Linear Systems and Signals

Causal and non-causal systems

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2020

Learning objectives

The learning objective for this section is:

- determine if a system is causal, memoryless, anticausal, or none of the above

Causal systems

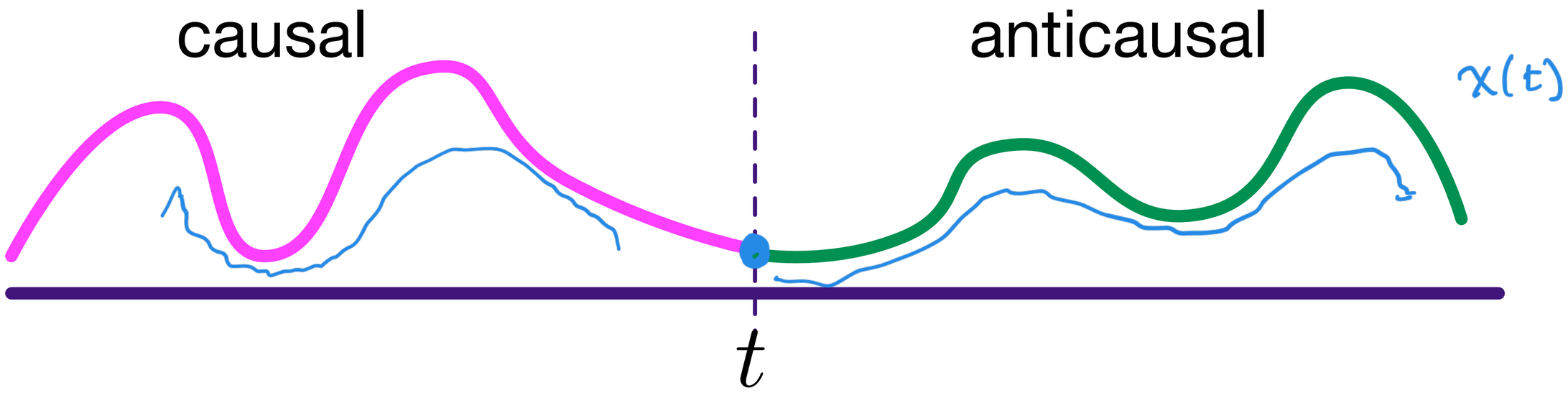
Definition

A system $\mathcal{H}$ is *causal* if the output at time t (or n) only depends on the input at times $\leq t$ (or $\leq n$).

A system $\mathcal{H}$ is *anticausal* if the output at time t (or n) only depends on the input at times $\geq t$ (or $\geq n$).

A system is *memoryless* if the output at time t (or n) only depends on the input at time t (or n).

Causality in pictures



- memoryless systems have outputs that depend on the current input
- causal systems have outputs that depend on the current and past inputs
- anticausal systems have outputs that depend on the current and future inputs

Some examples

Let $y(t) = \mathcal{H}(x(t))$ or $y[n] = \mathcal{H}(x[n])$ denote a system.

- $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$. This is *causal* because the integral goes over $\tau \in (-\infty, t)$.
- $y[n] = x[n]\cos(5\pi n)$. This is *memoryless* because the output at time n only depends on the input at time n.
- $y(t) = x(t) + 2x(t+2)$. This is *anticausal* because the output at time t depends on the input at time $t+2$.
- $x[n] = x[-n]$. This is neither causal nor anticausal. For $n > 0$ the output depends on the past but for $n < 0$ the output depends on the future.

Try it yourself

Problem

Determine if each of these systems is causal, memoryless, anticausal, or neither causal nor anticausal.

- *The instantaneous power:* $\mathcal{H}(x(t)) = x(t)^2$
- $y[n] = x[4n+1]$
- $\mathcal{H}(x(t)) = x(t-2) + x(2-t).$
- $\mathcal{H}(x(t)) = \int_{-\infty} 2tx(\tau)d\tau.$
- $\mathcal{H}(x[n]) = x_o[n].$