ECE 345: Linear Systems and Signals — Fall 2020
Professors Sarwate and El Rouayheb — Due: 5 PM, September 25, 2020

Homework 2

Problem 1 (SSTA Problem 2.23)**.** Determine whether or not each of the following LTI systems is: (1) causal and/or BIBO-stable. If the system is not BIBO stable, provide a bounded input that yields an unbounded output.

(a) $y(t) = \frac{dx}{dt}$

(b) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(c) $y(t) = \int_{-\infty}^{t} x(\tau)\cos(t-\tau)d\tau$

(d) $y(t) = x(t+1)$

(e) $y(t) = \int_{t-1}^{t+1} x(\tau)d\tau$

(f) $y(t) = \int_{t}^{\infty} x(\tau)e^{2(t-\tau)}d\tau$

Problem 2 (Time-invariance)**.** Determine whether the following systems are time-invariant or time-varying. Note the first two are CT and the second two are DT.

(a) $y(t) = \sin(x(t))$

(b) $y(t) = t\sin(x(t))$

(c) $y[k] = 3(x[k] - x[k-2])$

(d) $y[k] = kx[k]$

Problem 3 (Even and odd parts)**.** Express the following signal as the sum of an even and an odd signal

$$x(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

Plot the even and odd parts.

Problem 4 (ECE 345 Final Exam, Fall 2017)**.** Consider the system $y[n] = x_{\text{even}}[n]$ that outputs the even part of a function. Determine whether or not this system is memoryless, time-invariant, linear, or stable.

Problem 5 (SSTA Problem 2.23)**.** Consider the CT system with the following input-output relation:

$$y(t) = x(t)\cos(120\pi t) + x(t-3). \tag{2}$$

Determine if this system is stable/unstable, causal/noncausal, linear/nonlinear, and time-invariant or time-varying.

Problem 6 (Lathi and Green 1.7-16)**.** For the systems described by the following equations, with input $x(t)$ and output $y(t)$, determine which are invertible and which are noninvertible. For the invertible systems, find the input-output relationship of the inverse system.

(a) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(b) $y(t) = \frac{dx(t)}{dt}$ for differentiable $x(t)$.

(c) $y(t) = x(3t - 6)$

(d) $y(t) = \cos(x(t))$

Problem 7 (ECE 345 Final Exam, Fall 2017)**.** Consider the upsampler by K:

$$y[n] = \begin{cases} x[n/K] & n/K \text{ is an integer} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Is the upsampler system linear? Either prove that the upsampler is linear or provide a counterexample to show that it is not linear.

Problem 8. For the following two DT LTI systems, find the impulse response and the output of the system with input

$$x[n] = 2\delta[n] + 3\delta[n-1] \tag{4}$$

(a) $y[n] = x[n-1] + 2x[n-3]$

(b) $y[n+1] - 0.4y[n] = x[n]$

Problem 9. Consider the DT LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] \tag{5}$$

What is the output of the system to the following inputs?

(a) $x[n] = \delta[n-1] + 2\delta[n-3] + 2\delta[n-5]$

(b) $x[n] = \left(\frac{1}{3}\right)^n u[n-3]$