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## Homework 2

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**Problem 1** (SSTA Problem 2.23). Determine whether or not each of the following LTI systems is: (1) causal and/or BIBO-stable. If the system is not BIBO stable, provide a bounded input that yields an unbounded output.

(a)  $y(t) = \frac{dx}{dt}$

(b)  $y(t) = \int_{-\infty}^t x(\tau) d\tau$

(c)  $y(t) = \int_{-\infty}^t x(\tau) \cos(t - \tau) d\tau$

(d)  $y(t) = x(t + 1)$

(e)  $y(t) = \int_{t-1}^{t+1} x(\tau) d\tau$

(f)  $y(t) = \int_t^{\infty} x(\tau) e^{2(t-\tau)} d\tau$

**Problem 2** (Time-invariance). Determine whether the following systems are time-invariant or time-varying. Note the first two are CT and the second two are DT.

(a)  $y(t) = \sin(x(t))$

(b)  $y(t) = t \sin(x(t))$

(c)  $y[k] = 3(x[k] - x[k - 2])$

(d)  $y[k] = kx[k]$

**Problem 3** (Even and odd parts). Express the following signal as the sum of an even and an odd signal

$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

Plot the even and odd parts.

**Problem 4** (ECE 345 Final Exam, Fall 2017). Consider the system  $y[n] = x_{\text{even}}[n]$  that outputs the even part of a function. Determine whether or not this system is memoryless, time-invariant, linear, or stable.

**Problem 5** (SSTA Problem 2.23). Consider the CT system with the following input-output relation:

$$y(t) = x(t) \cos(120\pi t) + x(t - 3). \quad (2)$$

Determine if this system is stable/unstable, causal/noncausal, linear/nonlinear, and time-invariant or time-varying.

**Problem 6** (Lathi and Green 1.7-16). For the systems described by the following equations, with input  $x(t)$  and output  $y(t)$ , determine which are invertible and which are noninvertible. For the invertible systems, find the input-output relationship of the inverse system.

(a)  $y(t) = \int_{-\infty}^t x(\tau) d\tau$

(b)  $y(t) = \frac{dx(t)}{dt}$  for differentiable  $x(t)$ .

(c)  $y(t) = x(3t - 6)$

(d)  $y(t) = \cos(x(t))$

**Problem 7** (ECE 345 Final Exam, Fall 2017). Consider the upsampler by  $K$ :

$$y[n] = \begin{cases} x[n/K] & n/K \text{ is an integer} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Is the upsampler system linear? Either prove that the upsampler is linear or provide a counterexample to show that it is not linear.

**Problem 8.** For the following two DT LTI systems, find the impulse response and the output of the system with input

$$x[n] = 2\delta[n] + 3\delta[n - 1] \quad (4)$$

(a)  $y[n] = x[n - 1] + 2x[n - 3]$

(b)  $y[n + 1] - 0.4y[n] = x[n]$

**Problem 9.** Consider the DT LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] \quad (5)$$

What is the output of the system to the following inputs?

(a)  $x[n] = \delta[n - 1] + 2\delta[n - 3] + 2\delta[n - 5]$

(b)  $x[n] = \left(\frac{1}{3}\right)^n u[n - 3]$