Linear Systems and Signals
Even and odd signals

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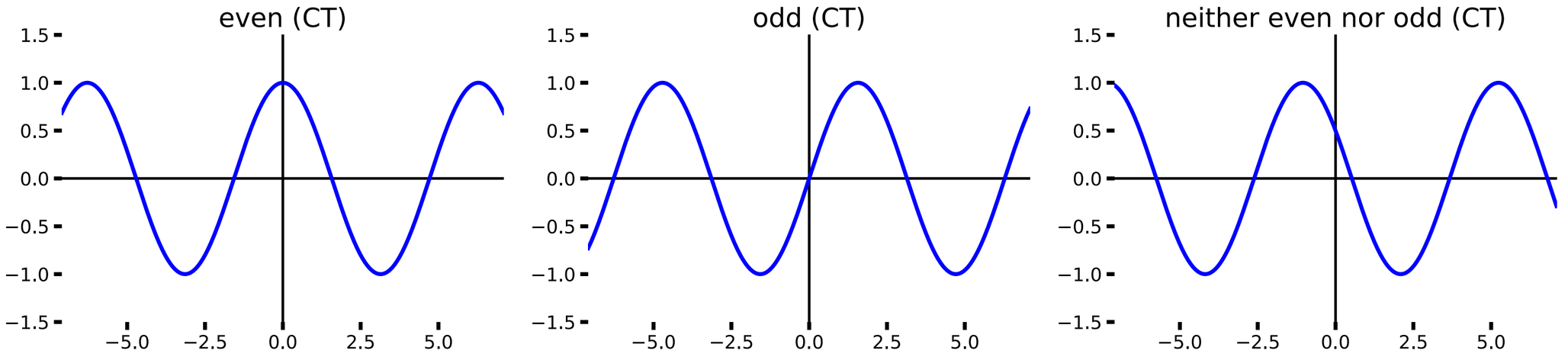
Learning objectives

The learning objectives for this section are:

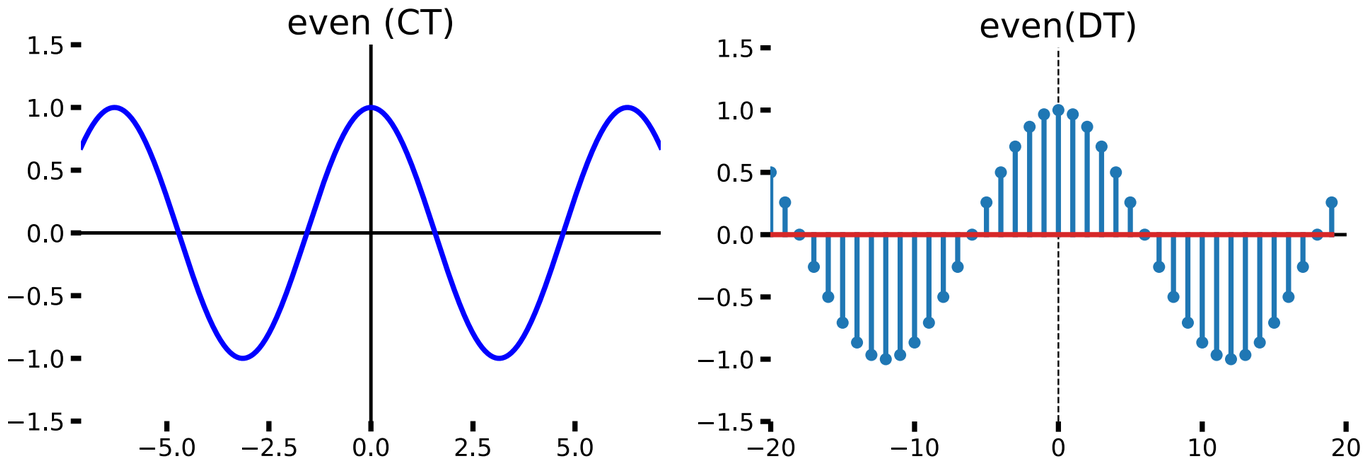
- determine whether a signal is even, odd, or neither
- calculate the even and odd parts of a signal

Symmetries in signals



- *Symmetry* turns out to be important when understanding signals using mathematics.
- We can think of these symmetries both algebraically and graphically.
- These properties are important for both CT and DT signals.

Even signals



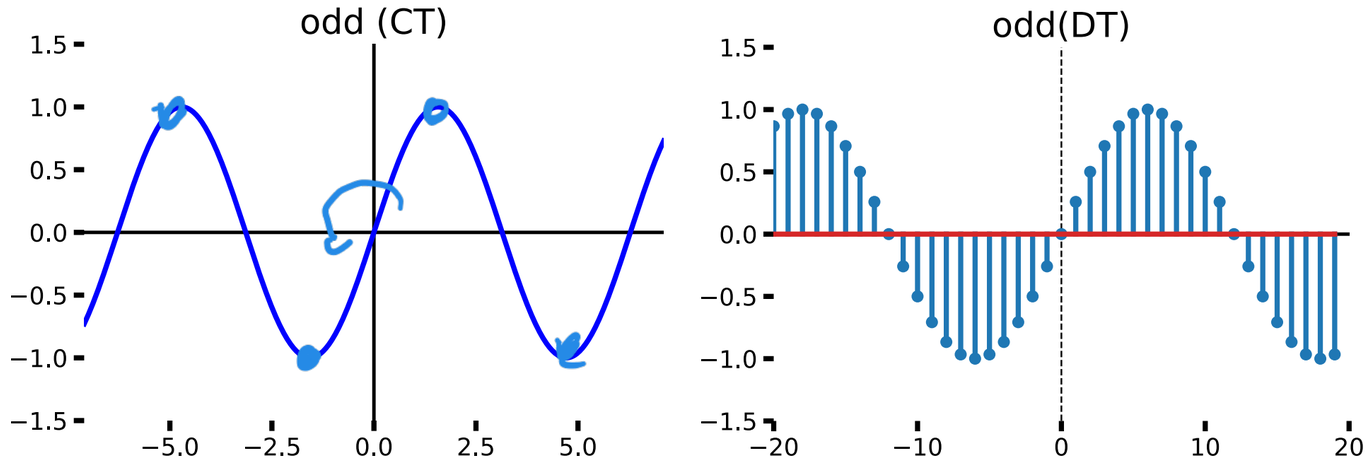
A signal is *even* if

$$x(t) = x(-t) \qquad x[n] = x[-n] \tag{1}$$

That is, if we *flip* the time axis on the graph then the signal looks the same.

To check if a signal is even just verify that the condition (1) holds.

Odd signals



A signal is *odd* if

$$x(t) = -x(-t) \qquad x[n] = -x[-n] \tag{2}$$

That is, if we *rotate* the graph by 180° then the signal looks the same. To check if a signal is even just verify that the condition (2) holds.

Decomposition

All signals can be written as a sum of even and odd signals! In CT, we have

$$x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t) \tag{3}$$

$$x_{\text{even}}(t) = \frac{x(t) + x(-t)}{2} \tag{4}$$

$$x_{\text{odd}}(t) = \frac{x(t) - x(-t)}{2} \tag{5}$$

And in DT we have

$$x[n] = x_{\text{even}}[n] + x_{\text{odd}}[n] \tag{6}$$

$$x_{\text{even}}[n] = \frac{x[n] + x[-n]}{2} \tag{7}$$

$$x_{\text{odd}}[n] = \frac{x[n] - x[-n]}{2} \tag{8}$$

Example 1

Problem

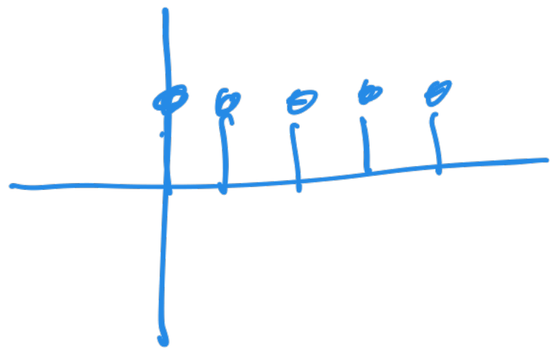
Is the signal $x(t) = \sin(t)\cos(2t)$ *even, odd, or neither even nor odd?*

Just compute $x(-t)$:

$$x(-t) = \sin(-t)\cos(-2t) = -\sin(t)\cos(2t) = -x(t). \qquad (9)$$

So the signal is odd. Try plotting it to verify.

Example 2



Problem

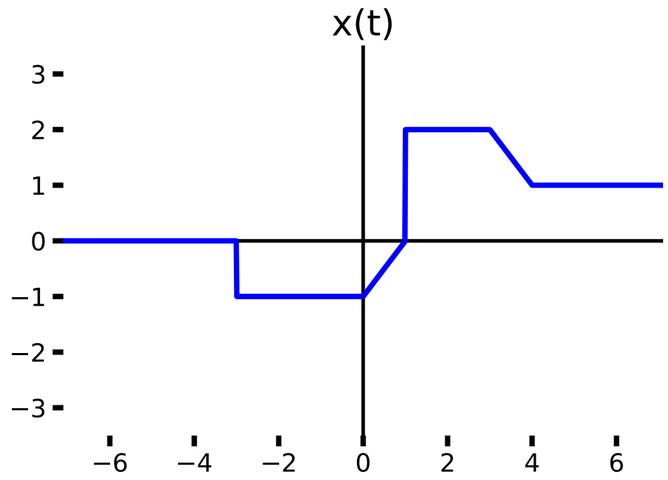
Is the signal $x[n] = u[n]$ even, odd, or neither even nor odd?

If we graph $u[n]$ we can see that neither flipping nor rotating the graph will map $u[n]$ to itself. So the signal is neither even nor odd.

$u[-n] \neq u[n]$

$u[-n] \neq -u[n]$

Example 3



Problem

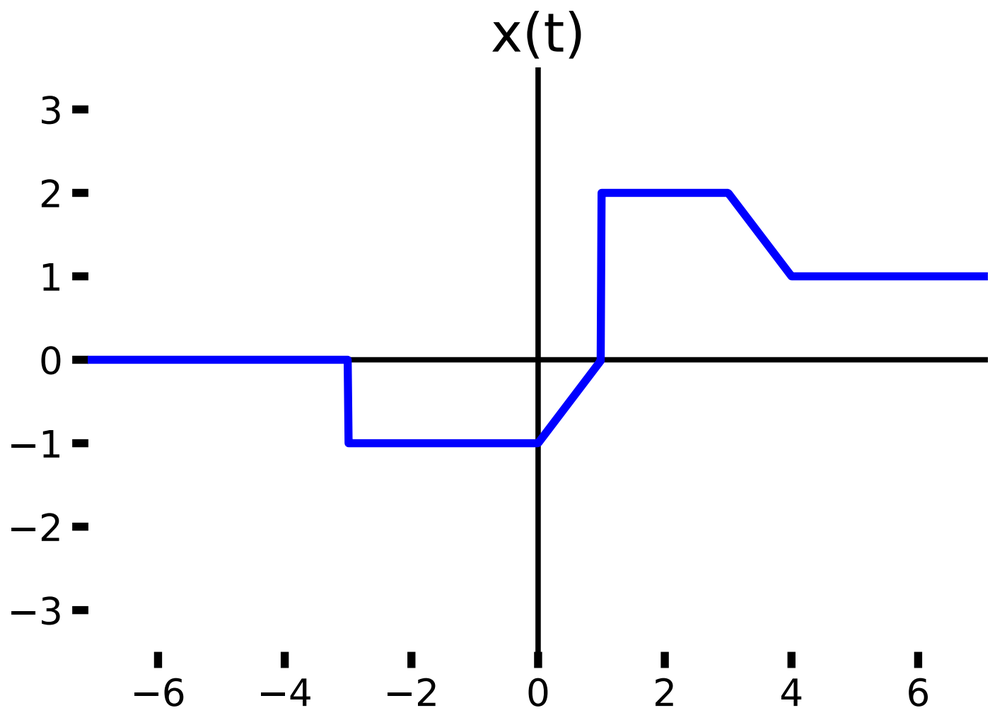
Graph the even and odd parts of the signal shown above.

The signal is

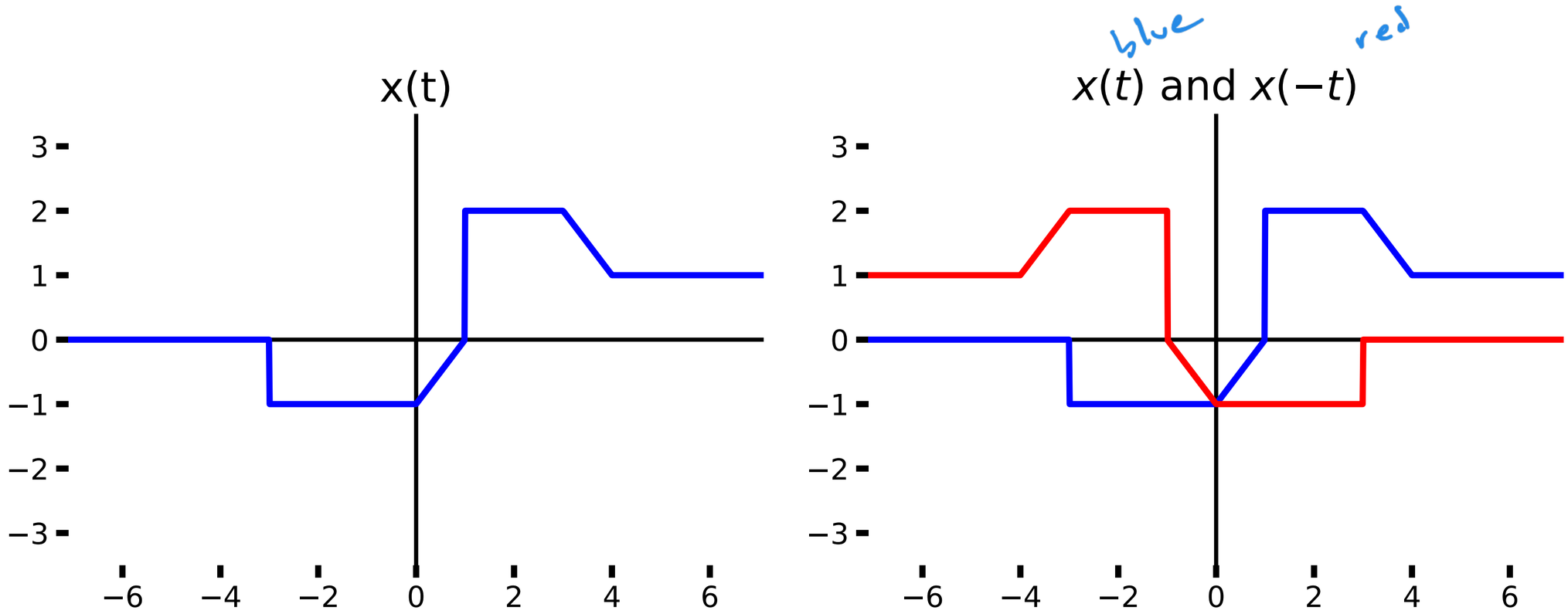
$$x(t) = -u(t+3) + 2u(t-1) + r(t) - r(t-1) - r(t-3) + r(t-4)$$

We can use the formulas to compute the even and odd parts or we can do this graphically. Let's try the latter approach.

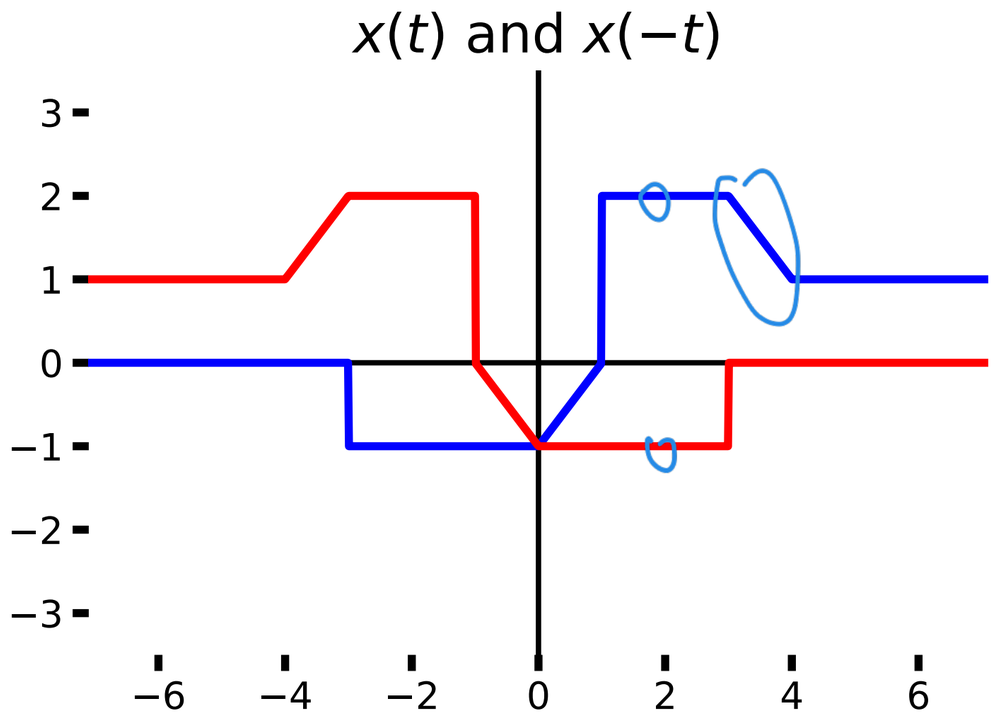
Example 3, continued

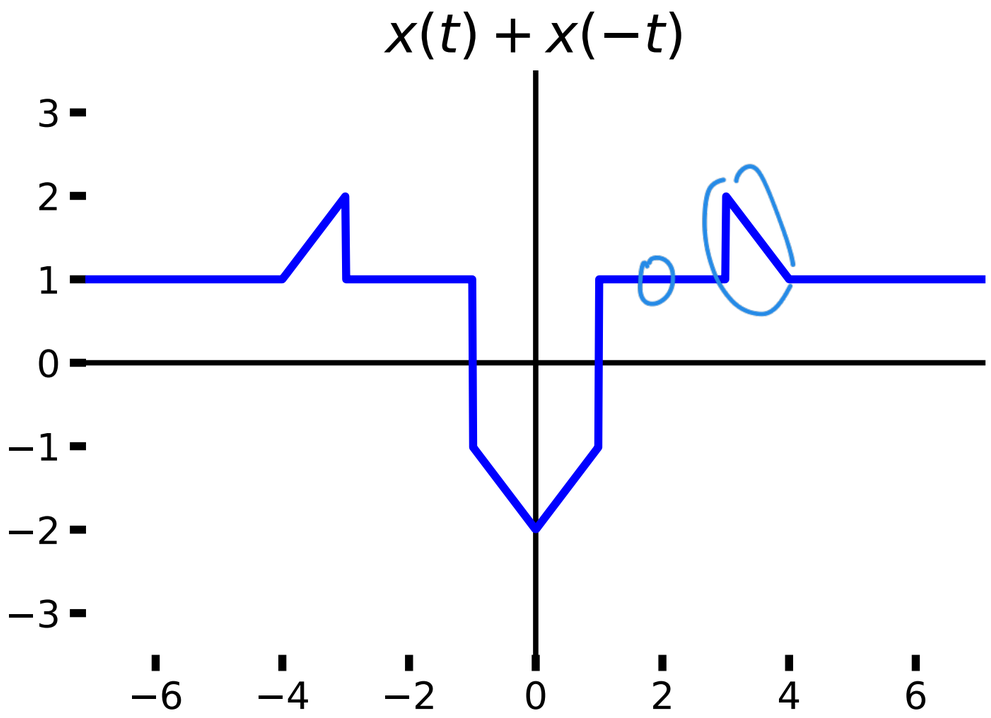


Example 3, continued

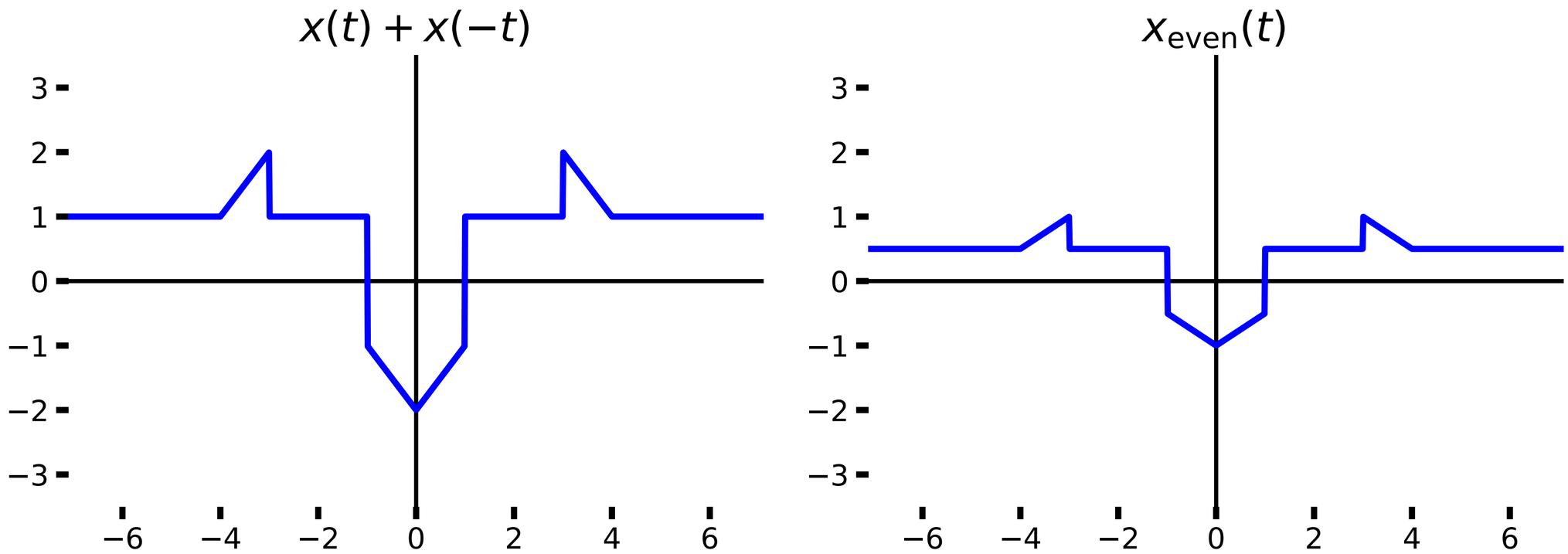


Example 3, continued

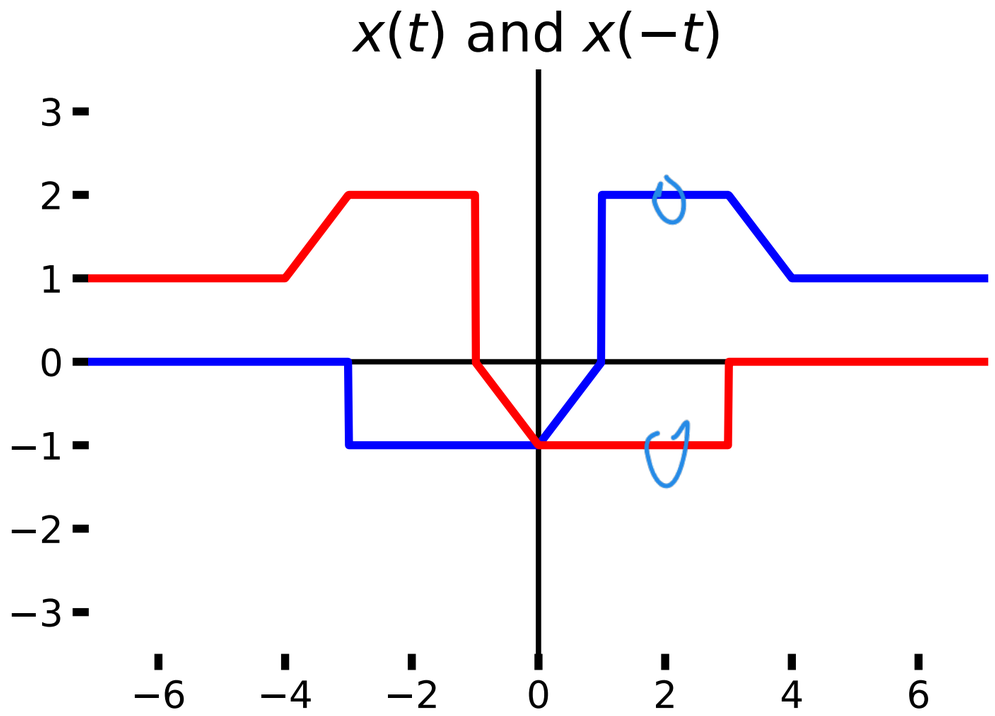


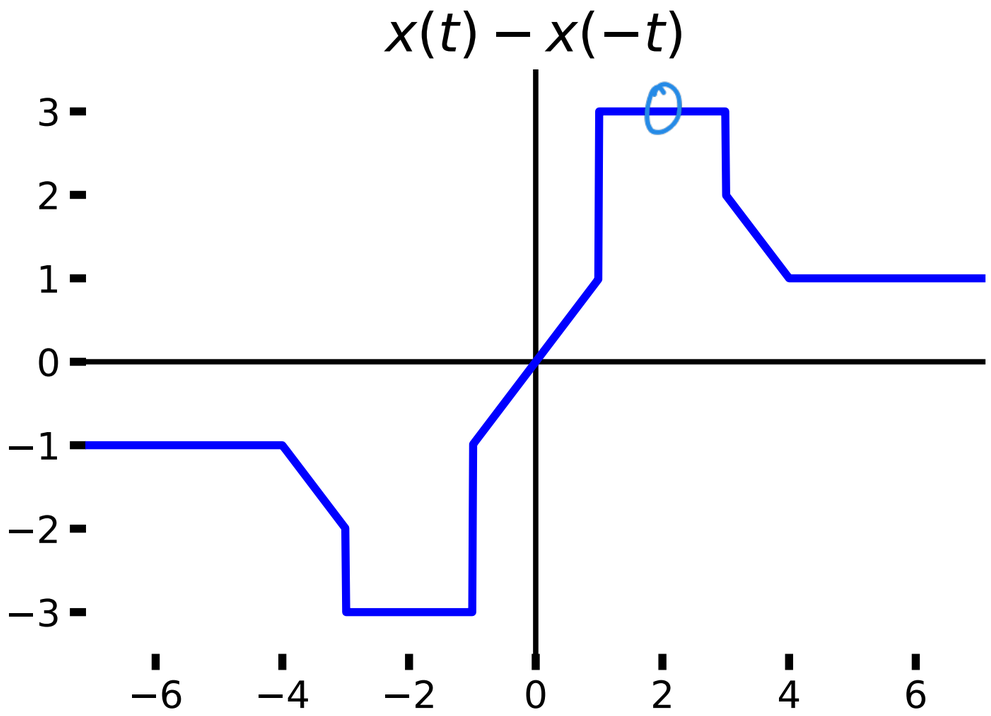


Example 3, continued

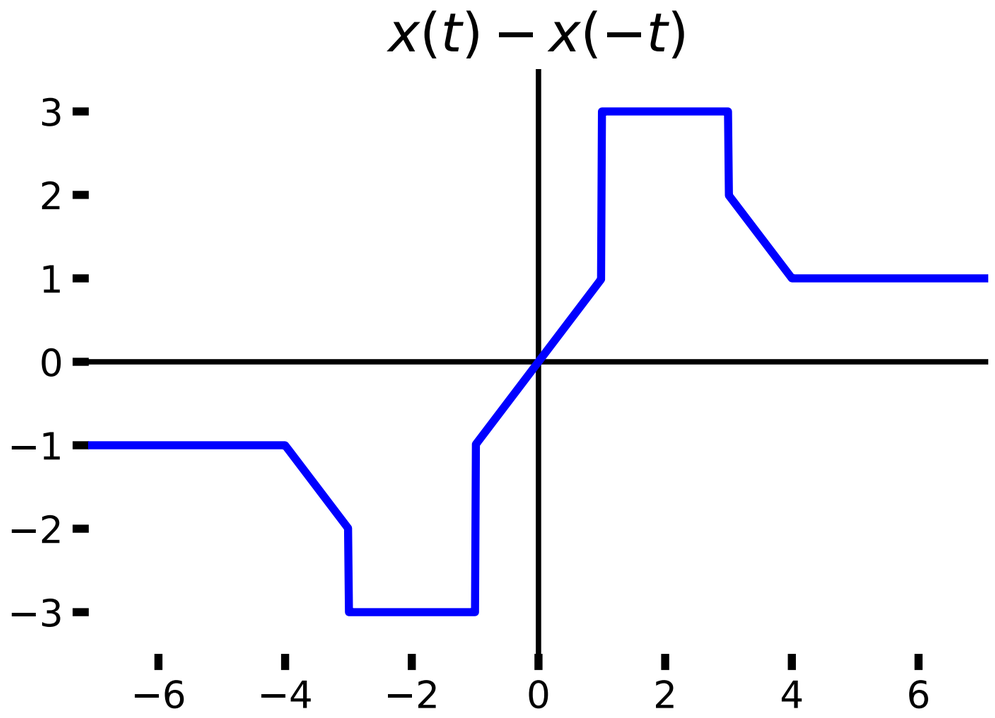


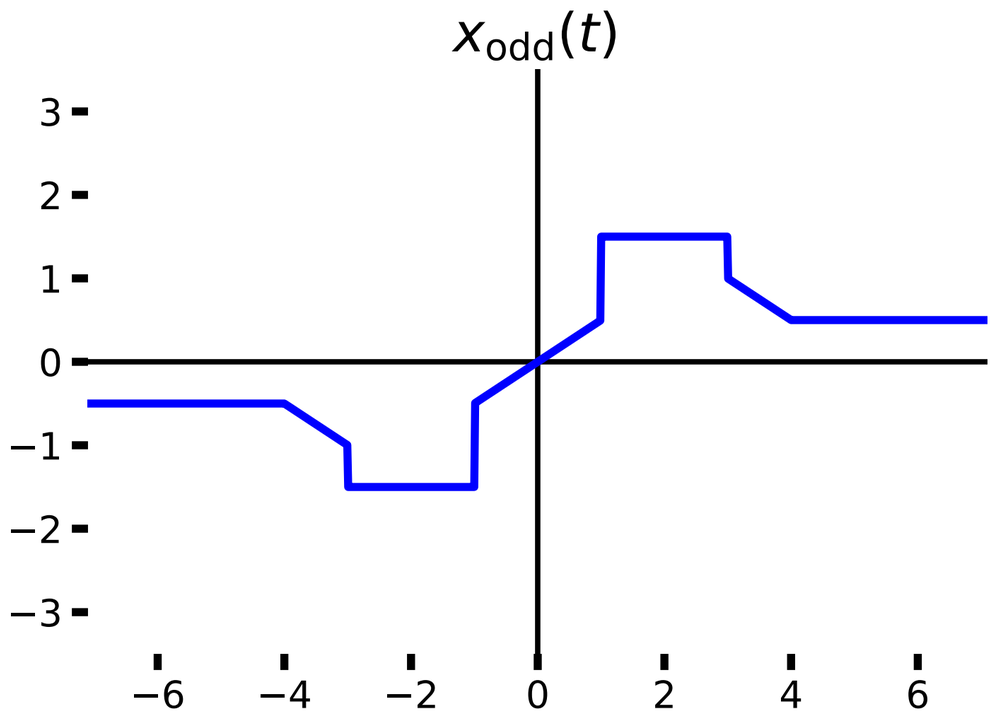
Example 3, continued



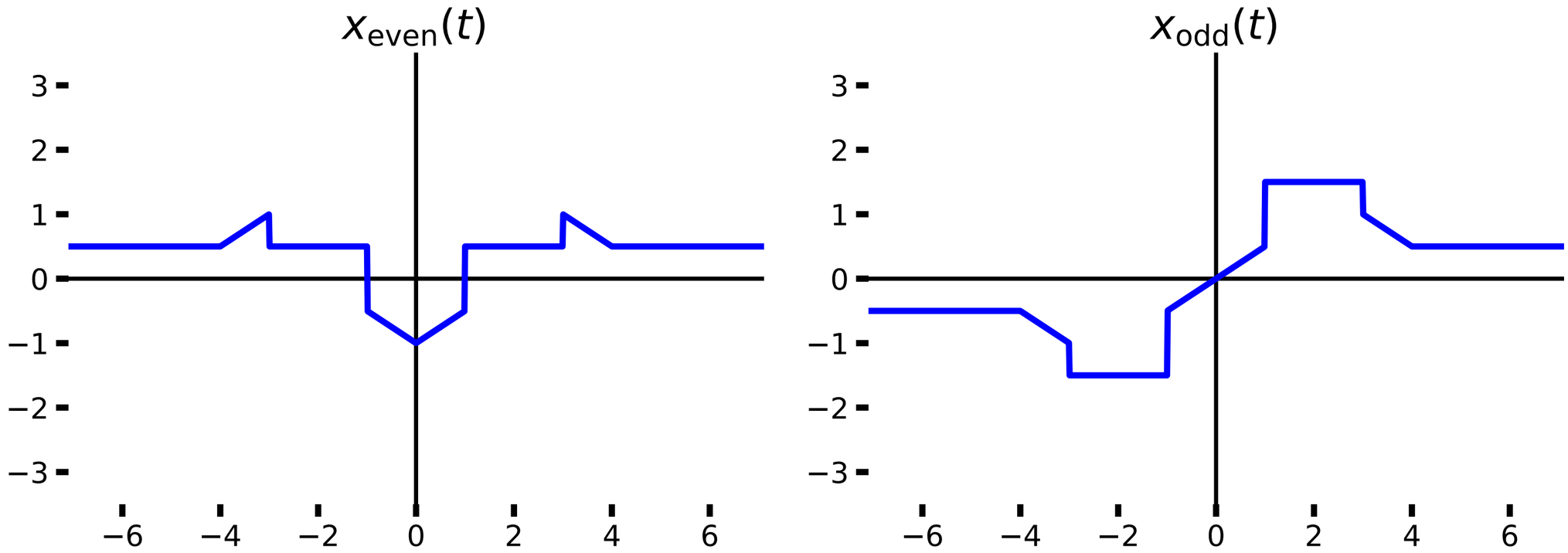


Example 3, continued





Example 3, continued



Try it yourself

Problem

Look at the signals we have seen so far and check if they are even, odd, or neither even nor odd. If they are neither even nor odd, compute or graph the even and odd parts.