Linear Systems and Signals

Block diagram basics

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Learning objectives

The learning objectives for this section are:

- represent basic signal manipulations in terms of block diagrams
- translate a block diagram into a mathematical formula

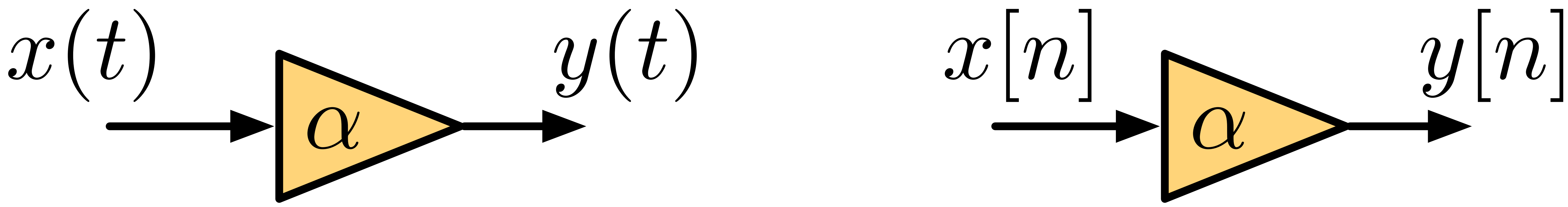
Block diagrams

In Signals and Systems we look at engineering problems from a "boxes and arrows" perspective:

- arrows are like wires that carry signals
- boxes are systems that transform signals

We can represent these in diagrams that show how signals are being processed and manipulated by systems.

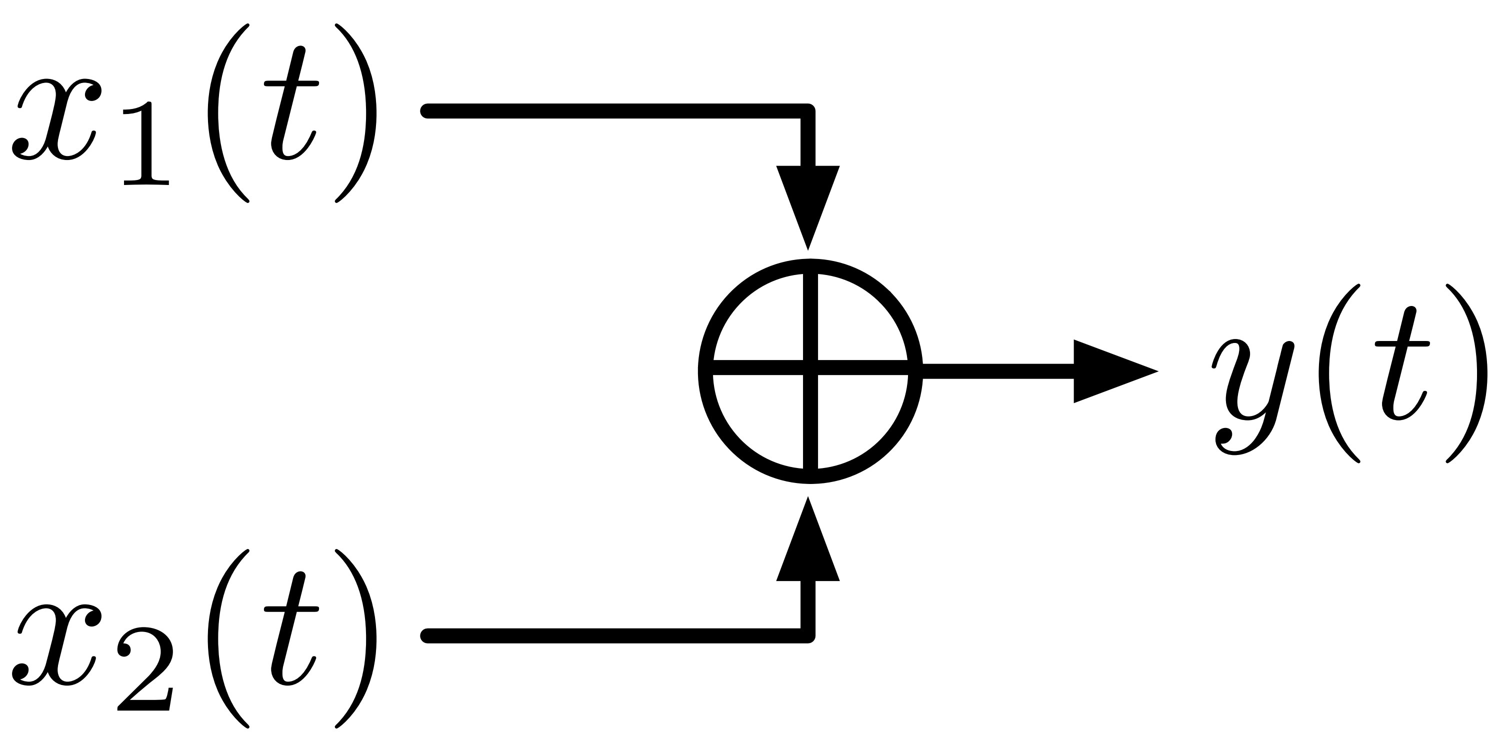
Gain

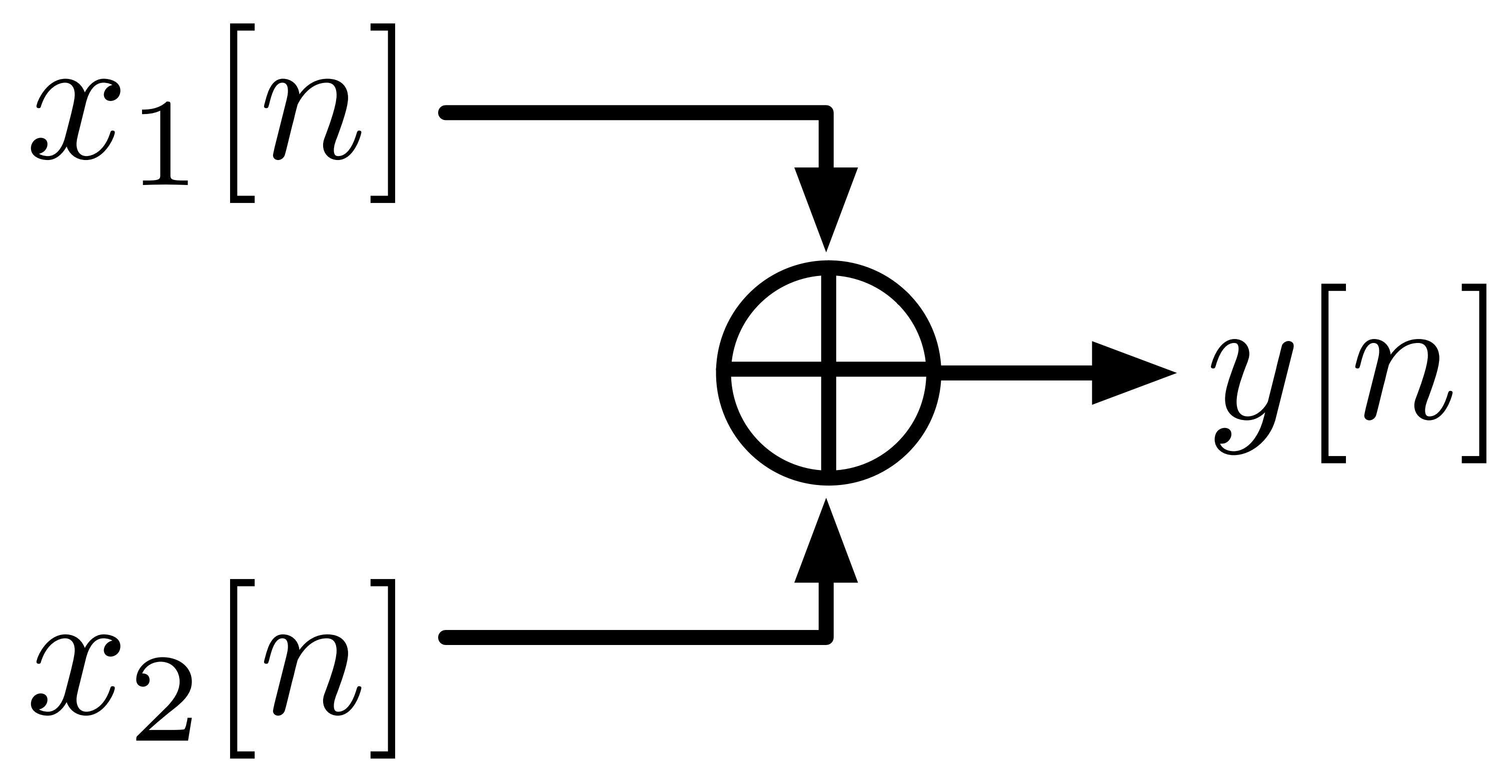


A *gain* is just scalar multiplication. We call this a gain even if $|\alpha| < 1$ (which is an attenuation).

$$y(t) = \alpha x(t) \tag{1}$$

Addition

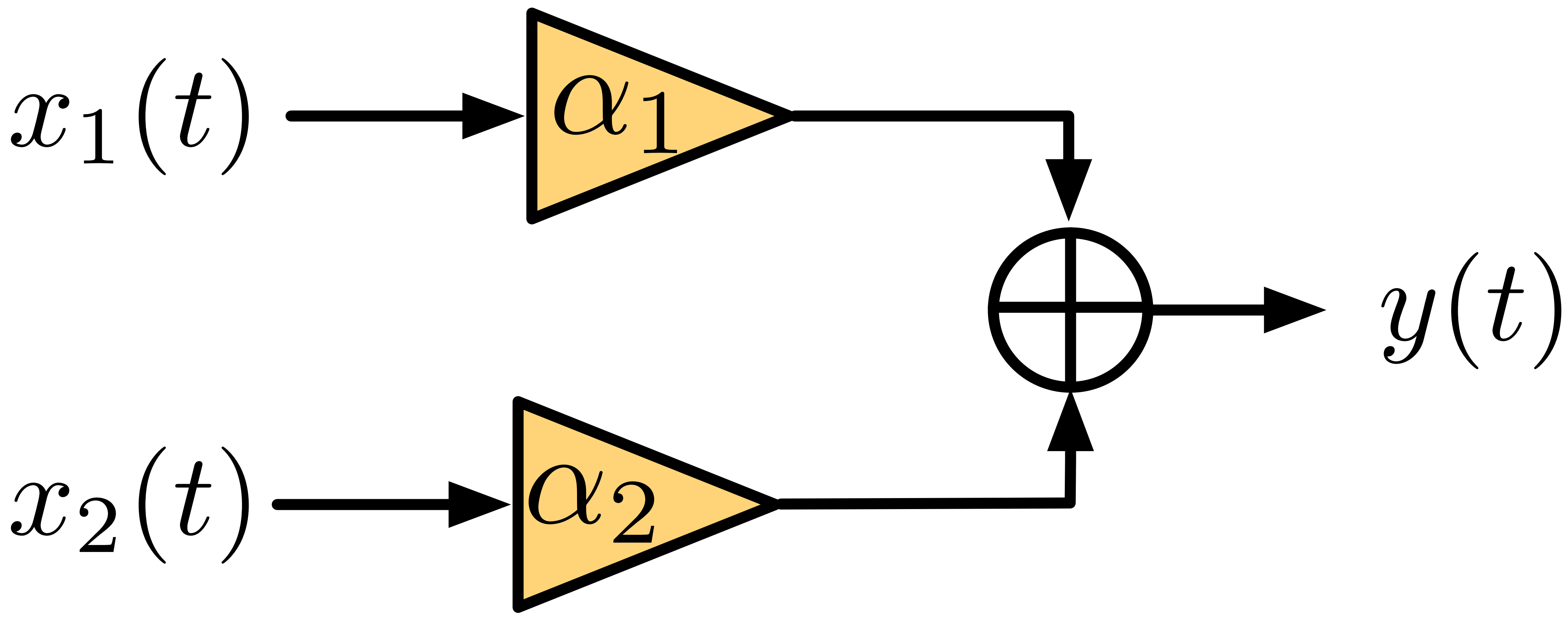




An *adder* just sums its inputs:

$$y(t) = x_1(t) + x_2(t) \tag{2}$$

Combining them

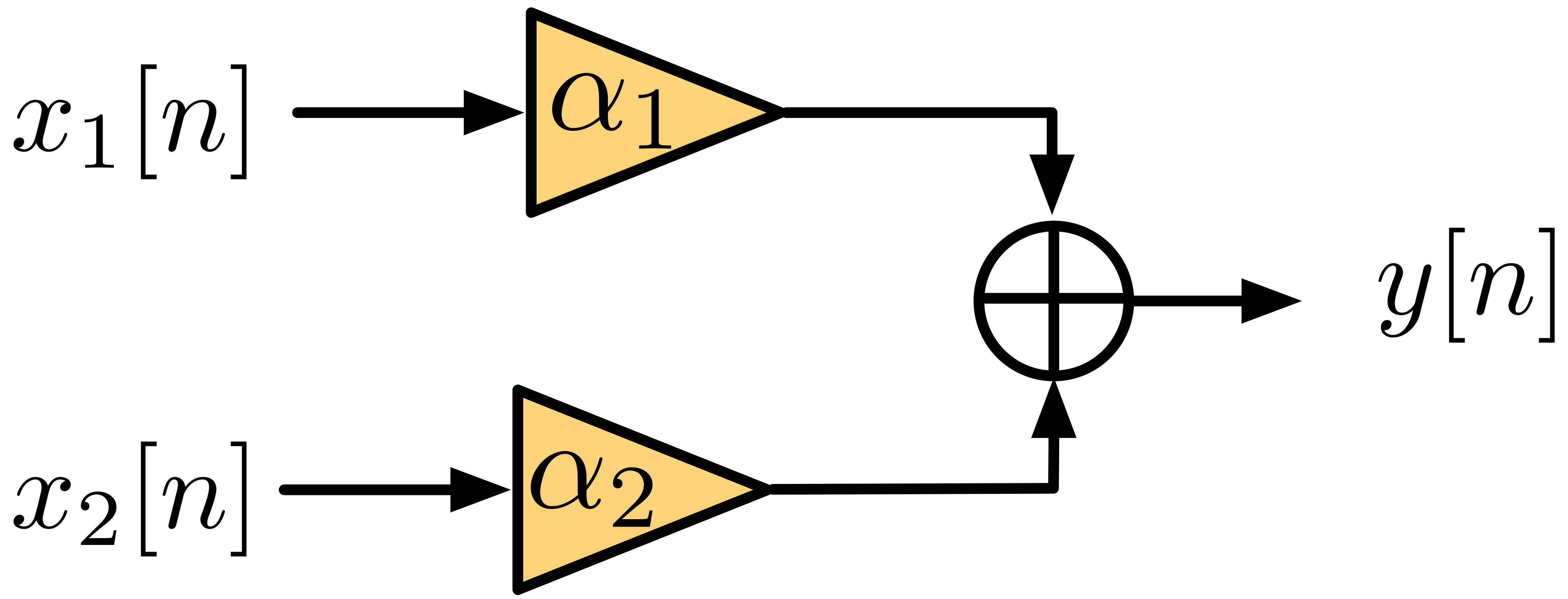


We can combine adders and gains:

$$y(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t) \tag{3}$$

$$y[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \tag{4}$$

Combining them



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$$x[n] \longrightarrow \boxed{z^{-k}} \longrightarrow x[n-k]$$

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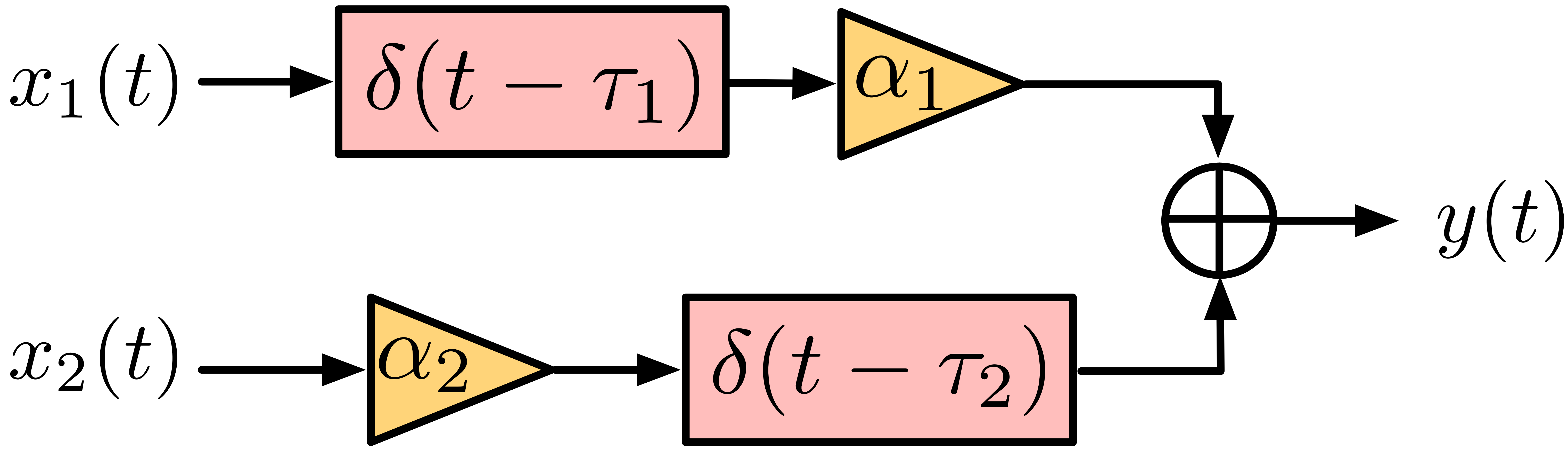
- Generically we can write a delay of τ as $\mathrm{delay}(\tau)$.
- We will see that $\delta(t-\tau)$ describes a CT system that delays its input by τ.
- For DT we write a delay of k as z^{-k} (the reason will be clearer later).

Combining them



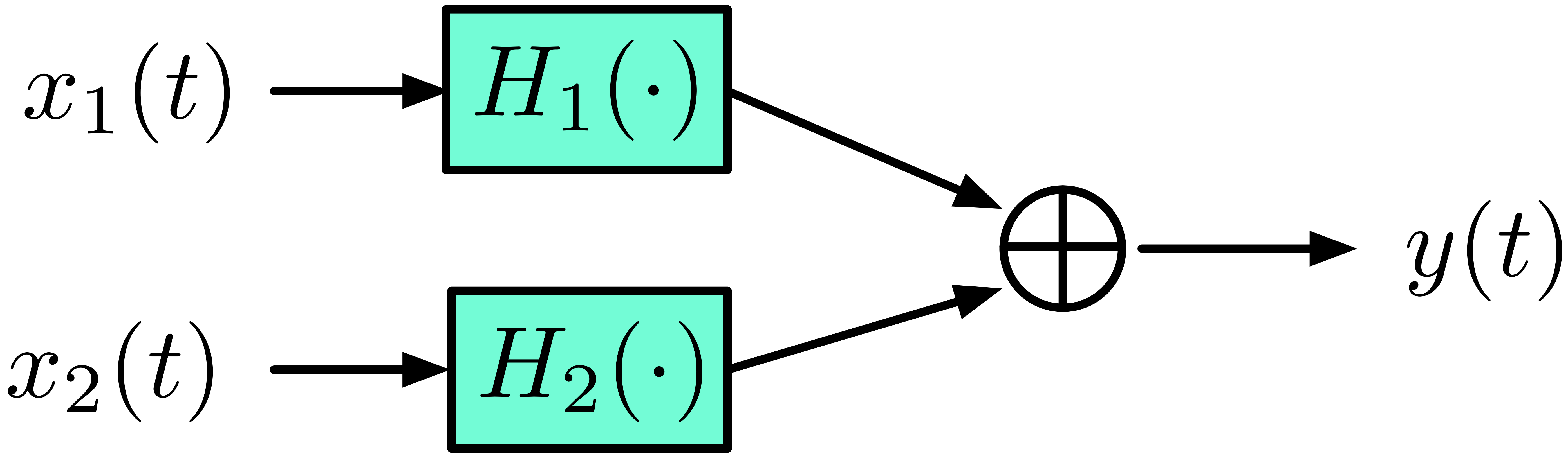
- delay and gains *commute*. That is, in CT

$$x(t) \to 3x(t) \to 3x(t-1) \leftarrow x(t-1) \leftarrow x(t), \qquad (5)$$

and in DT

$$x[n] \to -2x[n] \to -2x[n-4] \leftarrow x[n-4] \leftarrow x[n]. \qquad (6)$$

Systems



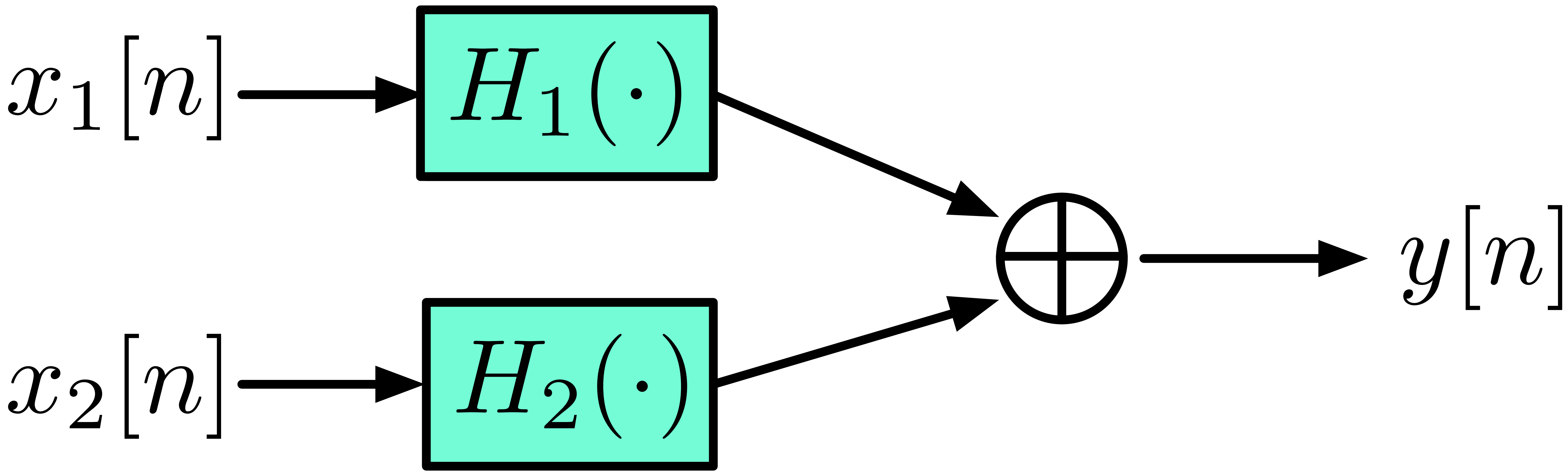
A system is a function that maps one signal to another. Here we have

$$y[n] = H_1(x_1[n]) + H_2(x_2[n]) \tag{7}$$

If $H_1(x[n]) = x[n]^2$ and $H_2(x[n]) = 3x[n+1] - 3x[n-1]$ then

$$y[n] = x_1[n]^2 + 2x_2[n+1] - 3x[n-1]. \tag{8}$$

Systems



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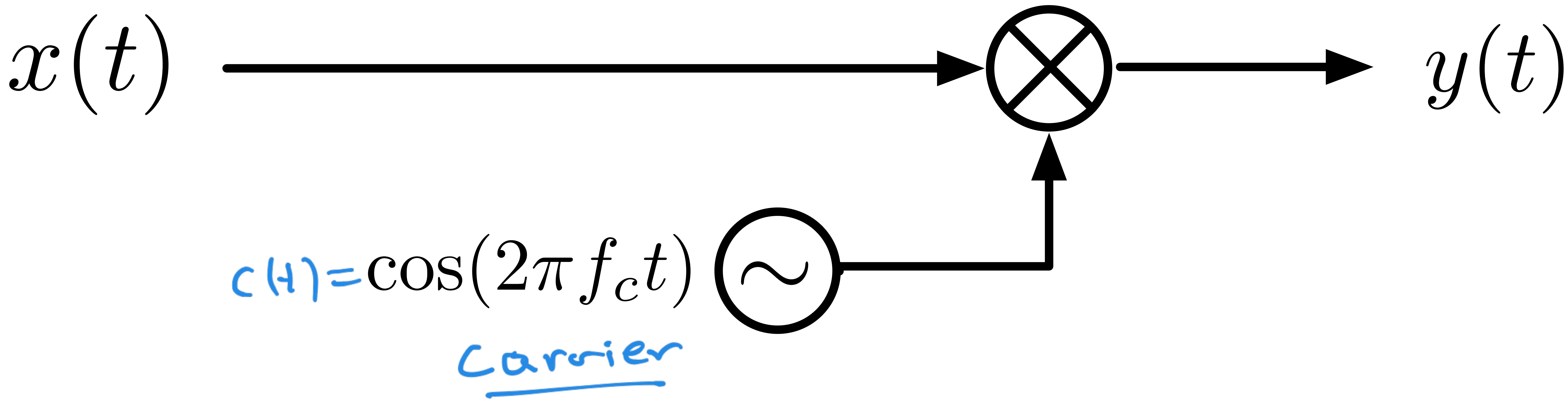
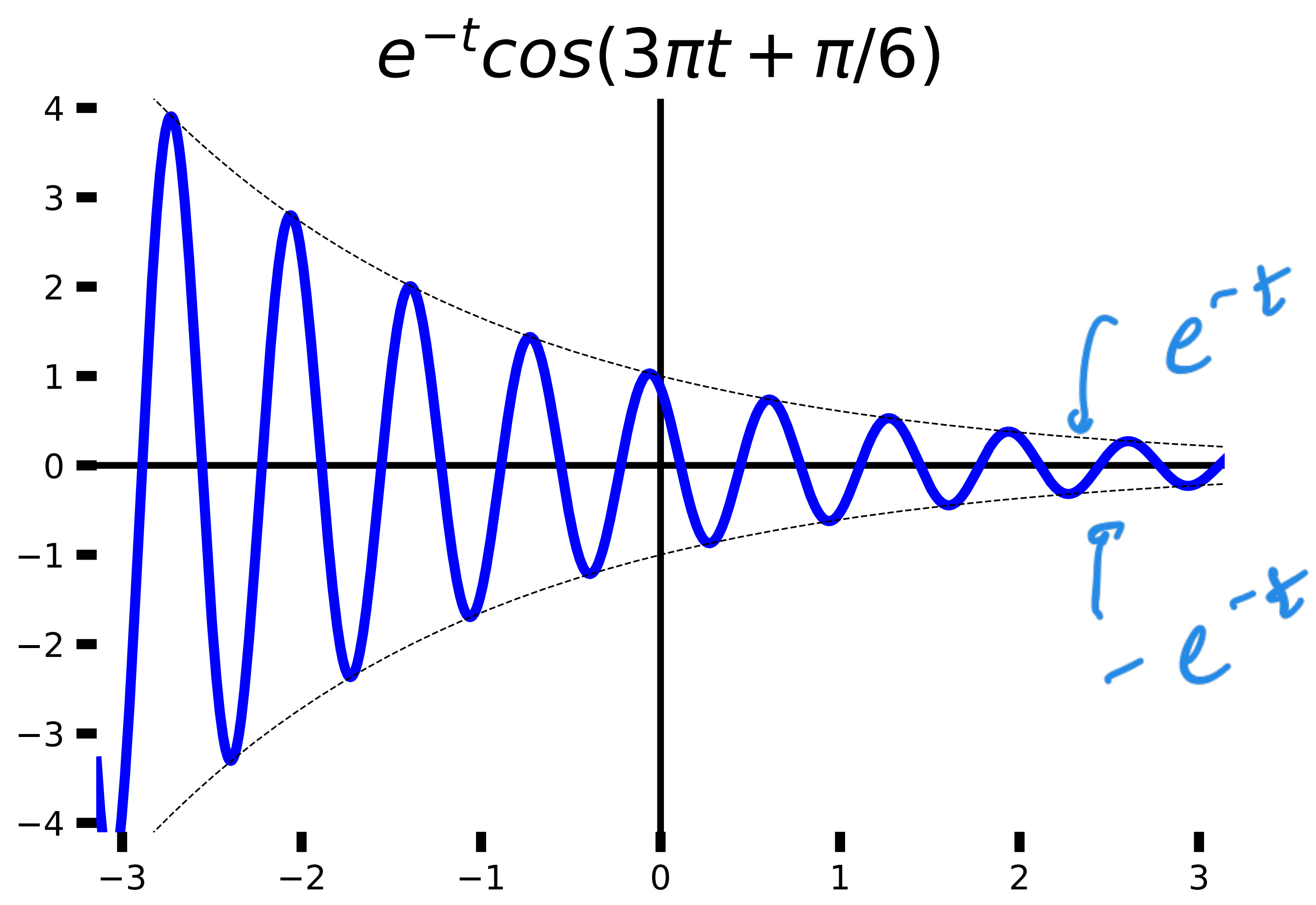
Modulation



One particular system that appears a lot in communication systems is a *modulator*. You modulate a signal $x(t)$ onto a signal $c(t)$ by multiplying the two signals pointwise. Typically $c(t)$ is a high-frequency sinusoid.

$$y(t) = x(t) \cos(2\pi f_c t) \tag{9}$$

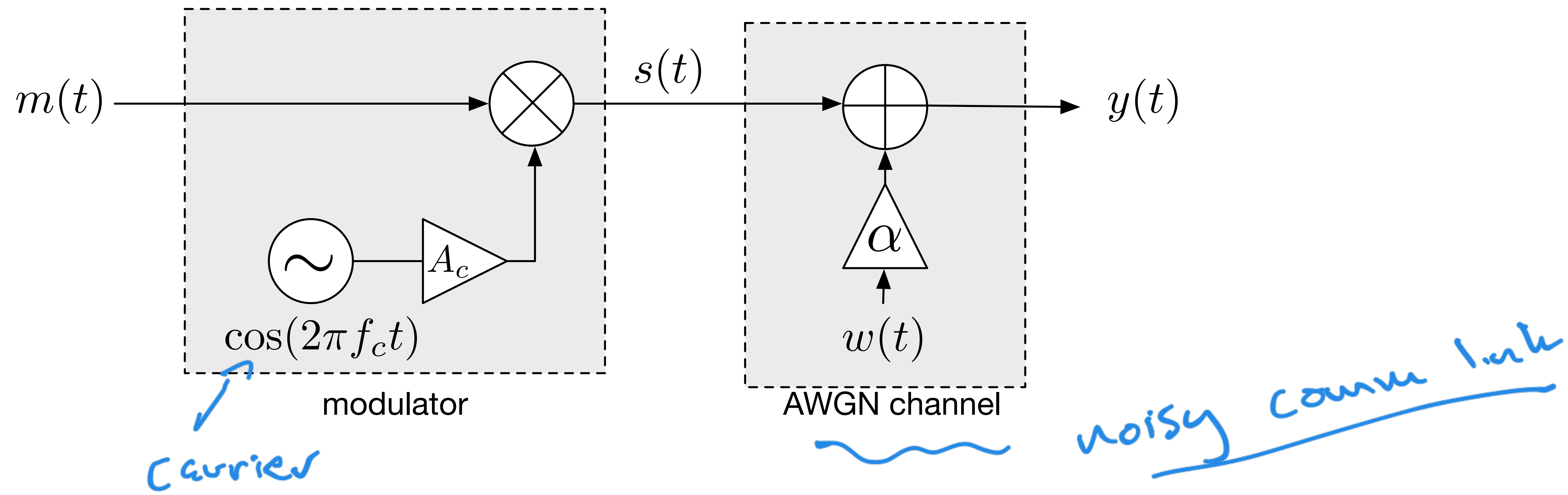
Modulation example



$x(t) = e^{-t}$

If there is some delay you might get $y(t) = x(t)\cos(2\pi f_c(t - \tau))$. We saw examples of modulated signals before. The signals $x(t)$ and $-x(t)$ form the *envelope* of the modulated signal.

AM radio transmission and channel

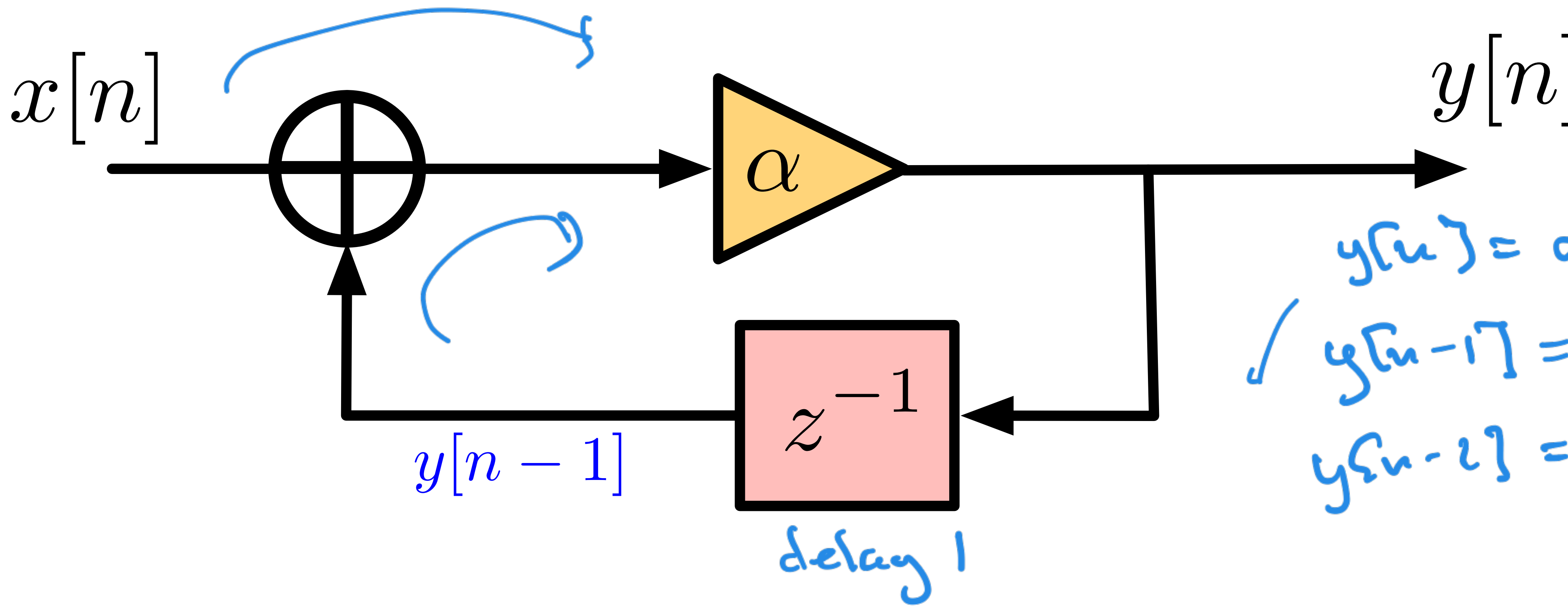


This is an AM transmitter – a signal is modulated onto a cosine and sent over the air. The *communication channel* between the transmitter and receiver is noisy, so the received signal is

$$y(t) = A_c m(t) \cos(2\pi f_c t) + \alpha w(t), \tag{10}$$

where f_c is called the *carrier frequency*, A_c is called the *carrier gain* and α^2 is the *noise power*. We assume here that $w(t)$ is *additive Gaussian noise* (white noise).

Feedback



$y[n] = \alpha x[n] + \alpha y[n-1]$
$y[n-1] = \alpha x[n-1] + \alpha y[n-2]$
$y[n-2] = \alpha x[n-2] + \alpha y[n-3]$

Feedback is a very important part of systems. How to we write the relationship between $y[n]$ and $x[n]$?

$$y[n] = \alpha x[n] + \alpha y[n-1] \tag{11}$$
$$= \alpha x[n] + \alpha^2 x[n-1] + \alpha^2 y[n-2] \tag{12}$$
$$= \alpha x[n] + \alpha^2 x[n-1] + \alpha^3 x[n-2] + \alpha^3 y[n-3] \tag{13}$$
$$= \sum_{k=0}^{\infty} \alpha^{k+1} x[n-k] \tag{14}$$

Try it yourself

Problem

Try drawing the block diagrams that generate the following signals:

$$y_1(t) = 4x(t) - x(t-3) + x(t-5) \tag{15}$$

$$y_2[n] = x[2n-1] + x[2n] \tag{16}$$

$$y_3(t) = 3x(t-2)\cos(440\pi t) - 4x(t)^3\cos(220\pi t) \tag{17}$$

$$y_4[n] = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k x[n-k] \tag{18}$$

~~Try playing $y_3(t)$ with $x(t) = \cos(880\pi t)$ in MATLAB. What do you hear?~~