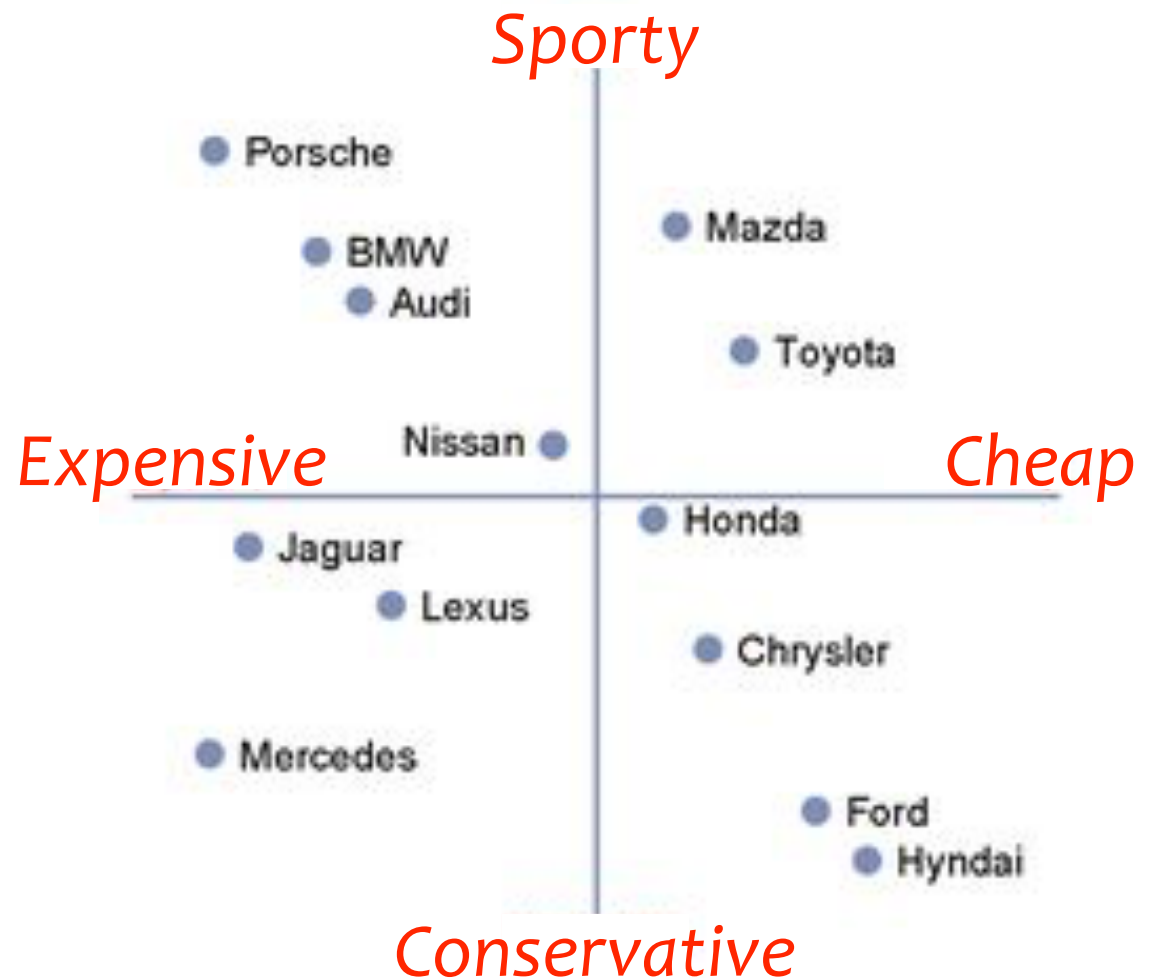# The Geometric Model of similarity

- One view of similarity is that it is analogous to proximity in some mental space

  - i.e. dissimilarity <-> distance

- That is, mental representation of the perceptual features takes the form of a mental space analogous to a physical space

- Multidimensional Scaling (MDS) is a statistical technique for visualizing this space

- Given a set of items and judgments of dissimilarity among the items, MDS finds positions in an imaginary space such that inter-item distances match judged dissimilarity as closely as possible

- Starting in about 1957, psychologists have plotted MDS spaces for thousands of types of items

Multidimensional scaling (MDS)

MDS solution:

• Subjects are asked to rate the similarity of pairs of objects

• Multidimensional Scaling (MDS) is used to reconstruct the corresponding distances in the mental space

Sporty

● Porsche

● Mazda

● BMW
● Audi

● Toyota

Nissan ●

Expensive

Cheap

● Honda

● Jaguar

● Lexus

● Chrysler

● Mercedes

● Ford
● Hyndai

Conservative

Mechanics of MDS

Subjective similarity ratings:

tiger	lion	?
tiger	wolf	?
lion	wolf	?
tiger	deer	?
lion	cow	?
wolf	deer	?
lion	deer	?

deer
cow

wolf

tiger

lion

2D solution

more MDS examples

Actions
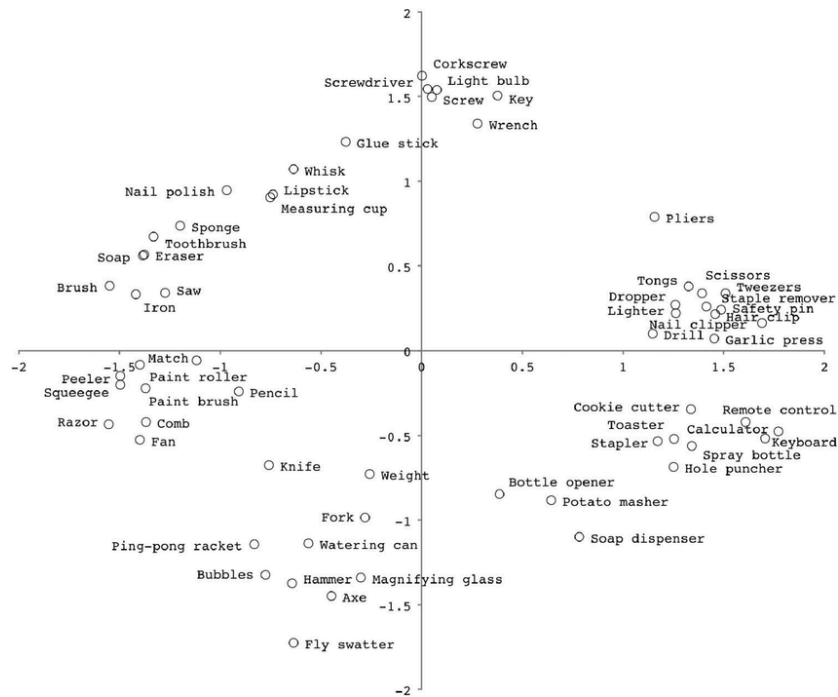
Soft Drink Consumer Perceptions
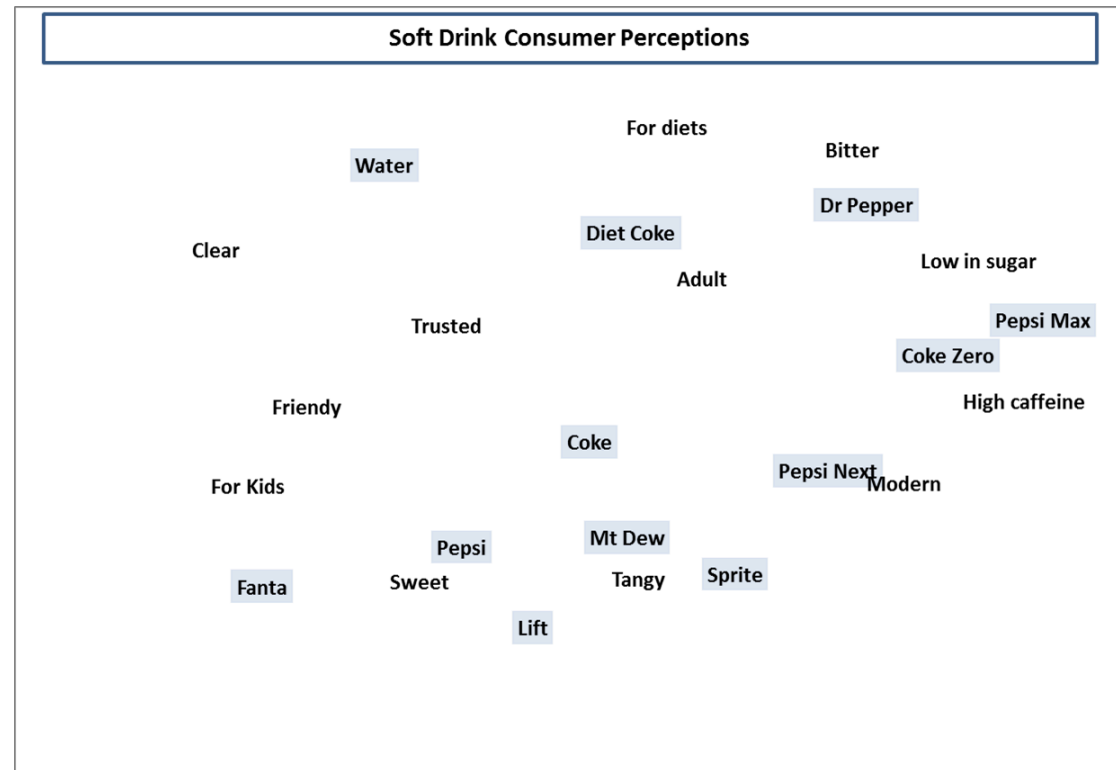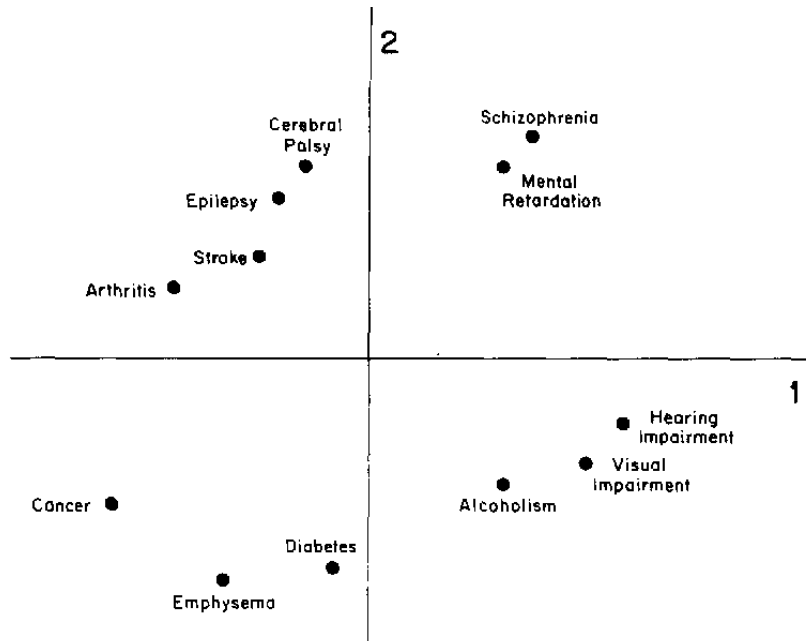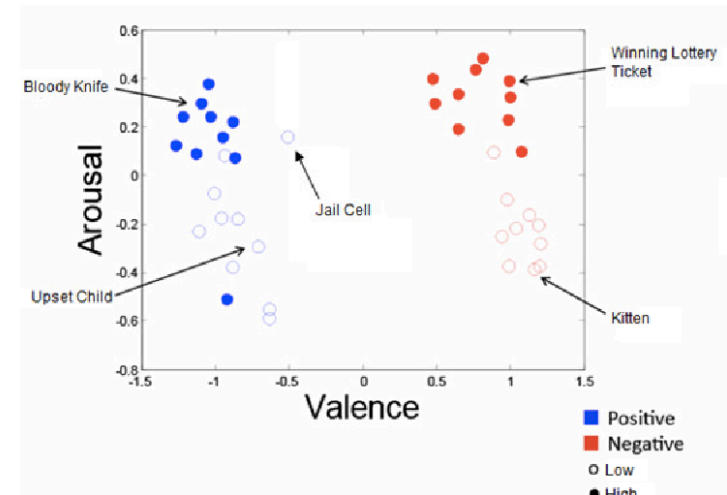
Soft drinks

more MDS examples

Stereotypes of disorders

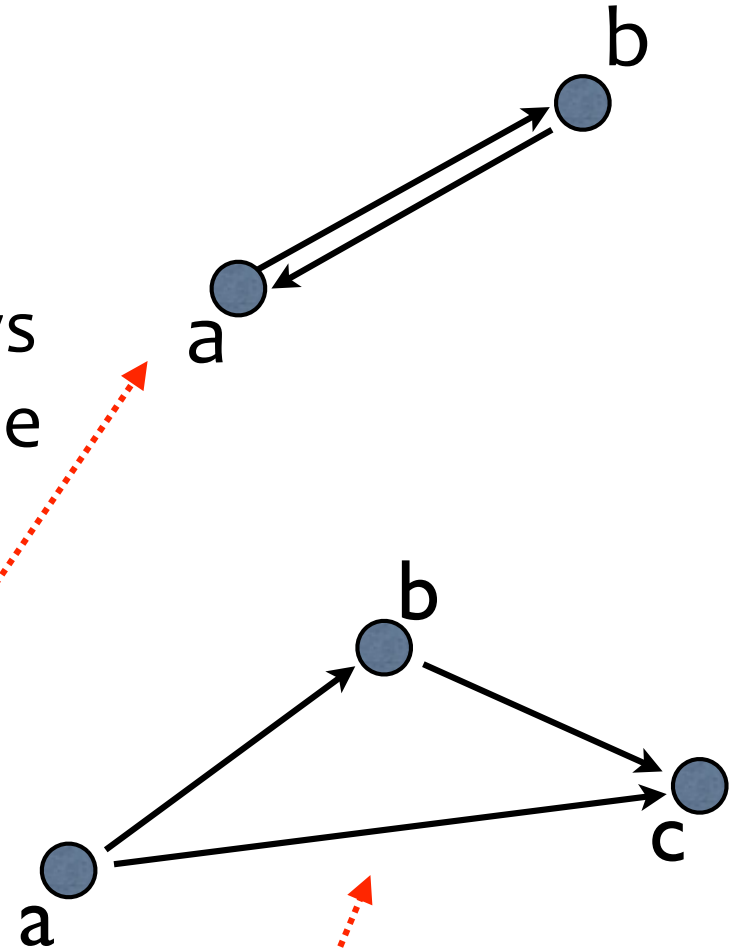Mental images

Questioning the geometric model

- But about 1980, Tversky & Kahneman questioned whether mental dissimilarity actually obeys the mathematical laws of distance

- These laws include the metric axioms or distance axioms, including:

  - Symmetry: $d(a,b) = d(b,a)$

  - Triangle inequality: $d(a,b) + d(b,c) \geq d(a,c)$
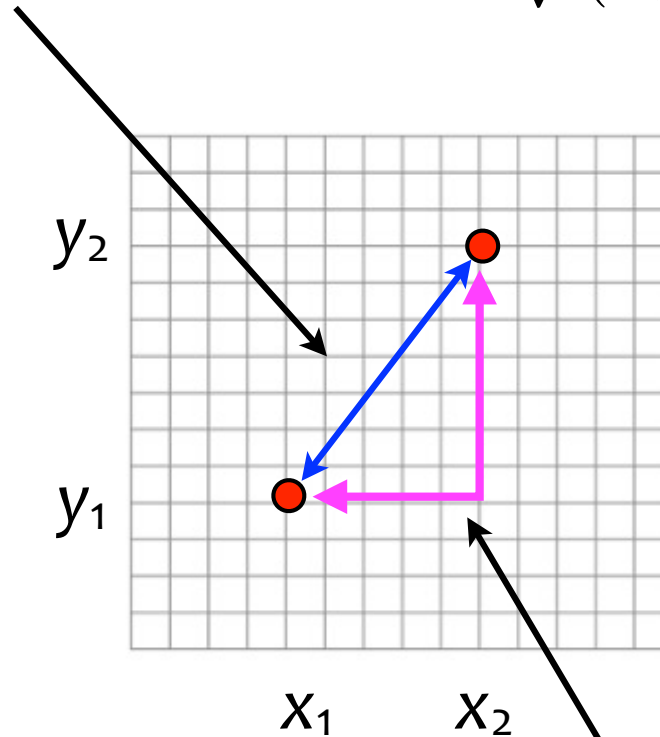
  i.e.: "the shortest distance between two points is a straight line"

There is more than one way to measure distance

Euclidean distance ("as the crow flies") = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Both metrics obey the distance axioms

y_2

y_1

x_1 x_2

City-block distance $= |x_1 - x_2| + |y_1 - y_2|$

General formula (Minkowski): $D = \left(\sum (\Delta F)^r \right)^{1/r}$

$r = 1 \rightarrow$ city-block $r = 2 \rightarrow$ Euclidean

Tversky & Kahneman: Does similarity obey the distance axioms?

- Symmetry: No.
  - "An apple is similar to a pomegranate" (less preferred) vs
  - "A pomegranate is similar to an apple" (more preferred)

- Triangle inequality: No.
  - Jamaica is similar to Cuba; (very similar)
  - Cuba is similar to North Korea (very similar)
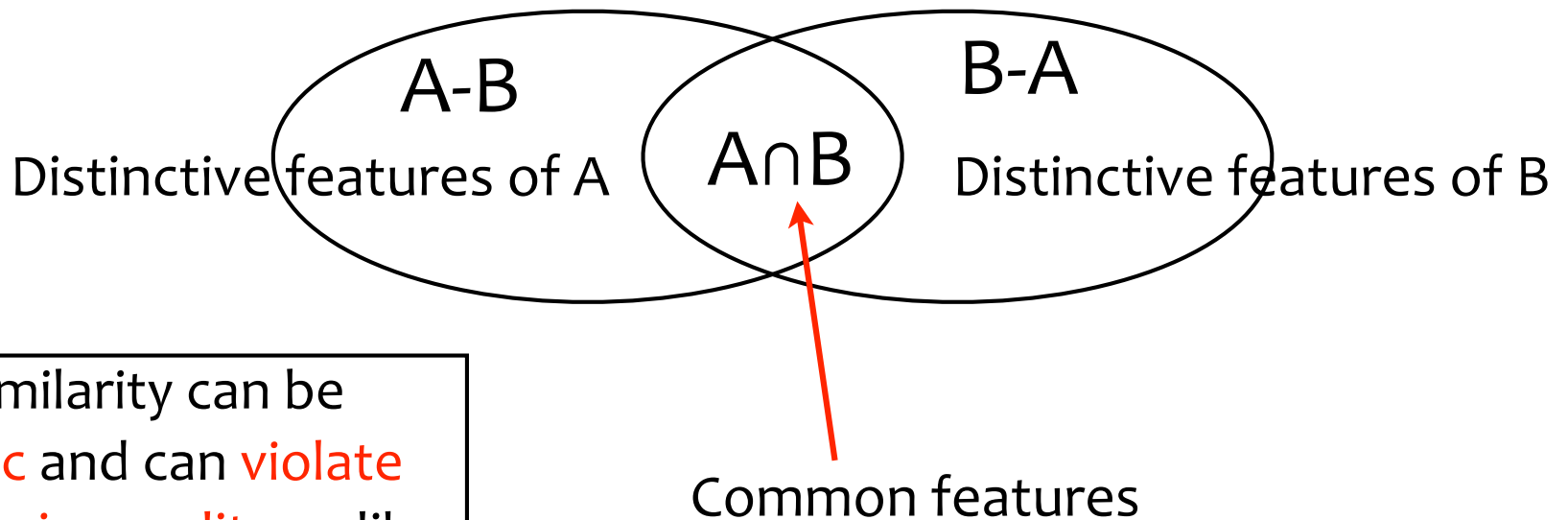 - Is Jamaica similar to North Korea? (very dissimilar)

Featural models of similarity

Features here are discrete attributes of objects

Kahneman & Tversky's "Contrast model" of similarity:

$$sim(A,B) = w_1\, f(A \cap B) - w_2\, f(A\text{-}B) - w_3\, f(B\text{-}A)$$

A-B

Distinctive features of A

A∩B

B-A

Distinctive features of B

Common features

Featural similarity can be asymmetric and can violate the triangle inequality — like human similarity judgments