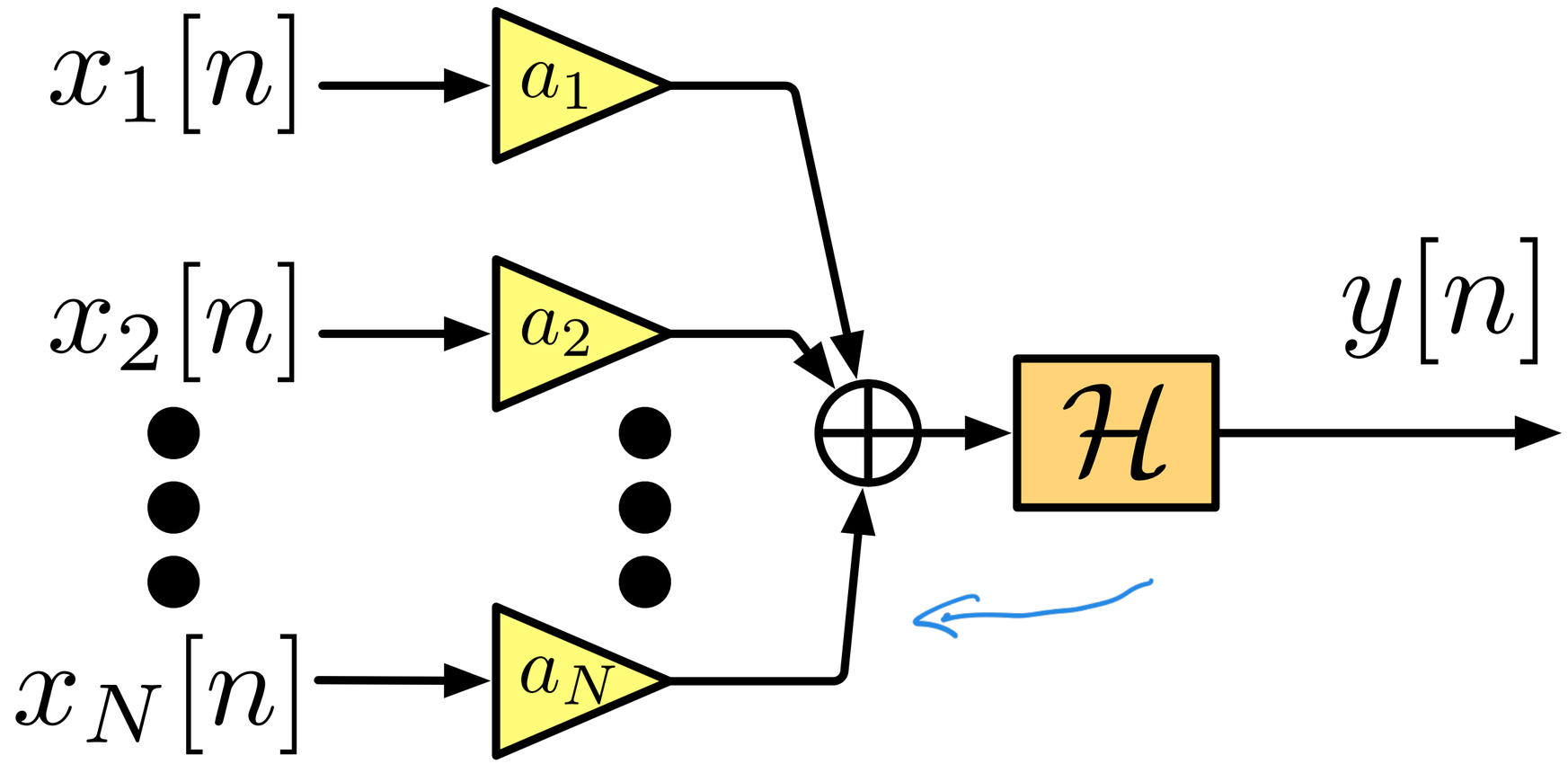


$$\mathcal{H}(a_1 x_1[n] + a_2 x_2[n]) = a_1 \mathcal{H}(x_1[n]) + a_2 \mathcal{H}(x_2[n])$$



In block diagrams

$$\mathcal{H}\left(\sum_{k=1}^{N} a_k x_k[n]\right) = \sum_{k=1}^{N} a_k \mathcal{H}(x_k[n])$$



In block diagrams

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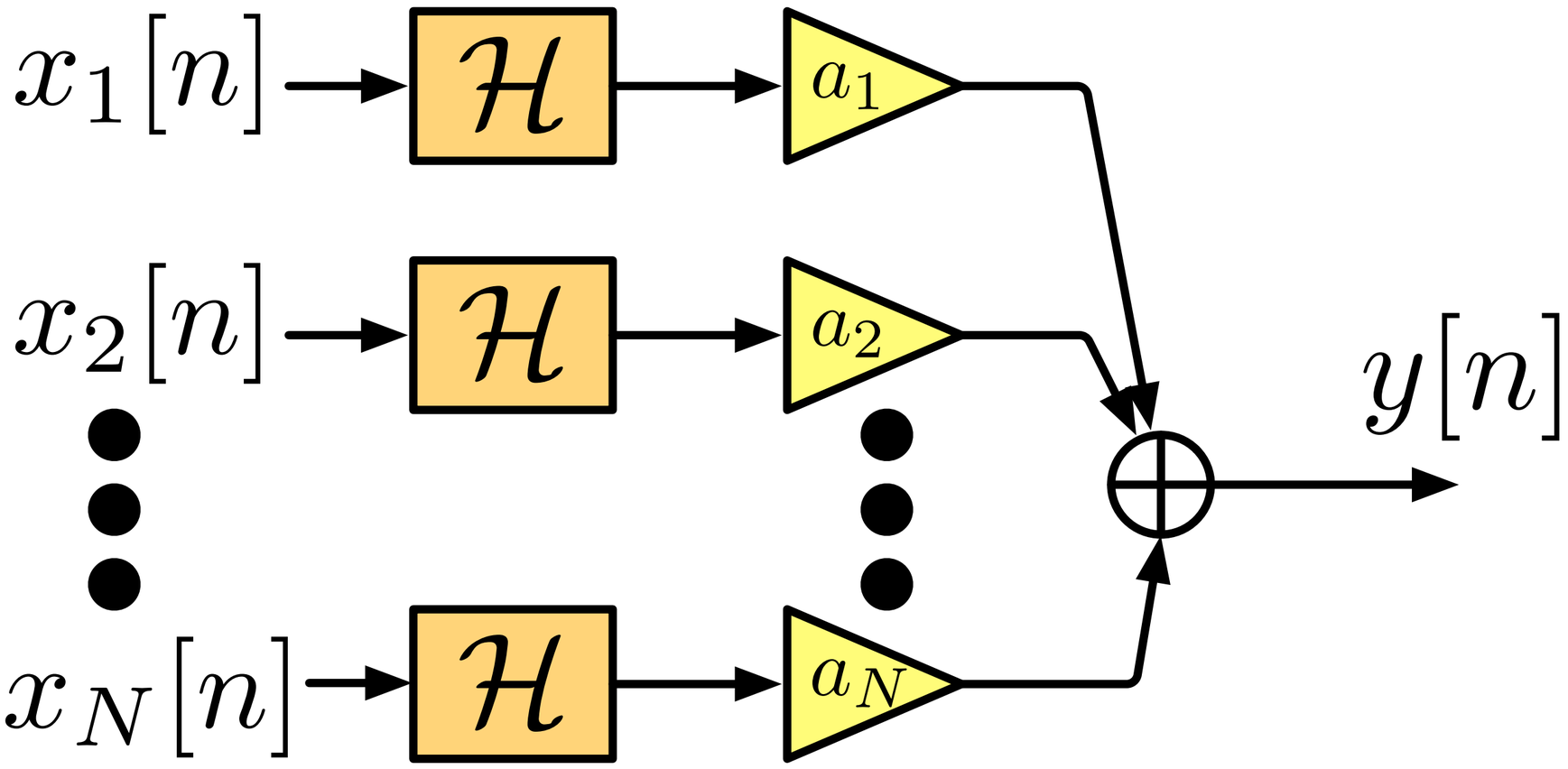
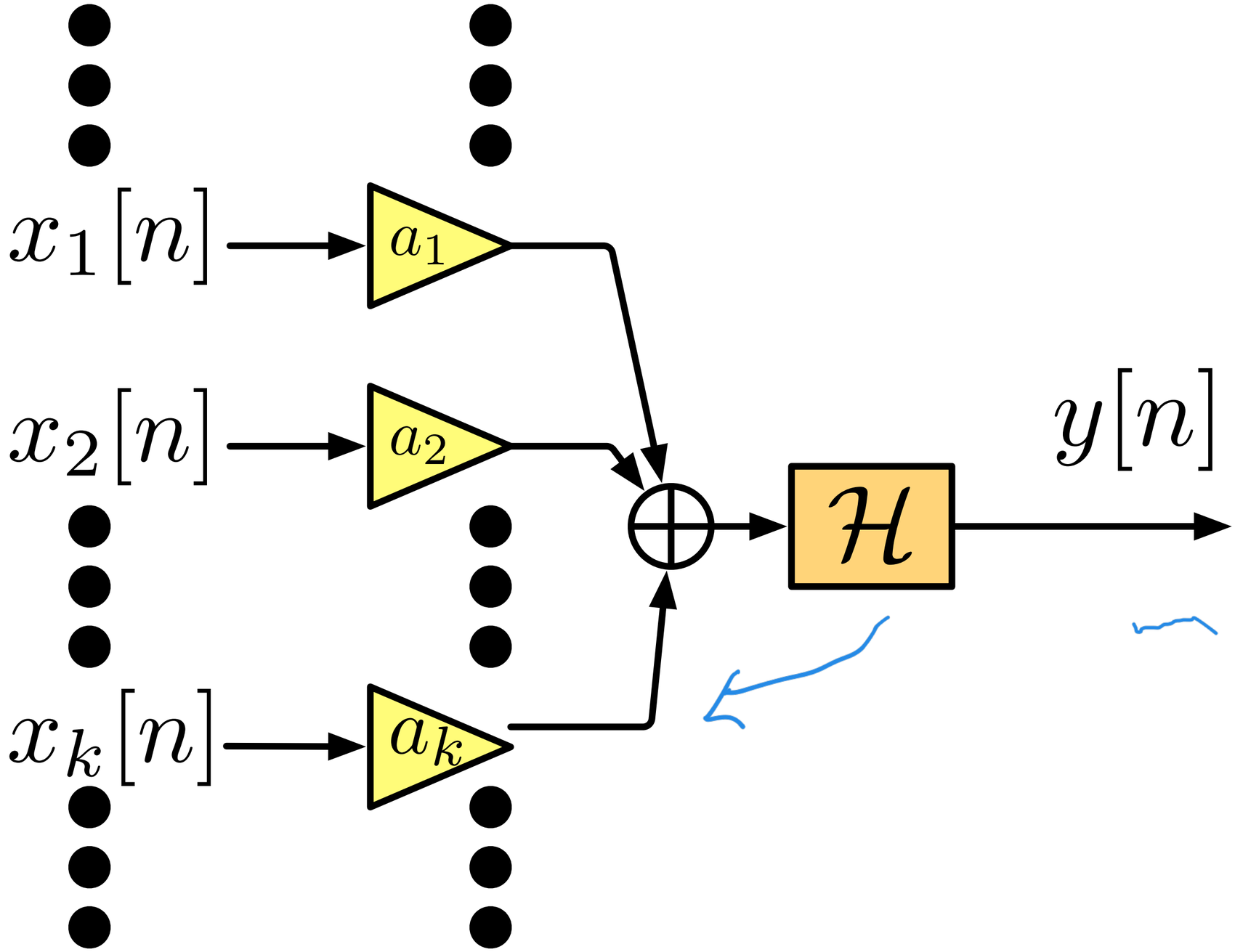
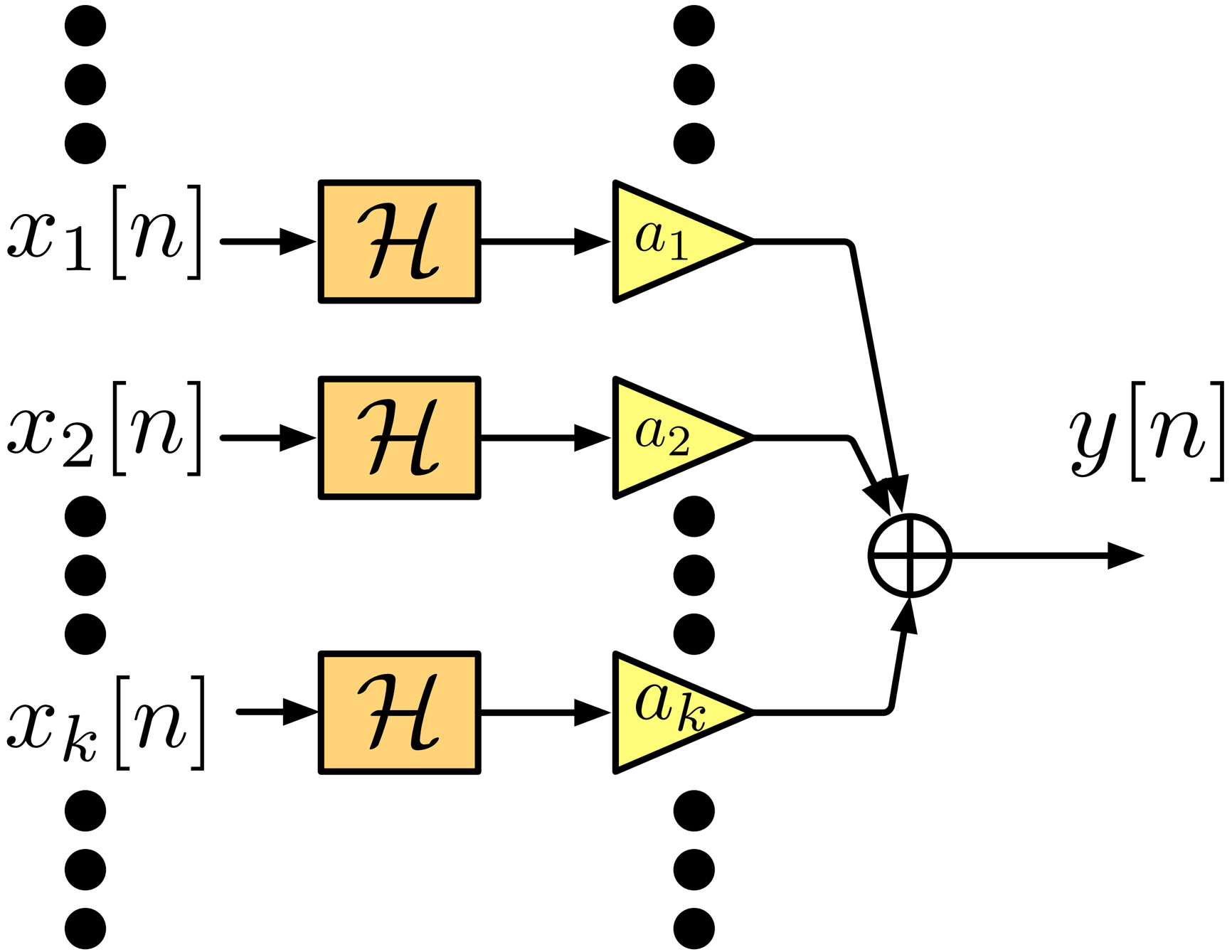


In block diagrams



In block diagrams



Checking for linearity

To check if a system is linear, check the definition:

- $\mathcal{H}(x(t)) = x(t)\cos(6000\pi t)$. Then

$$\mathcal{H}(a_1x_1(t) + a_2x_2(t)) = (a_1x_1(t) + a_2x_2(t))\cos(6000\pi t) \quad (3)$$

$$= a_1\mathcal{H}(x_1(t)) + a_2\mathcal{H}(x_2(t)). \quad (4)$$

linear!

- $\mathcal{H}(x(t)) = \cos(6000\pi x(t))$. Then

$$\mathcal{H}(a_1x_1(t) + a_2x_2(t)) = \cos(6000\pi((a_1x_1(t) + a_2x_2(t)))) \quad (5)$$

$$\neq a_1\cos(6000\pi x_1(t)) + a_2\cos(6000\pi x_2(t)) \quad (6)$$

The system has to be linear for all input signals x_1 and x_2.

Try it yourself

Problem

Determine if each of these systems is linear or nonlinear.

- *The instantaneous power:* $\mathcal{H}(x(t)) = |x(t)|^2$
- *Downsampler by* N *(a DT system).*
- $\mathcal{H}(x(t)) = x(t-2) + x(2-t)$.
- $\mathcal{H}(x(t)) = \frac{d}{dt}x(t)$.
- $\mathcal{H}(x[n]) = x_e[n]$.