Linear Systems and Signals

Steps, ramps, and rects

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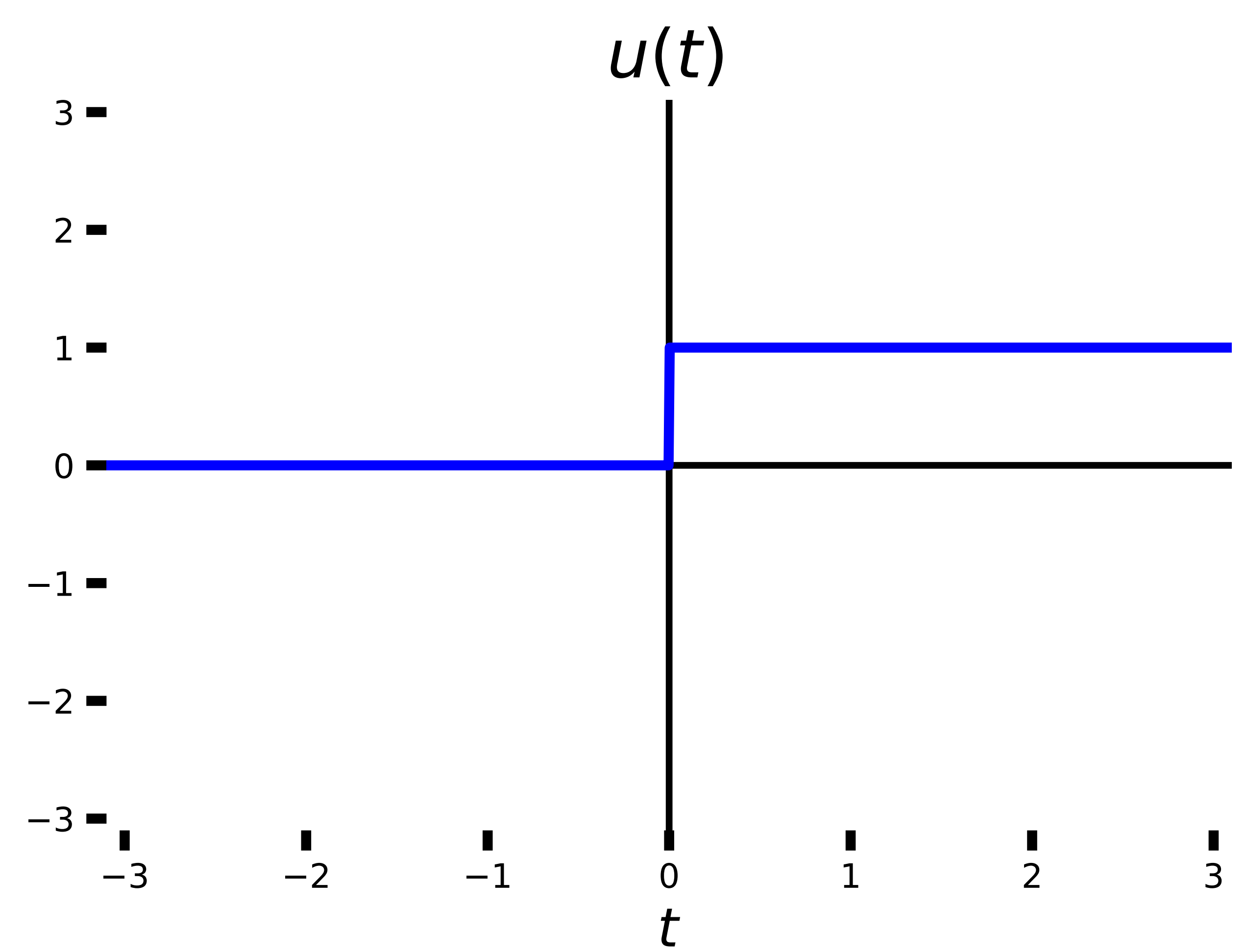
Learning objectives

The learning objectives for this section are:

- understand the definitions of the step, rect, and ramp functions
- use the unit step function to make signals right or left-sided
- use the rectangle function and its shifts/dilations to window signals

The unit step function



$t = 0$?

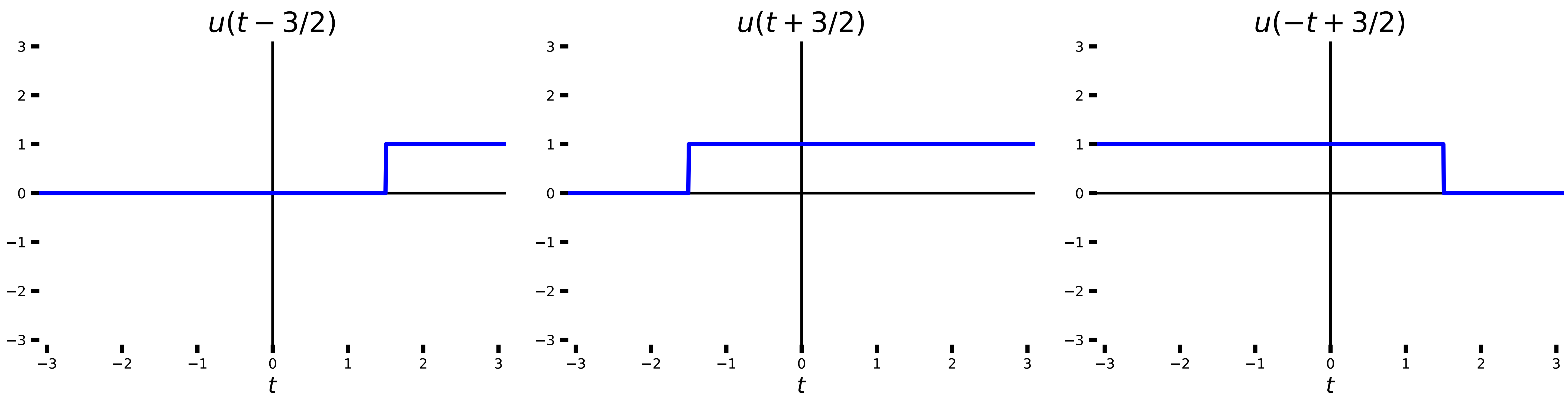
Heaviside Step func

$u(0) = 1/2$

One of the most useful functions we use is the *unit step function*

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \tag{1}$$

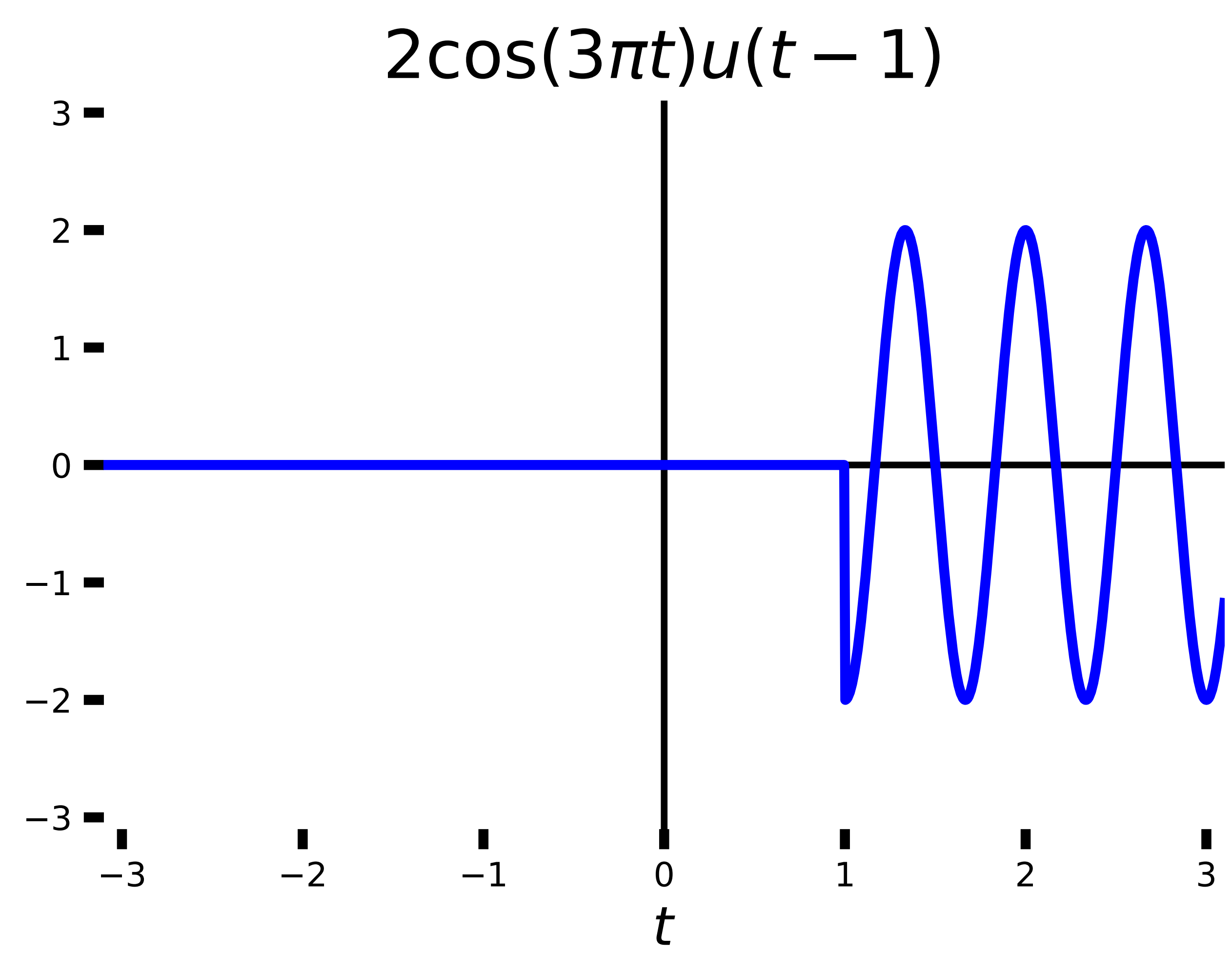
Manipulating the unit step



We can do all the same things with step functions as with other functions:

- time shifts
- time reversal
- expansion/contraction

Interpreting/using the step function

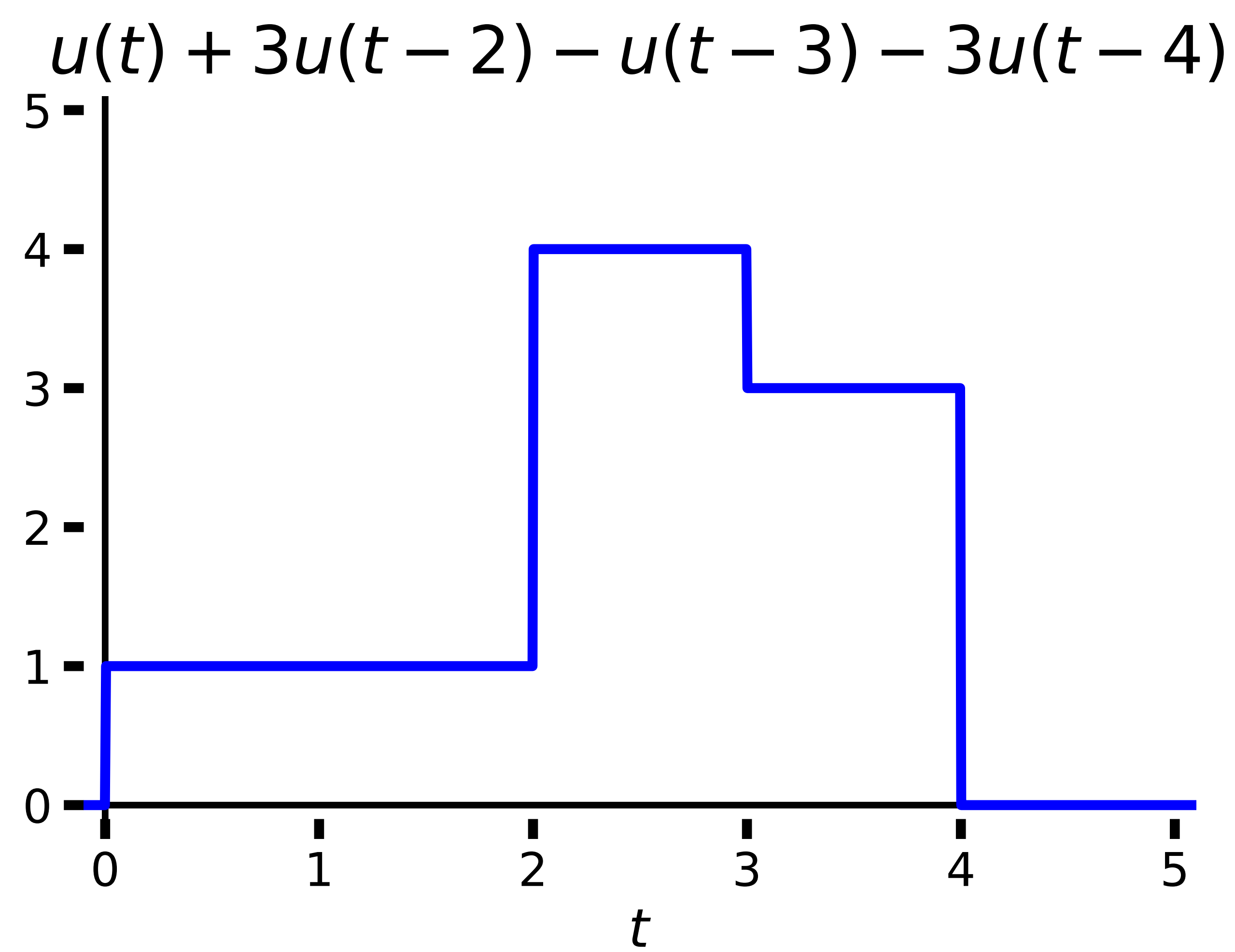


- Making a signal "start" at a certain time:

$$f(t)u(t-\tau) = \begin{cases} f(t) & t > \tau \\ 0 & t < \tau \end{cases} \tag{2}$$

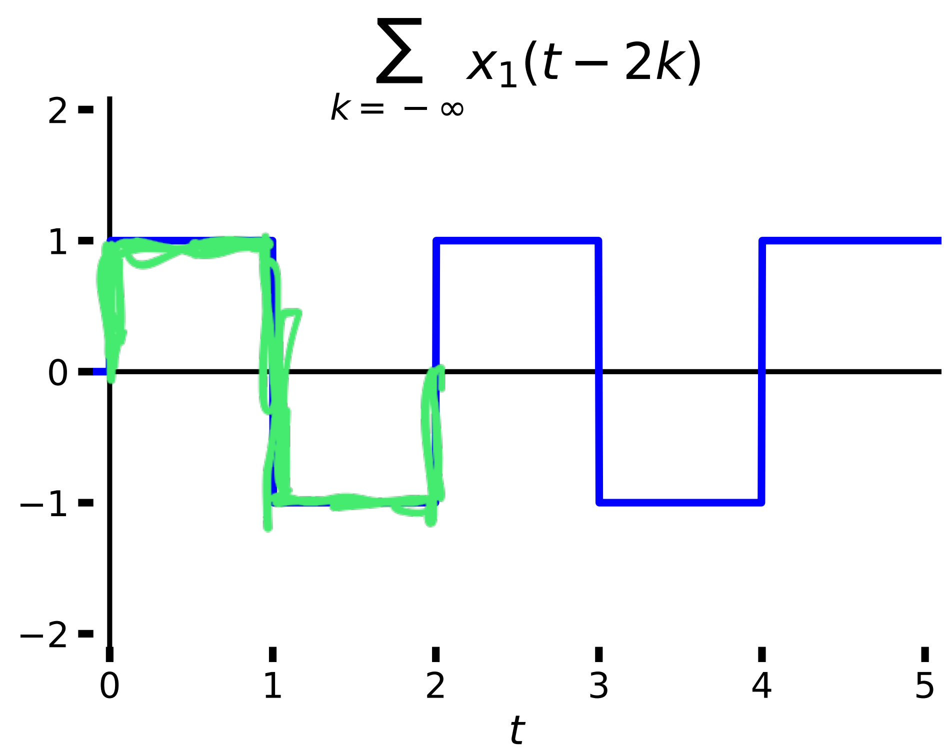
- "Switching on the system": e.g. a DC voltage starts a time 1.
- Effect of integrating an impulse function (more on that later).

Example: piecewise constant functions



We can use step functions to build piecewise constant functions.

Square waves

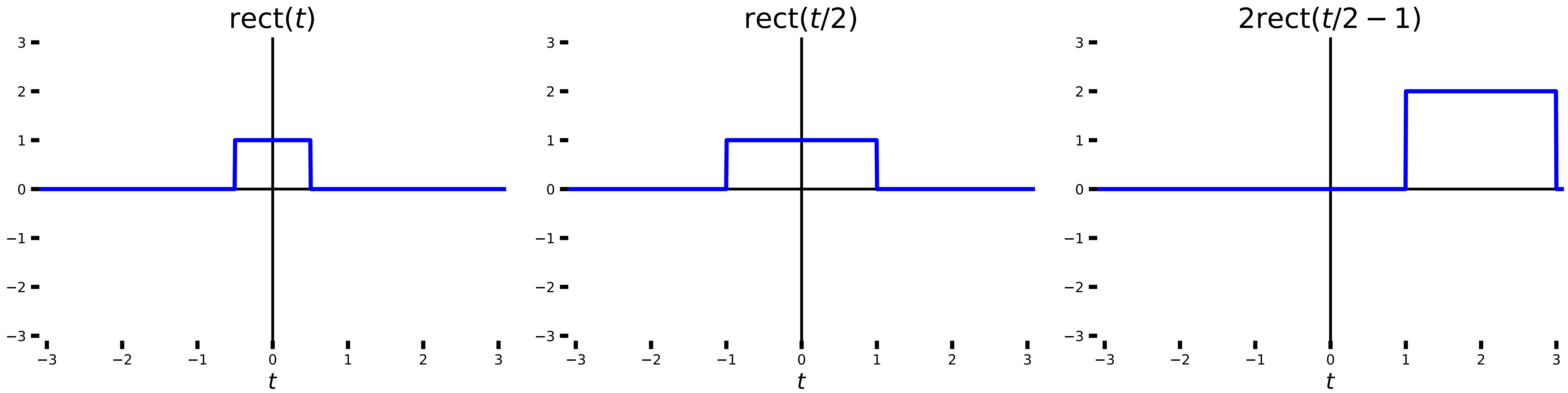


Another example is a square wave: we make one pulse and then repeat it:

$$x_1(t) = u(t) - 2u(t-1) + u(t-2) \tag{3}$$

$$x(t) = \sum_{n=-\infty}^{\infty} x_1(t-2n). \tag{4}$$

Rectangle function



A related function is the rectangle function:

$$\mathrm{rect}(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases} \tag{5}$$

$$= u(t + 1/2) - u(t - 1/2). \tag{6}$$

Since it's so useful it gets its own notation.

Windowing functions

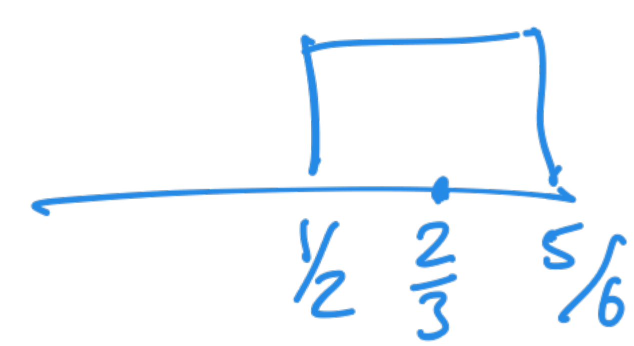
Rectangles are often used to *window* functions – this means only keeping the function inside a window. The function $\mathrm{rect}(at - b)$ keeps a window of total length $\frac{1}{a}$ centered at $\frac{b}{a}$. So

$$f(t)\,\mathrm{rect}(3t-2) = \begin{cases} f(t) & \frac{1}{2} < t < \frac{5}{6} \\ 0 & \text{elsewhere} \end{cases} \tag{7}$$

$\frac{b}{a} = \frac{2}{3} = \frac{4}{6}$

total $\frac{1}{3}$

$\frac{1}{6}$ in each direction



Non-rectangular windows

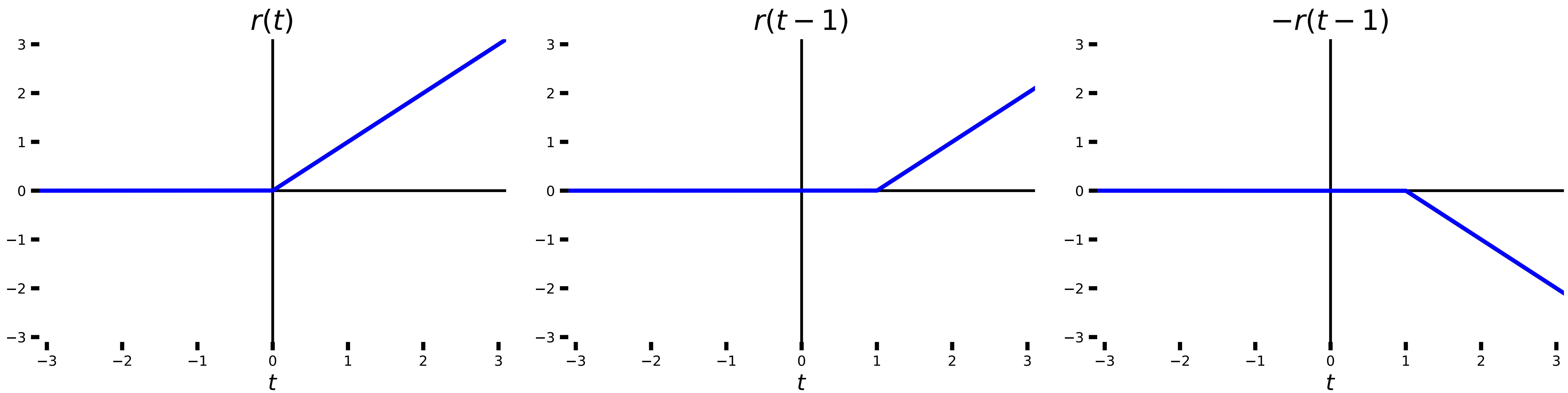
In some applications a rectangular window is not desirable due to the sharpness of the cutoffs. You will see in MATLAB documentation references to all sorts of window functions. The rect function is called a "Hamming window." Others include

- Hanning
- Kaiser
- Bartlett
- Parzen

There are a ton in the MATLAB documentation:

`https://www.mathworks.com/help/signal/windows.html`

The ramp function

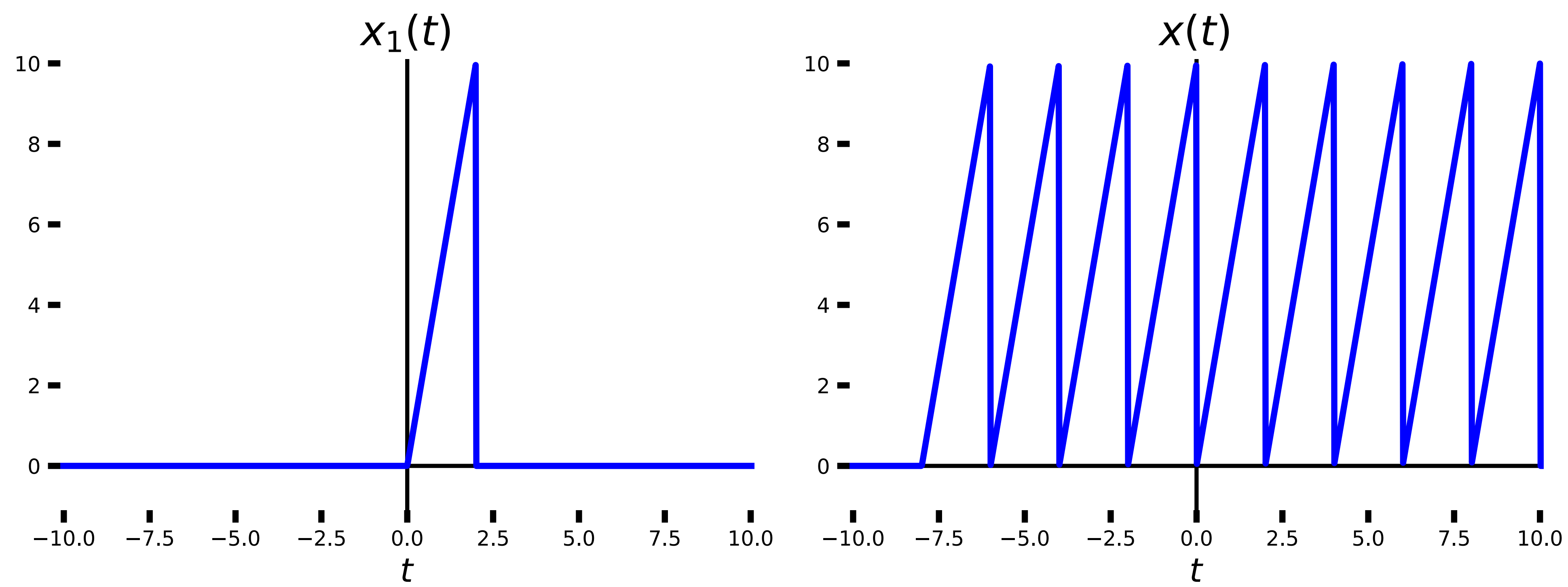


If we integrate the unit step function we get the ramp function:

$$r(t) = \begin{cases} 0 & t \leq 0 \\ t & t > 0 \end{cases} \tag{8}$$

If you feed a step function into an integrator you'll get a ramp – this shows that the integrator is *unstable*.

Sawtooth waves

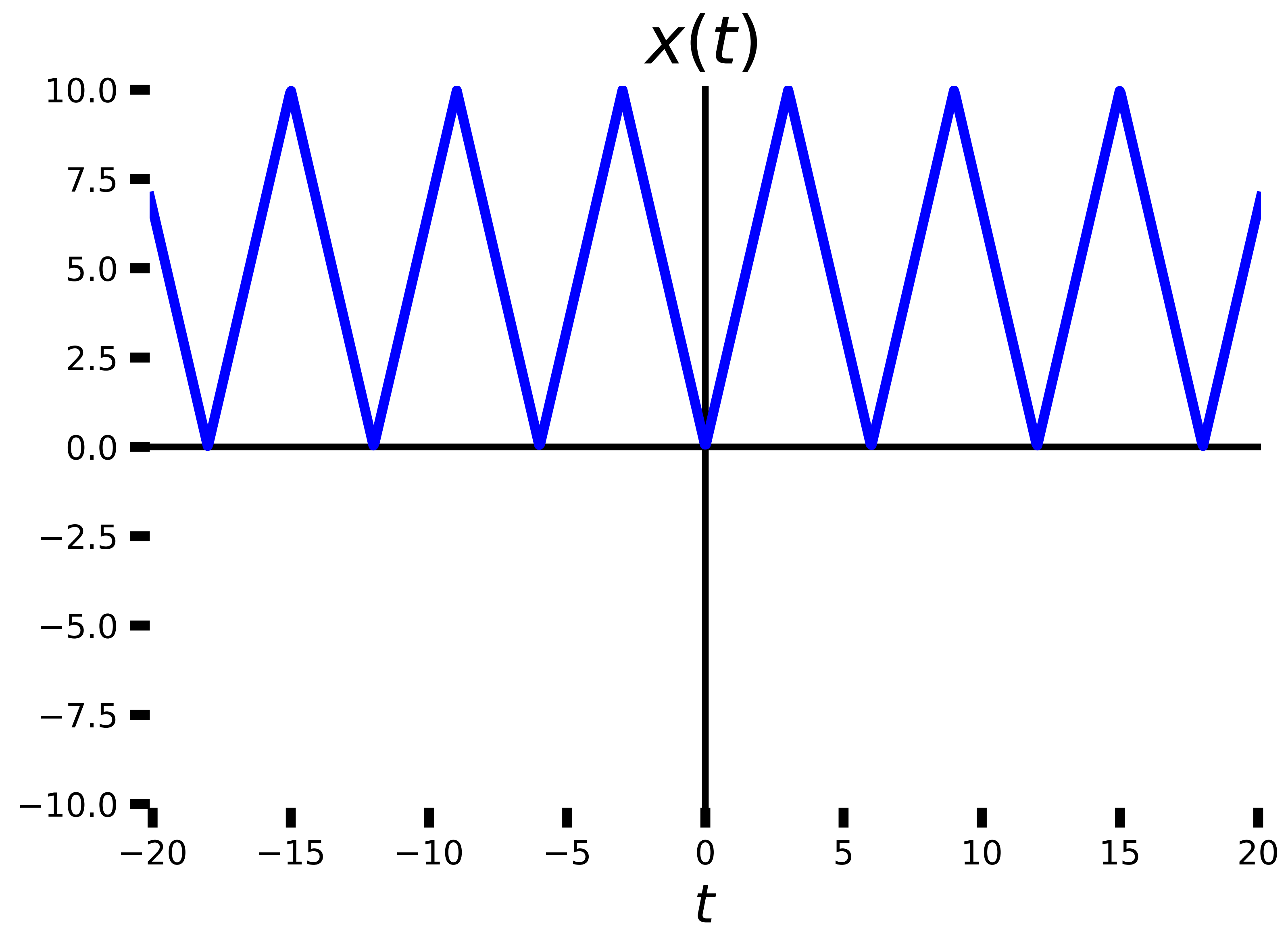


We can synthesize sawtooth waves with ramps and unit steps:

$$x_1(t) = 5r(t) - 5r(t-2) - 10u(t-2) \qquad 0 \leq t \leq 2 \tag{9}$$

$$x(t) = \sum_{n=-\infty}^{\infty} x_1(t - 2n). \tag{10}$$

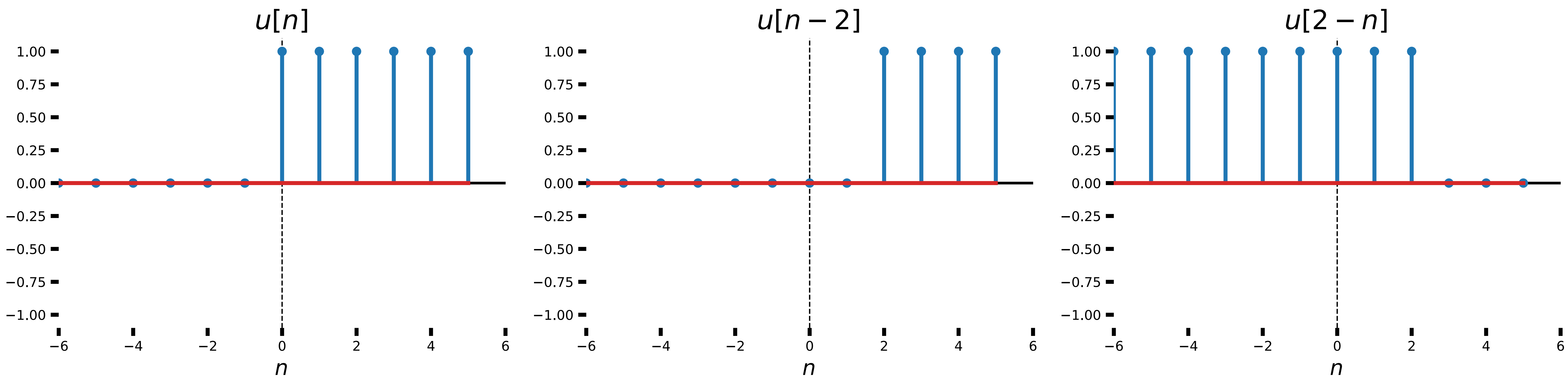
Wave synthesis: try it yourself



Problem

Write a formula for a triangular wave with period 6, minimum value 0 and maximum value 10. Try to plot it in MATLAB.

What about DT?

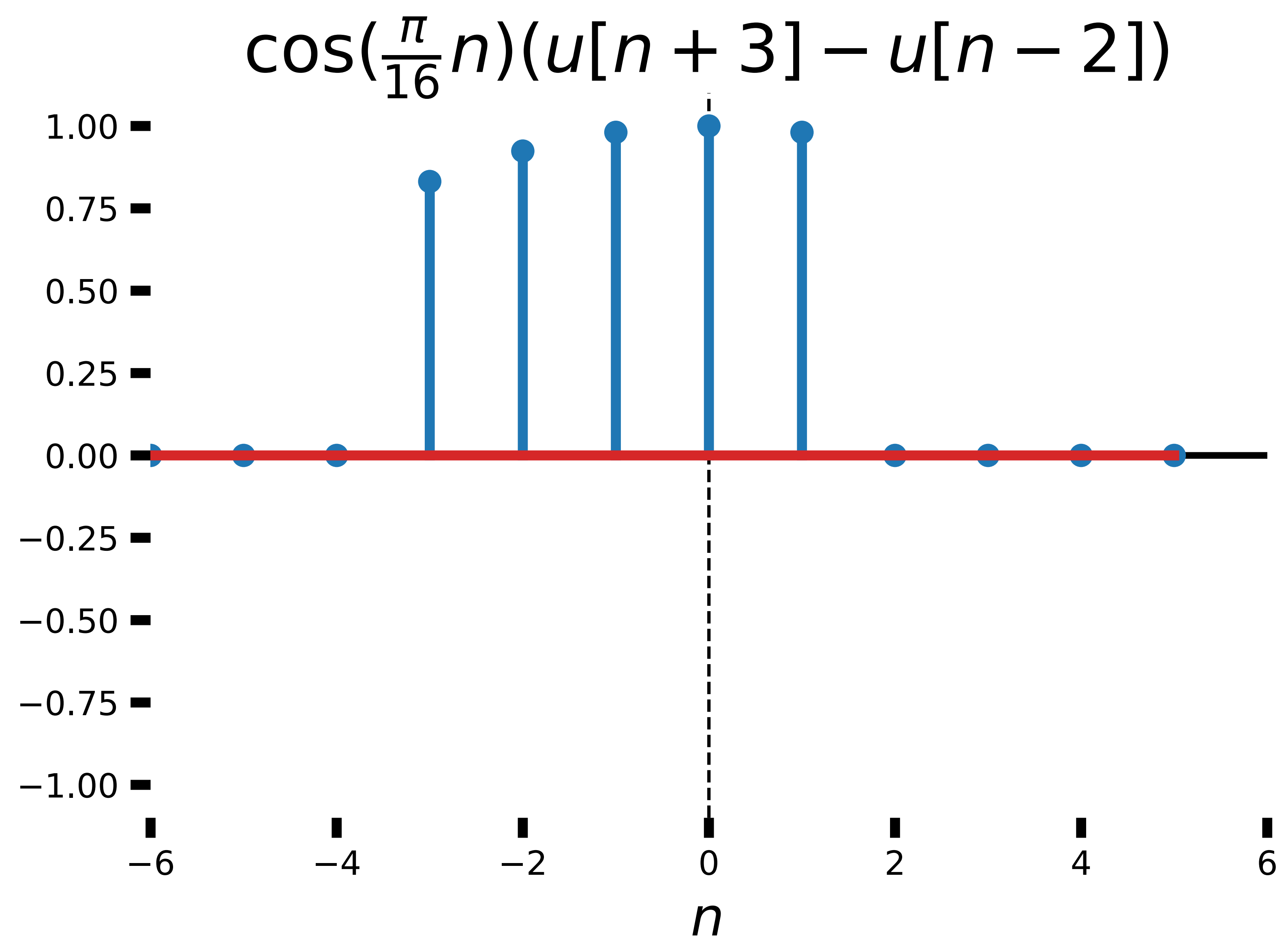


In DT we also use the unit step function

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

We don't tend to use the ramp and rect functions however.

What about DT?



We can use unit steps to make windowed versions of signals.

Recap

- introduced the unit step, unit ramp, and rect functions in CT
- introduced the DT unit step
- showed how to use steps to make signals start and stop at different times
- constructed infinite waves using steps and ramps