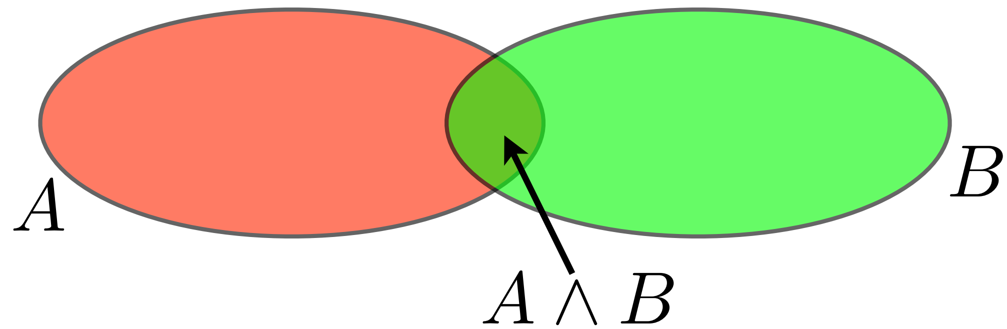
Conditional probability



Conditional probability of A given B:
probability of A given that B is true.
Example:
  A = "It is snowing"
  B = "It is January"

$$p(A|B) = \frac{p(A \wedge B)}{p(B)}$$

$p(A|B)$ is the probability of snow, given that it is January
$p(B|A)$ is the probability it is January, given that it is snowing

Note generally $p(A|B)$ and $p(B|A)$ are different quantities

Bayes' Rule

$$p(A|B) = \frac{p(A \wedge B)}{p(B)}$$

Conditional probability

$$p(B|A) = \frac{p(A \wedge B)}{p(A)}$$

$$p(A|B)p(B) = p(A \wedge B)$$

$$p(B|A)p(A) = p(A \wedge B)$$

$$p(B|A)p(A) = p(A|B)p(B)$$

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)}$$

Bayes' rule

Bayes and rationality

- Bayesian inference is a method for making rational inferences from data

  - In particular for solving induction problems rationally

- Bayesian inference is internally consistent

  - But no other method of probabilistic inference is

- Hence Bayesian inference is usually considered normative (i.e. “objectively correct”)

- Whether it is also descriptive of human inference is the subject of intense debate

Bayes' rule applied to categorization

- Bayes' rule: $$p(C|f) = \frac{p(C)p(f|C)}{p(f)}$$

f is a feature
C is a category

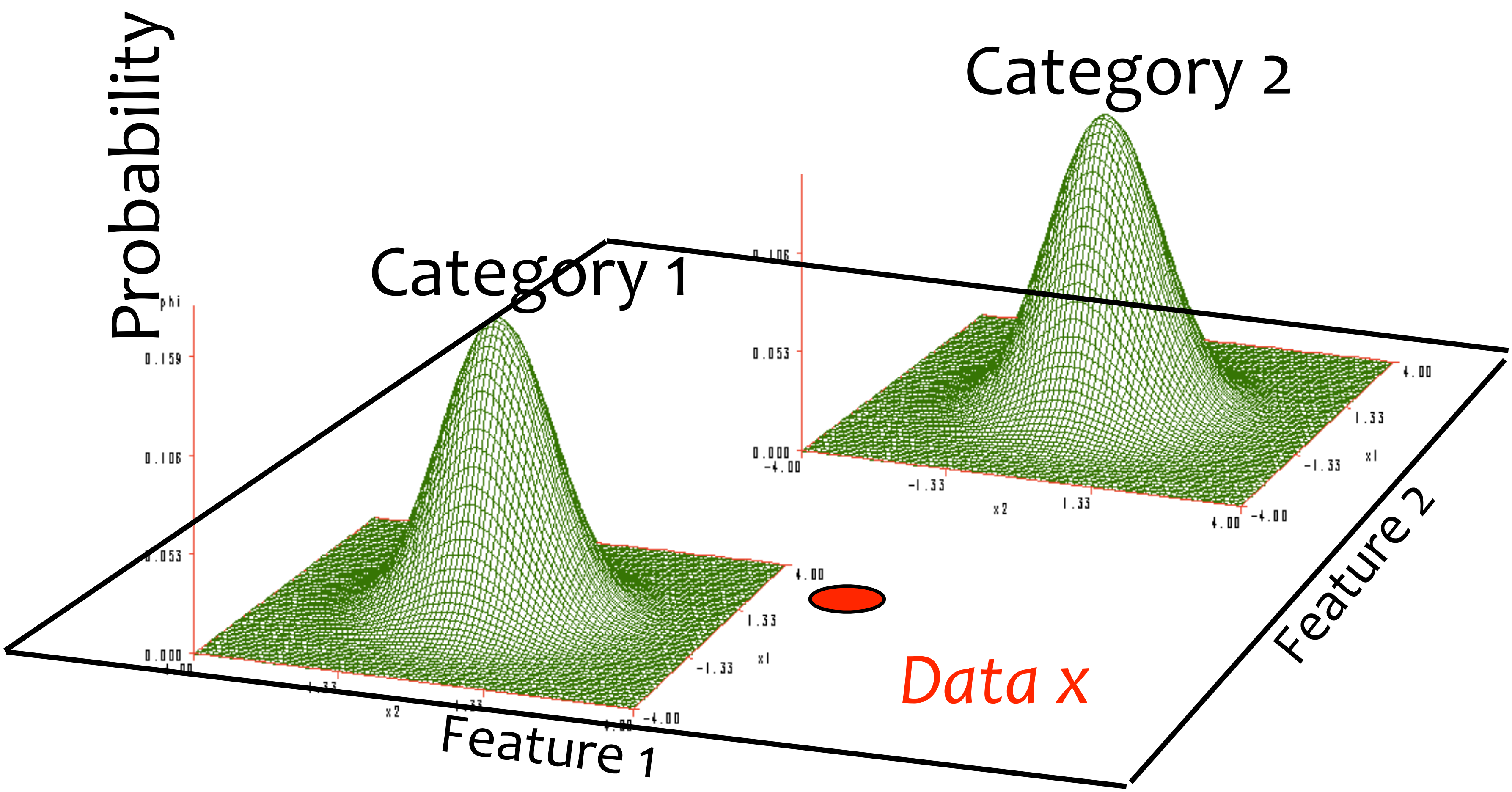
- $p(C)$ is the prior probability of a given category
- $p(f|C)$ is the conditional probability of feature *f* in category C, called the likelihood

  - a measure of fit between C and *f*

- $p(C|f)$ is the conditional probability of category C given feature *f*, called the posterior probability

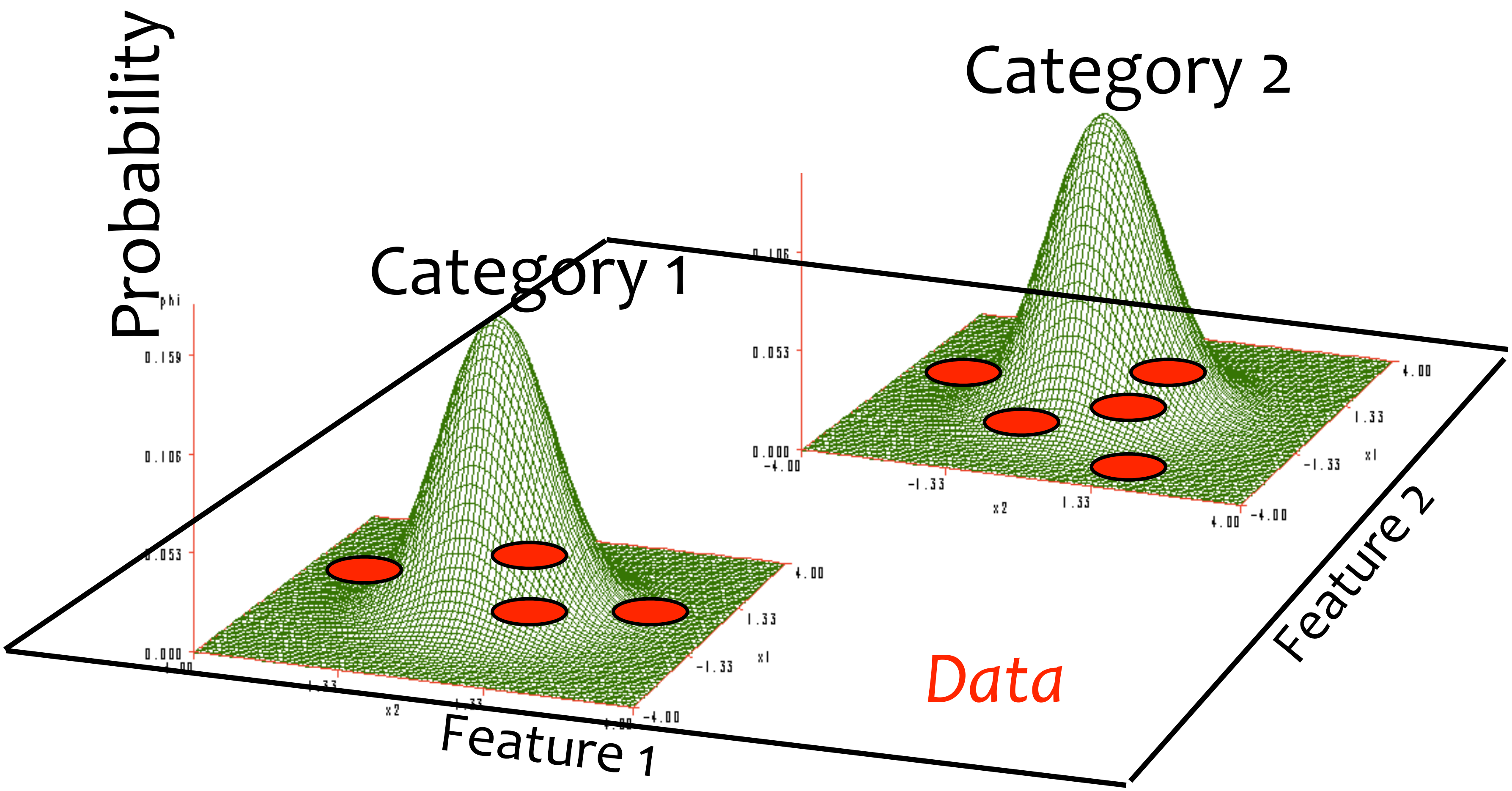
  - a measure of rational degree of belief that *f* is a C

Bayesian classification

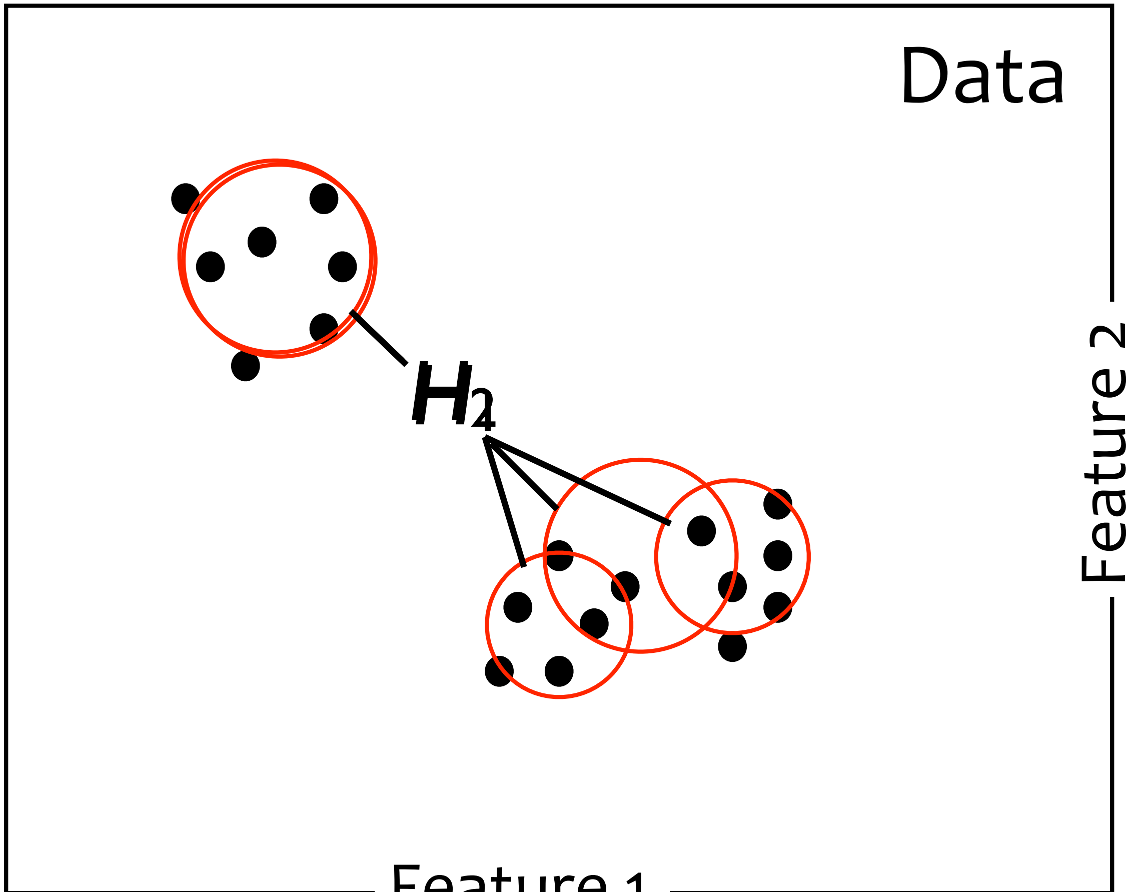


$$P(x \text{ in } C_i) = p(C_i|x) = \frac{p(C_i)p(x|C_i)}{\sum_i p(C_i)p(x|C_i)}$$

Bayesian concept learning



Complexity vs. data-fit in probabilistic concept learning



H_1: Two categories — simple

H_2: Three categories — more complex, but fits the data better

Bayes' rule tells you which makes more sense.

“Theory theory” (Murphy & Medin, 1985)

- All these models that treat classification as a “math problem” are missing the point
- Categories and concepts involve background knowledge and context
- They depend on a “theory” of the world
- Philosophical holism

  - Concepts can’t be understood in isolation

- Conceptual coherence

  - Why is *dog* a reasonable category but *women, fire and dangerous things* is not?

What is conceptual coherence?

- Prototype theory, exemplar theory, etc., don’t really address the problem of conceptual coherence
- “Less coherent” concepts may be harder to classify

  but all sets of examples have prototypes and exemplar representations!

- But what is conceptual coherence anyway?