Final Exam Equation Sheet 750:124 Spring 2022

Kinematic Equations

Linear, constant acceleration

$x = x_i + v_i t + \frac{1}{2}at^2$
$v = v_i + at$
$v^2 = v_i^2 + 2a(x_f - x_i)$

Rotational, constant acceleration

1 revolution = 2π radian = 360°

$\omega = \omega_i + \alpha t$

$\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$

Circular motion around the Center of Mass

$s = r\theta$
$v = r\omega$ $\quad \vec{\omega} = \vec{r} \times \vec{v}/r^2$ $\quad \vec{v} = \vec{\omega} \times \vec{r}$
$a = r\alpha$ $\quad \vec{\alpha} = \vec{r} \times \vec{a}/r^2$ $\quad \vec{a} = \vec{\alpha} \times \vec{r}$
$KE_{rotation,cm} = \frac{1}{2}I_{cm}\omega_{cm}^2$

Rolling without Slipping

$v_{CM} = R\omega$
$a_{CM} = R\alpha$
$KE_{rolling} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega_{cm}^2$

Vector Products

$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta$

$\vec{A} \times \vec{B} = (a_2b_3 - a_3b_2)\hat{\imath} - (a_1b_3 - a_3b_1)\hat{\jmath} + (a_1b_2 - a_2b_1)\hat{k}$

Newton's Laws for Rotational Motion

Moment of Inertia around the Center of Mass

$I = \Sigma\, m_i r_i^2$ for point masses

$I_{hoop} = mR^2$ for hoop

$I_{disk} = \frac{1}{2}mR^2$ for disk

$I_{rod} = \frac{1}{12}mL^2$ for rod of total length L

$I_{solid\ sphere} = \frac{2}{5}mR^2$ for solid sphere

$I_{hollow\ sphere} = \frac{2}{3}mR^2$ for hollow sphere

Torque and angular momentum

$\vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}|\sin(\varphi)$
$\vec{L} = \vec{r} \times \vec{p} = m|\vec{r}||\vec{v}|\sin(\varphi)$

$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$
$\vec{L} = I\vec{\omega}$ $\quad \vec{p} = m\vec{v}$
$\tau = I\alpha$ $\quad \vec{F} = m\vec{a}$

Conservation of Angular Momentum:

$I_i\omega_i = I_f\omega_f$ (if $\sum\vec{\tau}_{ext} = 0$)

Static Equilibrium Conditions:

$\sum\vec{F}_{ext} = 0$
$\sum\vec{\tau}_{ext} = 0$

Kepler's Laws and Universal Gravitation

$G = 6.67 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$

$\vec{a}_G(r) = -\frac{GM}{r^2}\hat{r}$

$\vec{F}_G(r) = -\frac{GMm}{r^2}\hat{r}$

$g_{Earth} = \frac{GM_{Earth}}{r_{Earth}^2} \approx 9.8\, m/s^2$

$U_G(r) = \int F_G(r)dr = -\frac{GMm}{r}$

Circular Orbit: $\vec{F}_G(r) = m\vec{a}_{cent}$

Kepler's 3rd Law for Circular Orbits: $T^2 = \frac{4\pi^2}{GM}r^3$

Simple Harmonic Oscillator

$F_{spring} = -kx$
$x(t) = A\cos(\omega t + \varphi)$
$T = \frac{2\pi}{\omega}$ $\quad f = \frac{1}{T}$
$\omega = \sqrt{\frac{k}{m}}$ (spring) $\quad \omega = \sqrt{\frac{g}{L}}$ (simple pendulum)
$v(t) = -\omega A\sin(\omega t + \varphi)$
$a(t) = -\omega^2 A\cos(\omega t + \varphi)$

Waves & Sound

Plane Wave:

$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x \pm vt)\right] = A\sin(kx \pm \omega t)$
$v = \pm\lambda f = \pm\frac{\omega}{k}$, $k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{T}$

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Sound Intensity and Level

Threshold of hearing: $I_o = 10^{-12}\,\frac{W}{m^2}$

$\beta(dB) = 10\,log\left(\frac{I}{I_0}\right)$

$I = \frac{P_{av}}{Area}$; $I = \frac{P_{av}}{4\pi r^2}$ for spherical waves

Travelling Wave on String

$v = \sqrt{\frac{F_T}{\mu}}$, μ is linear mass density (taut string)

$K_\lambda = \frac{1}{4}\mu A^2\omega^2\lambda$ $E_{tot,\lambda} = \frac{1}{2}\mu A^2\omega^2\lambda$

$P_\lambda = \frac{1}{2}\mu A^2\omega^2 v$

Interference

(Path Length Difference of Two In-Phase Sources)

Destructive: $|r_1 - r_2| = n\frac{\lambda}{2}, n = 1,3,5\ldots$

Constructive: $|r_1 - r_2| = n\lambda, n = 1,2,3\ldots$

Standing Sound Waves in Pipes (Air Tubes)

$s_n(x,t) = s_{0n}\,sin(k_n x)\,cos(\omega_n t)$

If both ends are open or closed,

$L = n\frac{\lambda_n}{2}, n = 1,2,3,\ldots$

$f_n = nf_1, n = 1,2,3,\ldots$

If one end is open and one is closed,

$L = n\frac{\lambda_n}{4}, n = 1,3,5,\ldots$

$f_n = nf_1, n = 1,3,5,\ldots$

$v_{air} = 331\frac{m}{s}\sqrt{1 + T_C/273°\text{C}}$

Thermodynamics

Linear Expansion: $\Delta L = \alpha L_o \Delta T$

$R \approx 8.314\,J/mol\cdot K$

$N_A \approx 6.02\,x\,10^{23} molecules/mole$

$k_B \approx 1.38\,x\,10^{-23} J/K$

$T_C = \frac{5}{9}(T_F - 32°)$

$T = T_C + 273.15K$

$KE_{transl} = \frac{3}{2}k_B T$; average for molecules in gas

$v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}}$

$PV = nRT = Nk_B T$

$Q = mc(T_f - T_i) = nc(T_f - T_i)$

$c_V = \frac{3}{2}R,\ c_P = \frac{5}{2}R$ monatomic ideal gas

Adiabatic Processes: $pV^\gamma = constant$; $\gamma = \frac{c_P}{c_V}$

1st Law of Thermodynamics

$\Delta E_{int} = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$ monatomic ideal gas

$\Delta E_{int} = Q + W_{on\ gas}$

$\Delta E_{int} = Q - W_{by\ gas}$

The work done **by the gas**:

$W_{by\ gas} = \int_{V_i}^{V_f} PdV$, $W_{isobaric} = P(V_f - V_i)$

$W_{isothermal} = nRT\ln\left(\frac{V_f}{V_i}\right)$

Heat Engines

$W = |Q_h| - |Q_c|$

$e = \frac{|W|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$ real efficiency

Ideal/Carnot cycle: $\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}$; $e_{ideal} = 1 - \frac{T_c}{T_h}$

Entropy

$dS = \frac{dQ}{T}$ $\Delta S = \int_i^f \frac{dQ}{T}$

$S = k_B \ln(\Omega_i)$, Ω_i number of microstates

2nd Law of Thermodynamics

$\Delta S_{universe} \geq 0$ for any real process

$\Delta S_{universe} = \Delta S_{system} + \Delta S_{surroundings}$

Entropy Change of an Ideal Gas

$\Delta S = \frac{f}{2}nR\ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$; *f = number of degrees of freedom*

$\Delta S = nc_V\ln\left(\frac{T_f}{T_i}\right)$ Isochoric Process

$\Delta S = \frac{Q}{T}$ Isothermal Process

$\Delta S = nc_P\ln\left(\frac{T_f}{T_i}\right)$ Isobaric Process

Entropy Change of a solid/liquid

$\Delta S = mc\ln\left(\frac{T_f}{T_i}\right)$ for temperature changes

$\Delta S = \frac{mL}{T}$ for phase changes

$Q_f = mL_f = T_f(S_{Liquid} - S_{Solid})$

$Q_v = mL_v = T_v(S_{Gas} - S_{Liquid})$