Linear Systems and Signals

Simple systems

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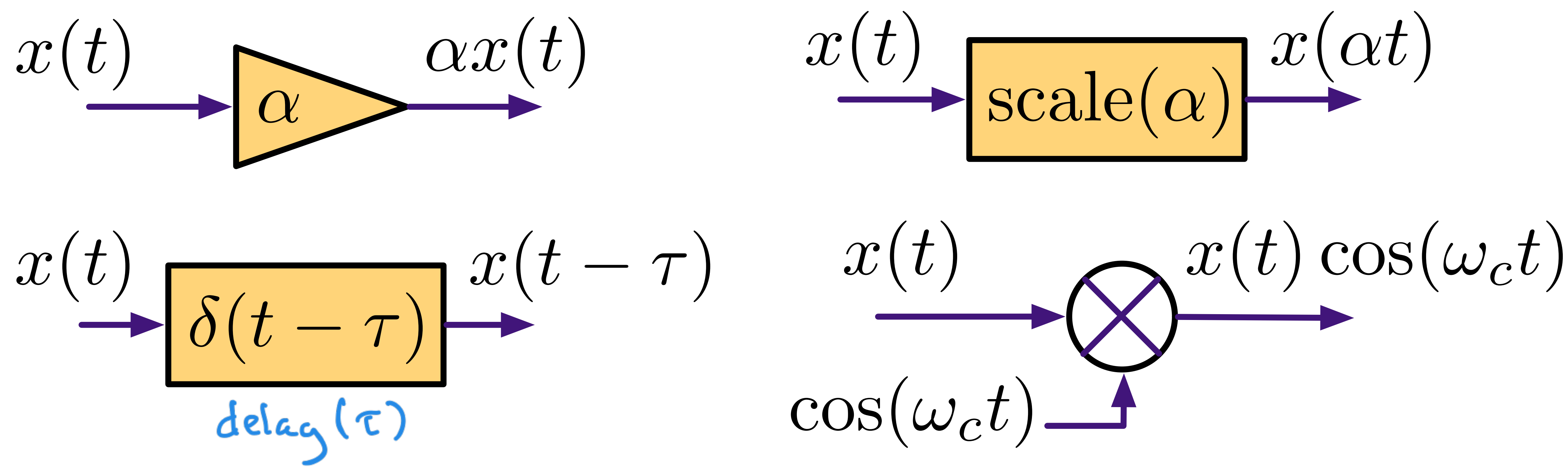
Learning objectives

The learning objectives for this section are:

- describe simple systems using blocks such as gains, delays, time scaling, and modulation
- describe gains, delays, time scaling, and modulation in terms of input/output relations
- use block diagrams to represent systems composed of basic elements

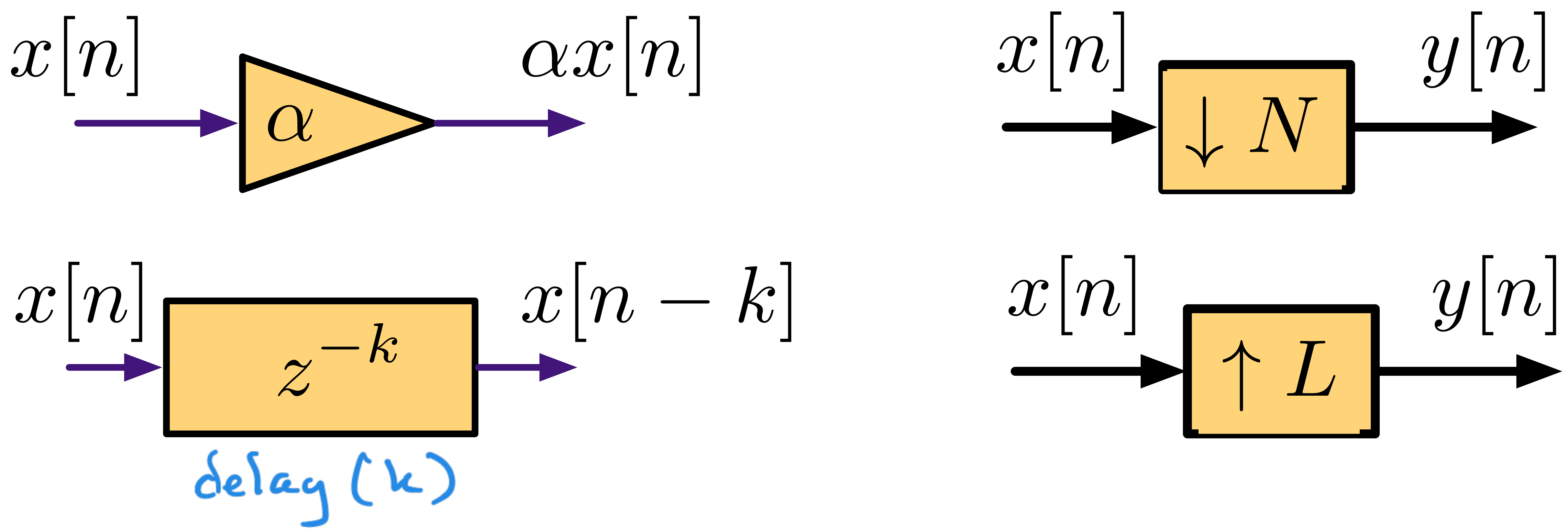
Simple systems in CT



There are a few simple systems that we have encountered before for CT systems

- gain
- delay
- time scaling
- modulation

Simple systems in DT



For DT systems we also saw some examples of systems:

- gain
- delay
- downsampler
- upsampler

Properties of systems

Systems are defined by their input/output relation. What kind of properties might we be interested in?

- causality: does the output at time t depend on past inputs for $t \leq 0$ or future inputs?
- stability: are there inputs which make the system output "blow up" to ∞ or $-\infty$?
- linearity: is the output a linear function of the system input?
- time-invariance: does the input-output relation vary with time?
- invertibility: can you recover the input signal from the output signal?

An example: a gain

Consider the gain where $0 < |A| < \infty$.

$$y(t) = Ax(t). \tag{1}$$

What properties does it have?

- At time τ, $y(\tau)$ depends only on $x(\tau)$. Causal!
- The only way $|y(t)| \to \infty$ is if $|x(t)| \to \infty$. Stable!
- Scalar multiplication is linear.
- The system doesn't behave differently when $t = 0$ versus $t = 10^3$. Needs more proof, but this is time-invariant.
- We can recover the input by using a gain of A^{-1}.

An example: delay

Consider the delay

$$y[n] = x[n-k]. \tag{2}$$

What properties does it have?

- At time n, $y[n]$ depends only on $x[n-k]$. This is the past if $k > 0$ and in the future if $k \cancel{>} < 0$.
- The only way $|y[n]| \to \infty$ is if $|x[n]| \to \infty$.
- Delay is also linear:
  $\text{delay}_k(\alpha_1 x_1[n] + \alpha_2 x_2[n]) = \alpha_1 x_1[n-k] + \alpha_2 x_2[n-k]$
- Note that $y[n-\ell] = \alpha x[n-k-\ell]$ so delaying the input introduces the same delay in the output.
- We can recover the input by using a delay of $-k$.

What's coming next

Mathematically, a system is a map whose input and output are both signals:

$$y(t) = \mathcal{H}\left(x(t)\right) \qquad y[n] = \mathcal{H}\left(x[n]\right) \tag{3}$$

We are going to dig into properties that a system $\mathcal{H}$ can have and see some more examples.

In the meantime, try checking the properties we looked at here for for DT gains and CT delays.