# Einstein's Relativity

- Modification of space and time themselves
  - Already discussed
- Modification of Maxwell E&M?
  - No, already OK.
- Modification of Newton mechanics? (Mass, force, acceleration, momentum, energy, …)
  - Yes, changes needed! … $\Rightarrow$ $E = mc^2$

## Clicker

It is said that Einstein, in his teenage years, asked the question: "What would I see in a mirror if I carried it in my hands and ran at very close to the speed of light?" How would you answer this question?

A. The mirror would be totally black.
B. You would see the same thing as if you were at rest.
C. The image would be distorted.
D. The image would be greatly red-shifted due to the Doppler effect.

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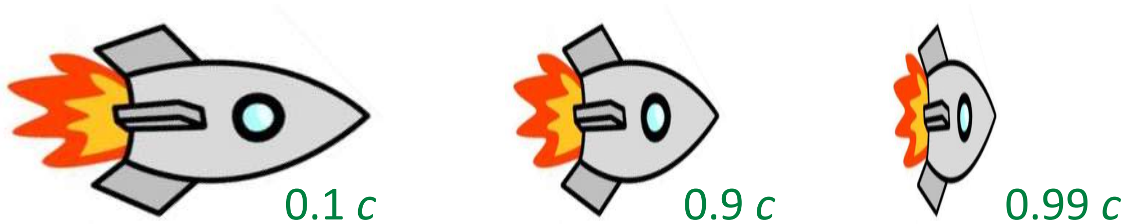
# Maximum speed *c*




- Time dilation becomes infinite as speed *v* approaches *c*.
- Length contraction becomes zero.
- This suggests that nothing can go faster than the speed of light *c.*
- Indeed: Objects accelerate less and less as they approach *c.*

- Let's keep

$$\boxed{\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}}$$

- This means mass increases for fast-moving objects!

# Mass in relativity theory

- Rest mass $m_0$ is the mass of an object measured when it is brought to rest. This quantity is always the same (a constant).
- Relativistic mass (inertial mass) $m_{\text{rel}}$ is the mass that determines the acceleration produced by a force. It depends on velocity.

$$m_{\text{rel}} = \gamma m_0 \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Mass and energy

- Relativistic Mass increases for fast-moving objects
- Energy increases for fast-moving objects
- So does this mean that mass and energy are related?
- Yes: Einstein's theory predicts

$$(\text{Change of } E) = (\text{Change of } m) \times c^2$$

$$(\Delta E) = (\Delta m) \times c^2$$

- Or, more generally,

Total energy:

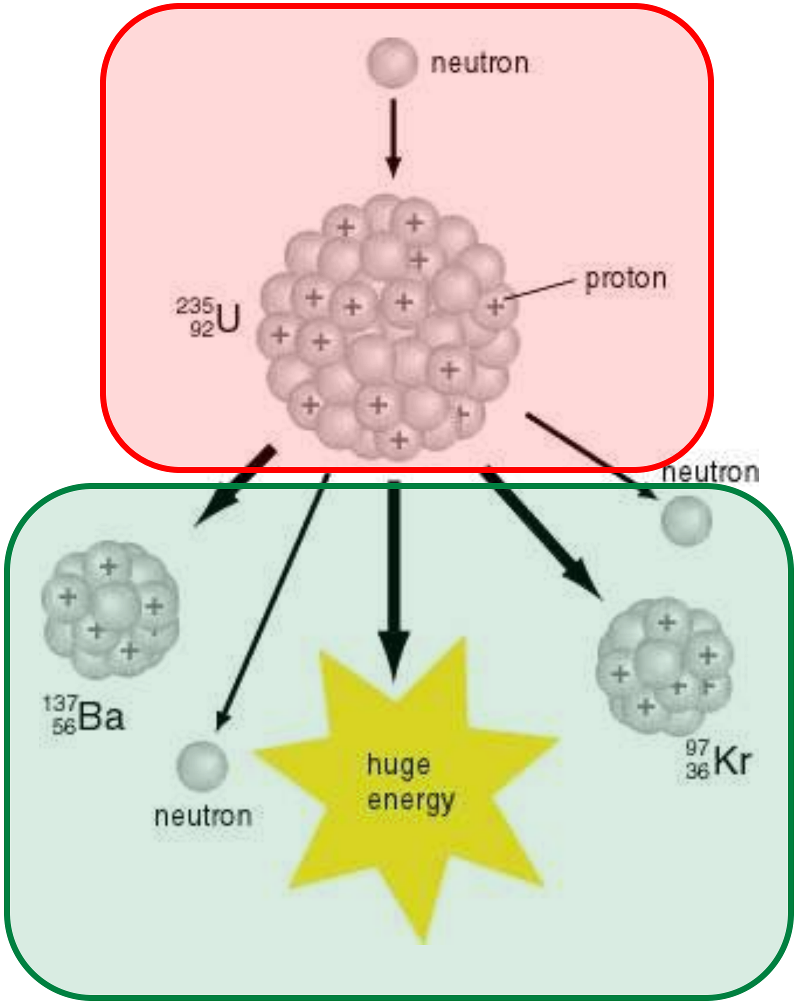
$$E = m_{rel}\, c^2$$

Kinetic energy:

$$K = E - m_0\, c^2$$

$$K = (m_{rel} - m_0)\, c^2$$

# Example: Nuclear fission

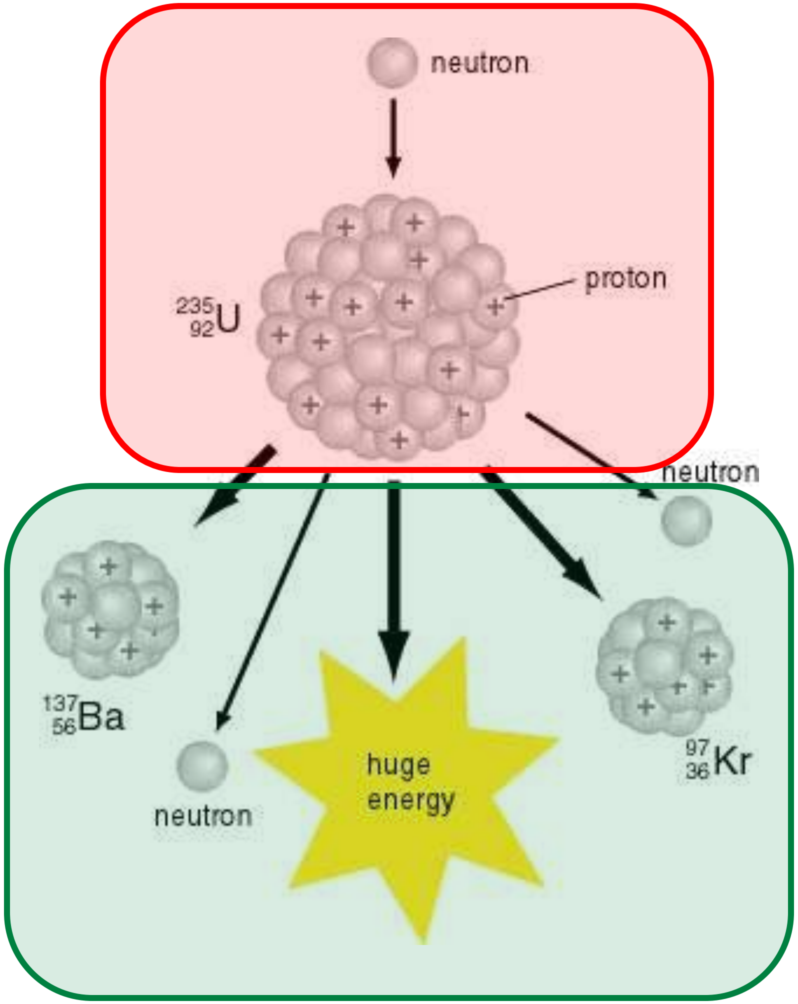



Total rest mass of ingredients

is greater than

Total rest mass of products

# Example: Nuclear fission




However,
total energy
is conserved!

(Therefore, relativistic mass is conserved.)

The “missing” rest energy has been converted to kinetic energy of the products, (plus radiation energy).

# Example: Nuclear fission

Suppose:

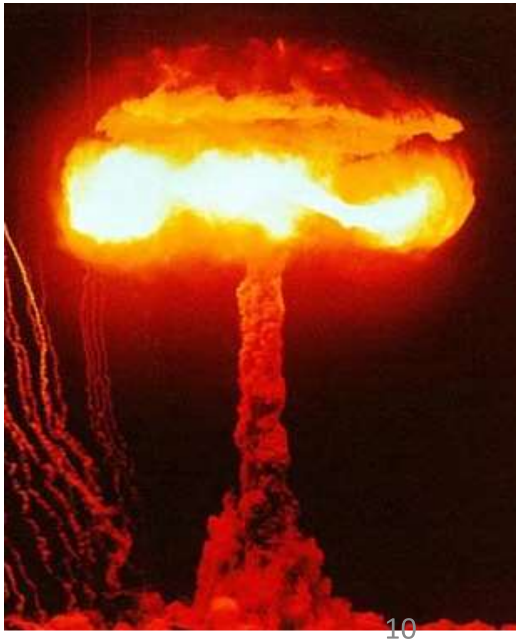
Rest mass of 1 kg $\Rightarrow$ 0.999 kg + Energy

Energy = $mc^2$

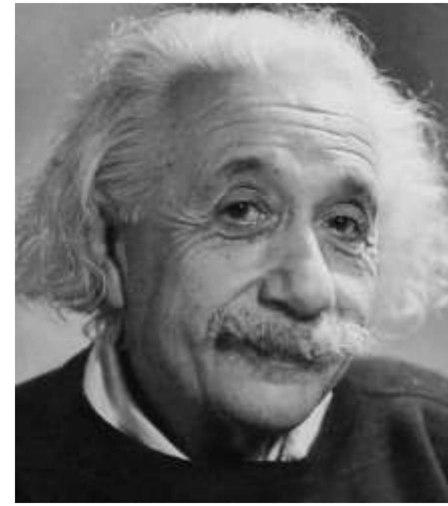
$= (10^{-3}\text{kg}) \times (3\times10^{8}\text{m/s})^2$

$= 10^{13}$ J

→ Equivalent to of 2.5 kilotons of TNT!



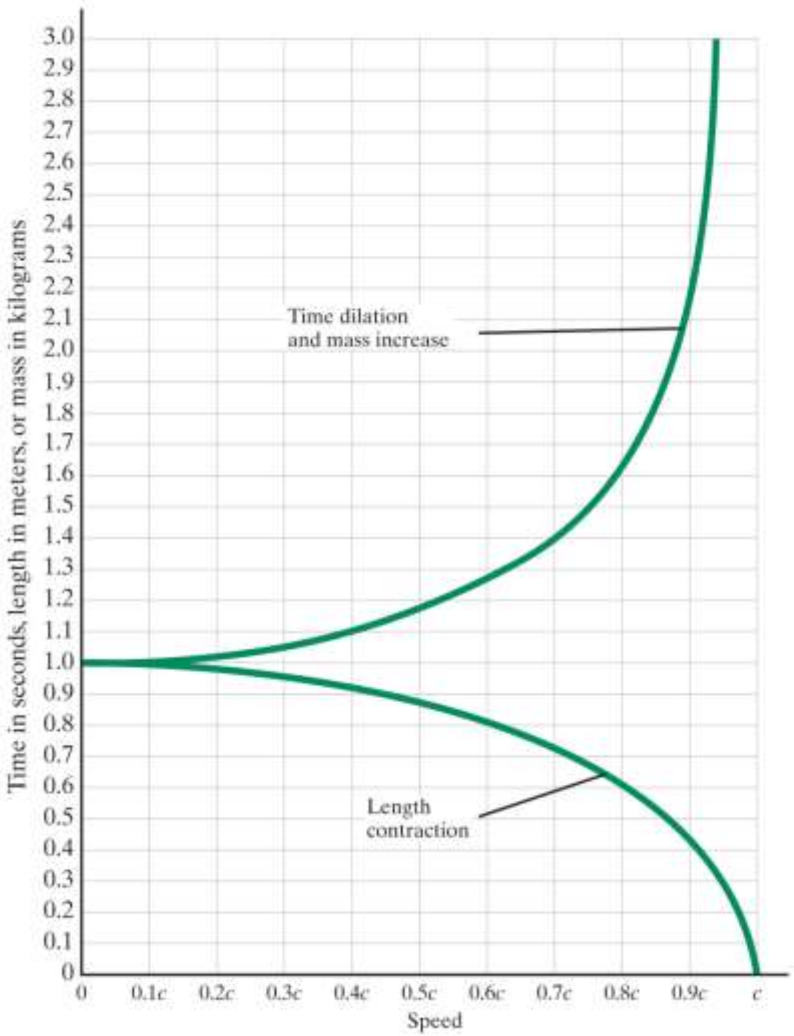
# Albert Einstein



"It followed from the special theory of relativity that mass and energy are both but different manifestations of the same thing—a somewhat unfamiliar conception for the average mind. Furthermore, the equation $E$ is equal to $mc^2$, in which energy is put equal to mass, multiplied with the square of the velocity of light, showed that very small amounts of mass may be converted into a very large amount of energy and vice versa. The mass and energy were in fact equivalent."

# The Relativity of Space, Time and Mass

- This diagram shows time dilation, mass increase, and length contraction as the speed approaches the speed of light.
- Close to the speed of light, the mass increases without limit; this is why no mass can move as fast as light.



Clicker

To what form or forms of energy does the relationship $E=mc^2$ apply?

A) Kinetic energy only.

B) Thermal energy only.

C) Nuclear energy only.

D) Kinetic and nuclear energy only.

E) All forms of energy.

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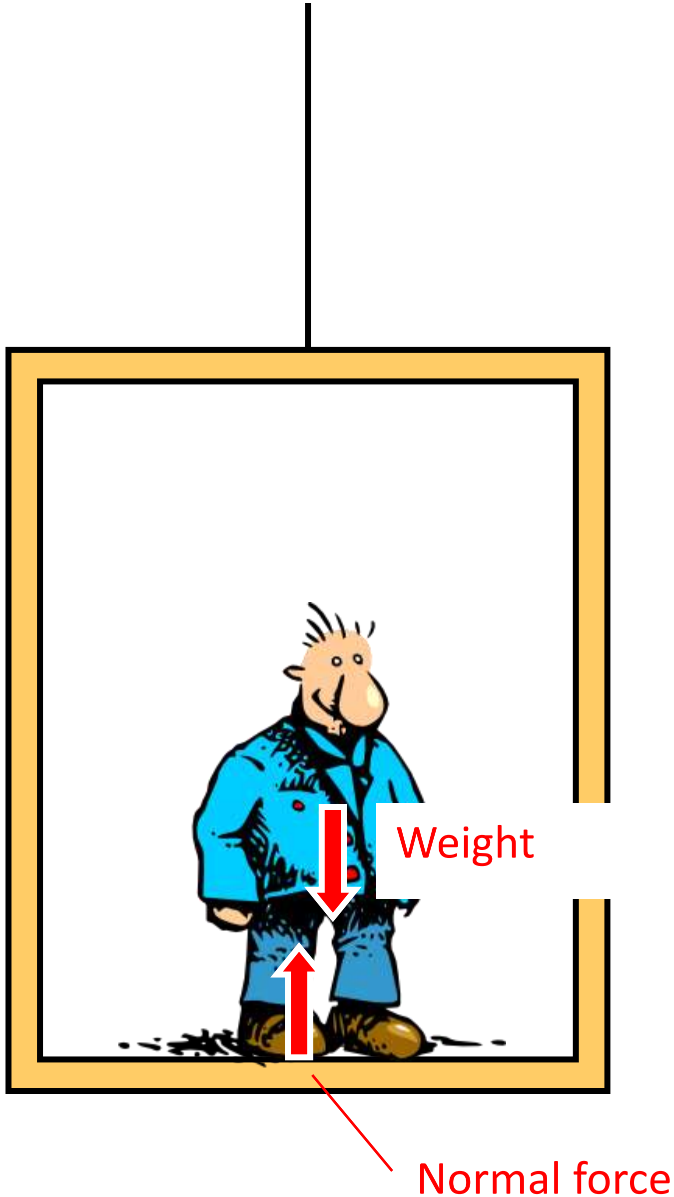
# General Relativity

"Special Relativity" (Einstein 1905):

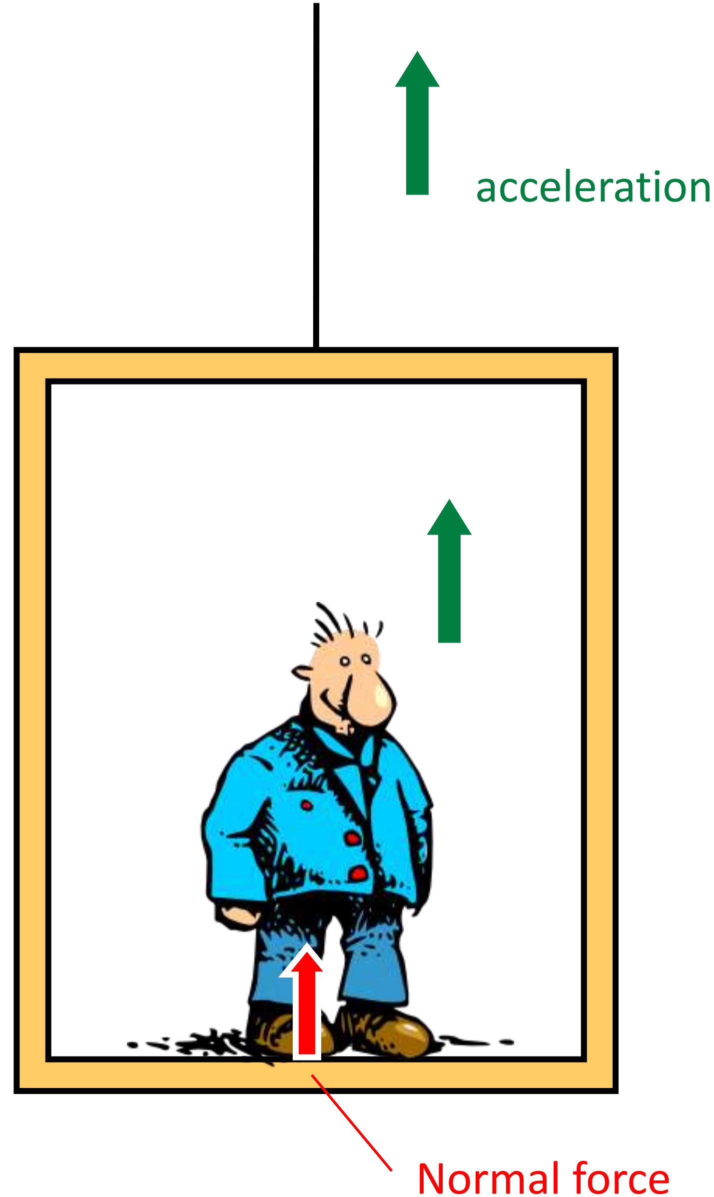
- How to relate physics as observed from two **inertial** (non-accelerating) **reference frames.**

"General Relativity" (Einstein 1915):

- How to relate physics as observed from accelerating reference frames.
- **Theory of gravity.**




Gravity is present




Gravity is absent

# Equivalence Principle

*You cannot do any experiment whatsoever that can distinguish between the effects of being in an accelerating frame of reference vs. the effects of gravity!*

# Clicker

You are in a spaceship with no windows, radios, or other means to check outside. How would you determine if the spaceship is at rest or moving at constant velocity?

A. By determining the apparent velocity of light in the spaceship.
B. By checking your precision watch. If it’s running slow, then the ship is moving.
C. By measuring the lengths of objects in the spaceship. If they are shorter than usual, then the ship is moving.
D. You should give up because you’ve taken on an impossible task.

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# Clicker

You are in a spaceship with no windows, radios, or other means to check outside. How would you determine if the spaceship is resting on the surface of a planet or using its thrusters to accelerate in outer space?

A. By determining the apparent velocity of light in the spaceship.
B. By checking your precision watch. If it's running slow, then the ship is in the gravitational field of the planet.
C. By measuring the path of a beam of light. If it bends, then you are in a gravitational field.
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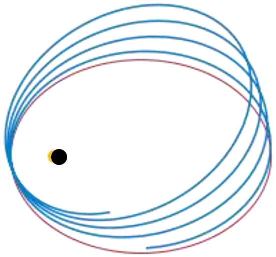
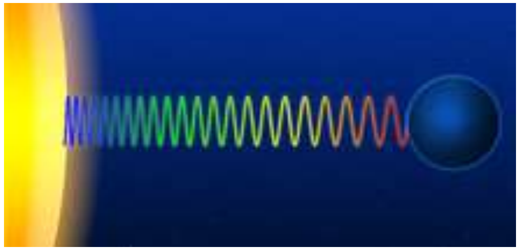
# Predictions of General Relativity

- Elliptical orbits are not *exactly* elliptical



Observed: Precession of orbit of mercury

- Clocks run slower in a gravitational field



Observed: GPS systems

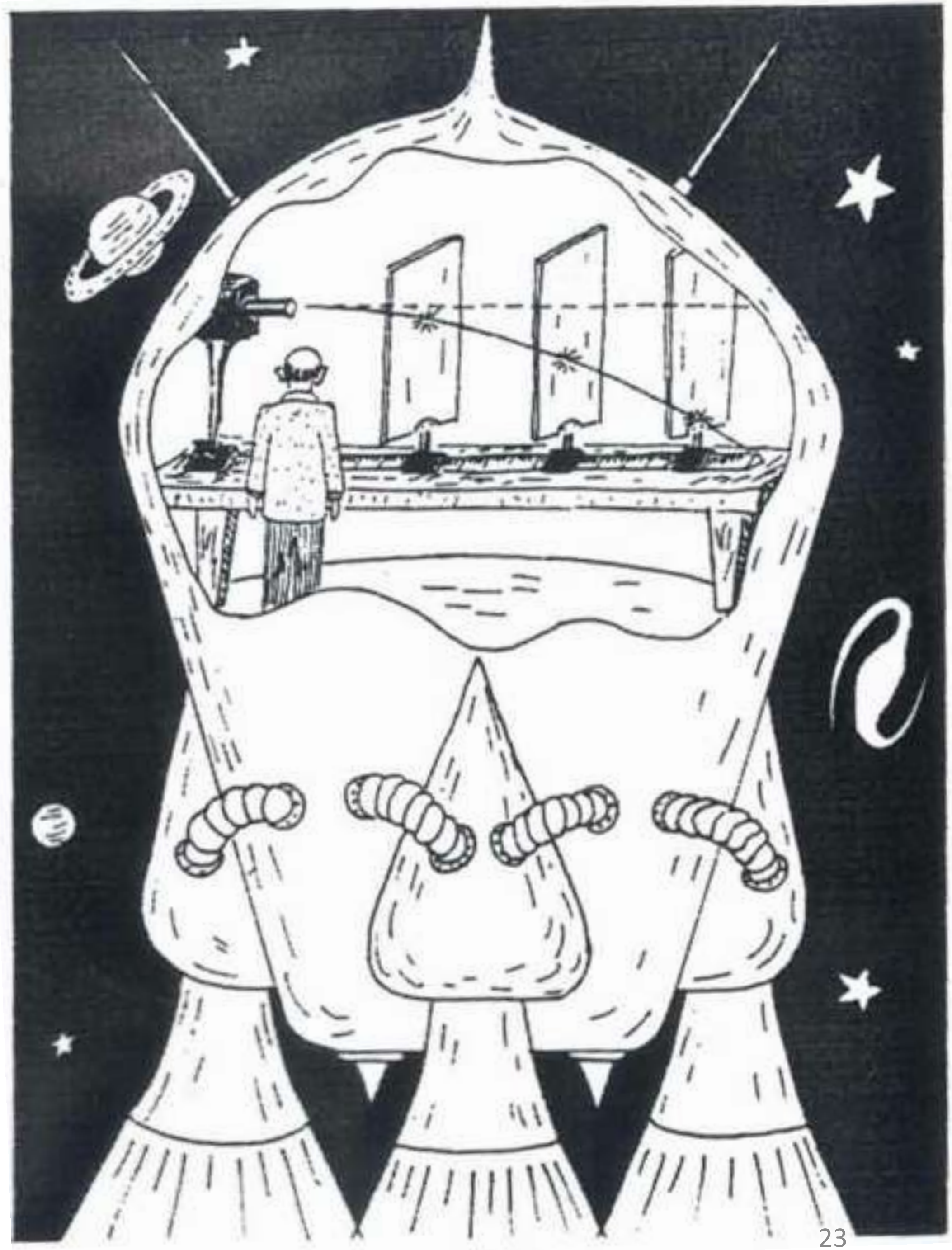
- Light is bent by gravity

Observed: Eclipses; Gravitational lenses

## Even optics!

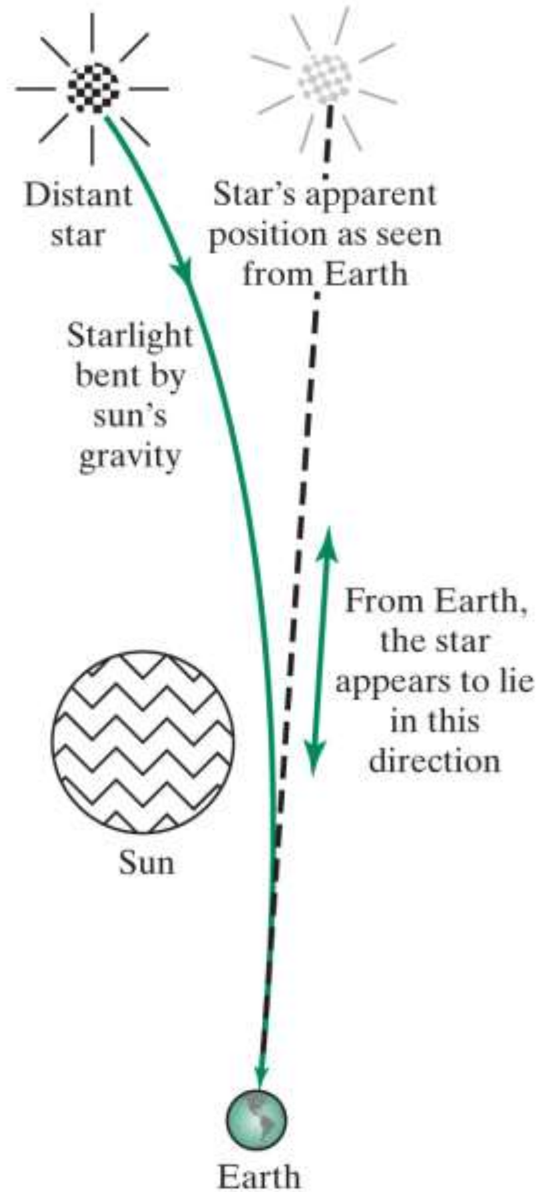
## Light bends in a gravitational field!

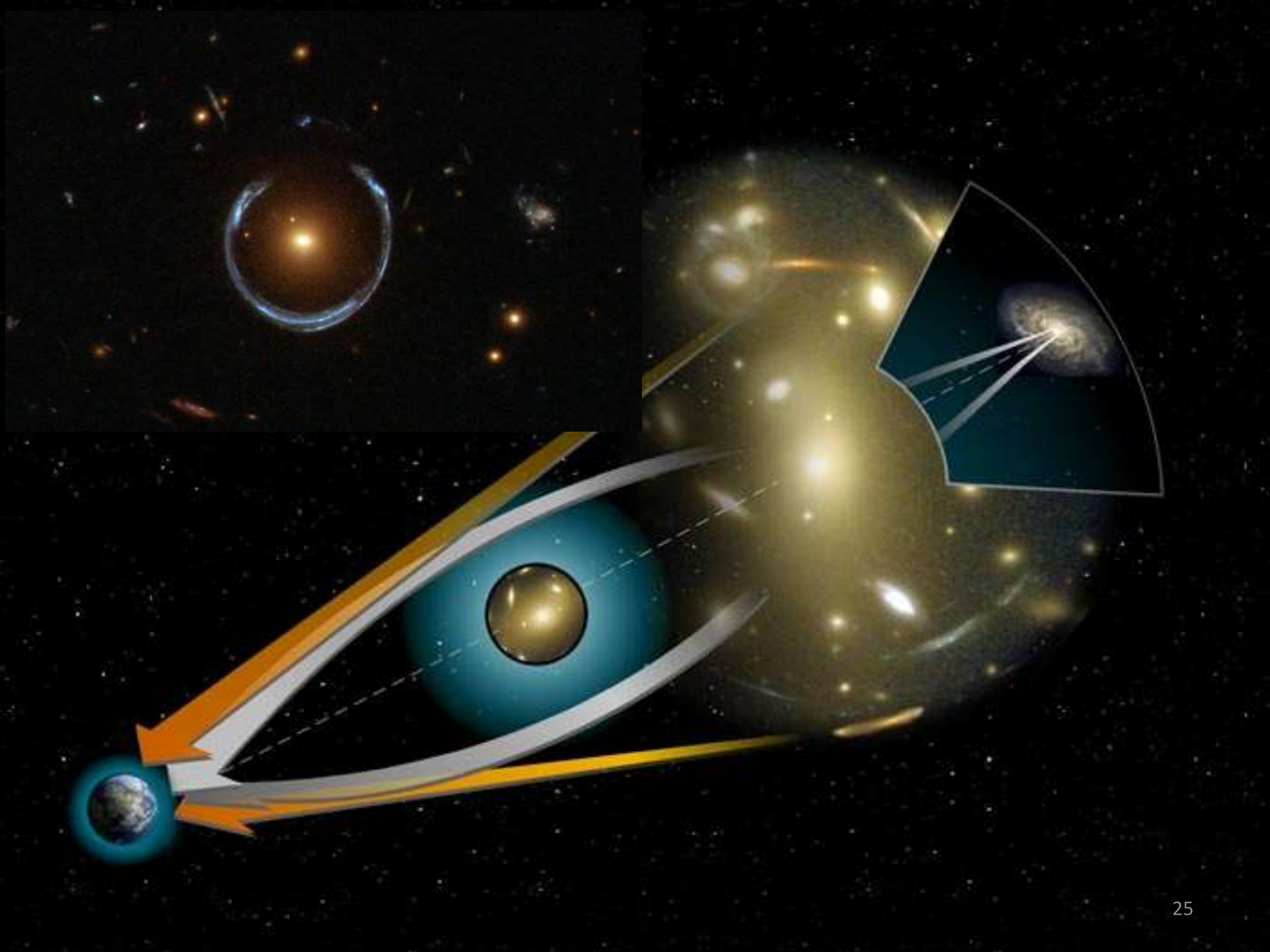
- Consider a rocket in motion **accelerating upwards**.
- Suppose a beam of light enters from the left.
- In an inertial frame, the light is traveling in a straight path.
- In the frame of the rocket, the light appears to bend downward.
- But gravitation and acceleration are equivalent concepts. Therefore, **light will be bent by gravity.**



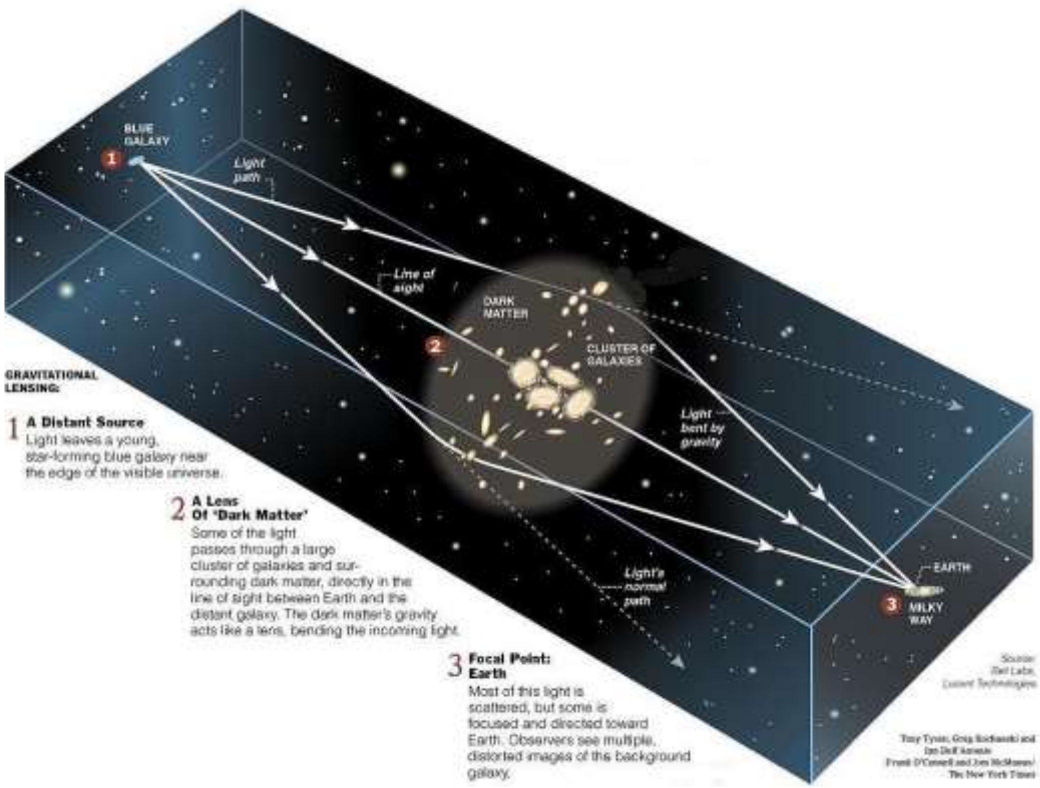
# Test of GR: Bending of light

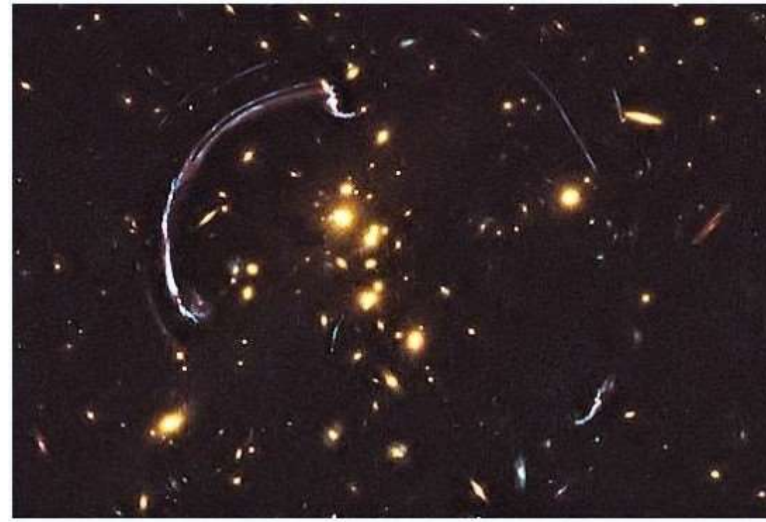
- The bending of light by a gravitational field has been well confirmed, starting in 1919 when stars were seen during a total solar eclipse that should have been behind the sun.
- During a solar eclipse of the sun by the moon, most of the sun's light is blocked on Earth. During a 1919 eclipse, starlight was bent as it passed near the sun which caused the star to appear displaced.
- Einstein's general theory predicted a deflection of 1.75 arc seconds, and two measurements in South America and in Africa found $1.98 \pm 0.16$ and $1.61 \pm 0.40$ arc seconds.
- Since then, many experiments have confirmed Einstein's predictions about the bending of light with ever increasing accuracy.

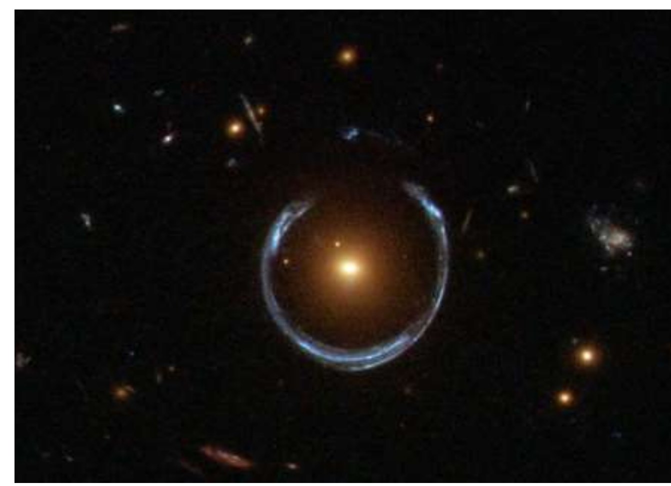




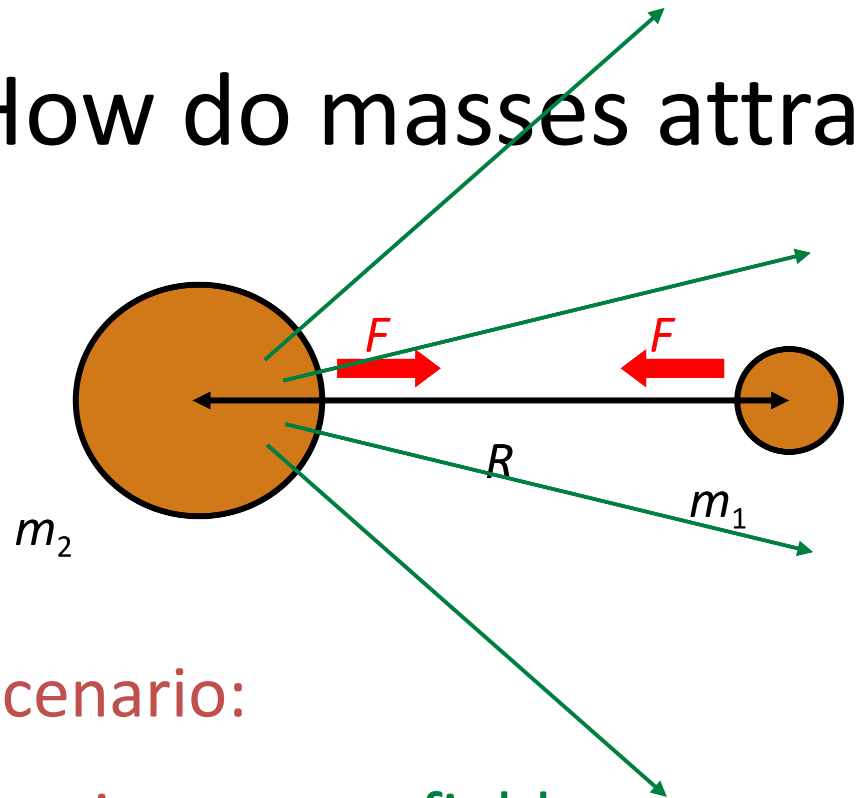
# Gravitational Lensing





# How do masses attract?

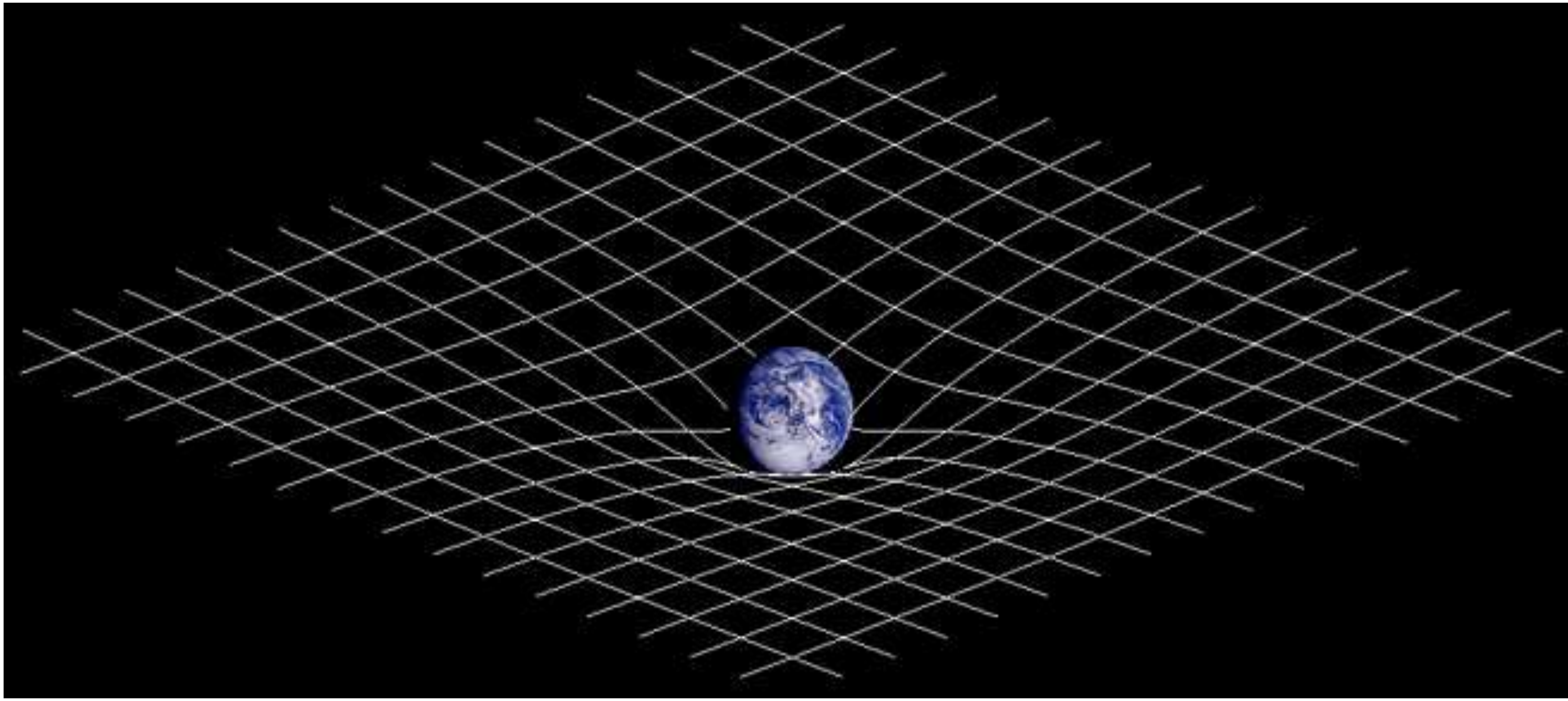



Two-step scenario:

- $m_2$ gives rise to grav field
- Grav field applies force to $m_1$

Yes, but according to Einstein, there is a better way to think about it:

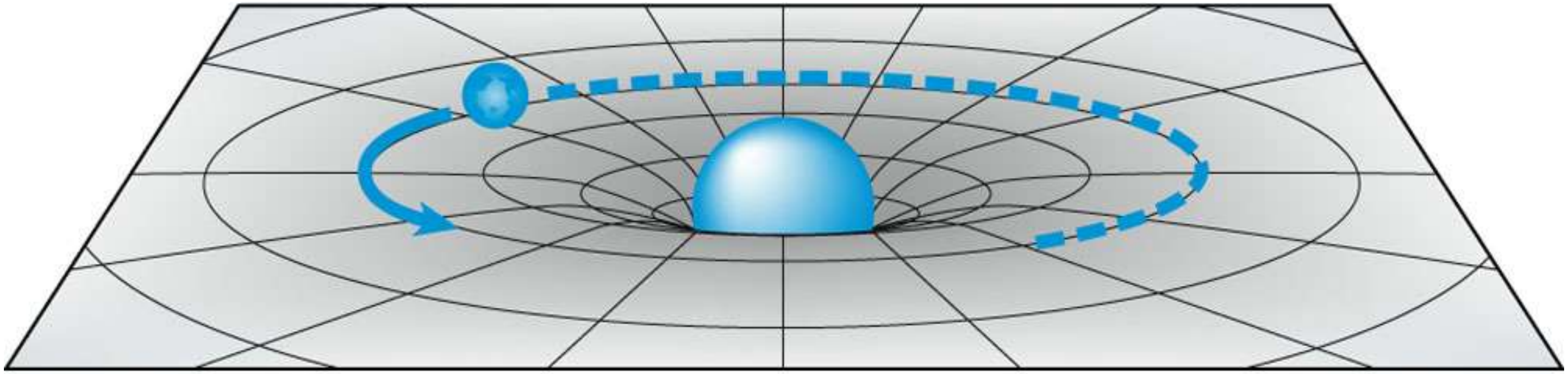
# Geometrical interpretation of general relativity



- This is a two-dimensional representation of what warped space might look like.
- Actually, it is four-dimensional space-time that is warped.
- All objects, including light, follow so called “geodesics”, which is the closest thing you have to a straight line in curved space-time.

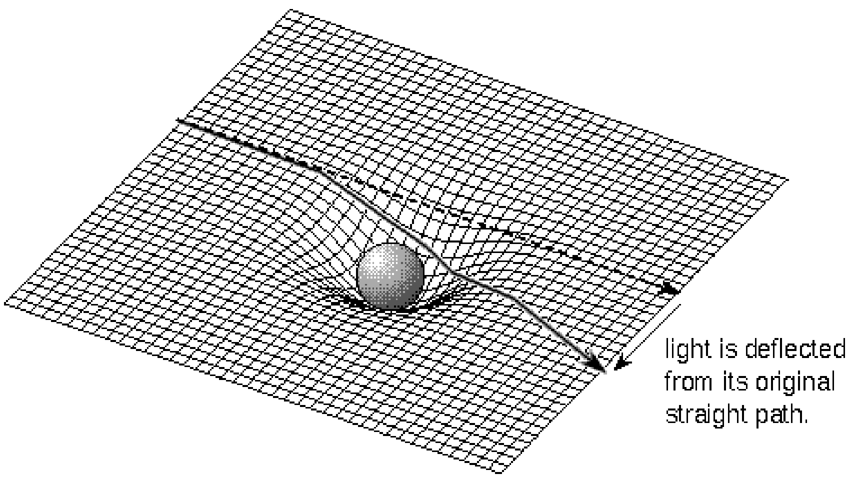
# Geometrical interpretation of general relativity

Spacetime is curved by massive bodies.
All objects, including light, follow so called “geodesics”, which is the closest thing you have to a straight line in curved space-time.



# Curved Spacetime

- **In curved spacetime, we must generalize our definition of a *straight line.***
- **What is a straight line in curved spacetime?** It is the shortest distance between two points!
- Matter (mass) warps spacetime. The effects of "gravity" are due to motion in a curved spacetime.




**General Relativity**: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.

2D Analogy:

- Earth's surface is a 2D curved space!
- Shortest flight path (=geodesic) connecting two cities on Earth looks like a curved path on the map:

Analogy:

- But when looked at differently, the shortest path looks more “straight” than the longer path:

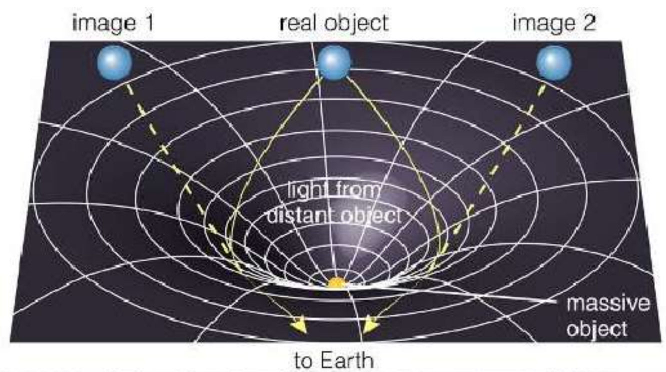
Analogy:

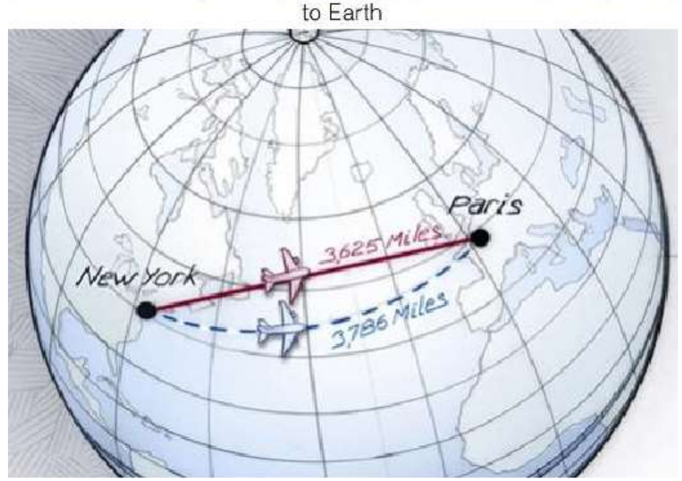
- However, neither of these paths is truly "straight". You would have to burrow through the Earth for that!
- If confined to the surface, the best we can do is to choose the shortest path, which is the "geodesic".

In the same way, light follows geodesic paths in curved spacetime.

Gravitational lensing in the geometric picture of gravity:

- It is not so much the light that is “deflected” by gravity into a curved path, but spacetime itself that is curved by the gravitating mass.
- Light just keeps doing what it always does, i.e., travel along the shortest path!



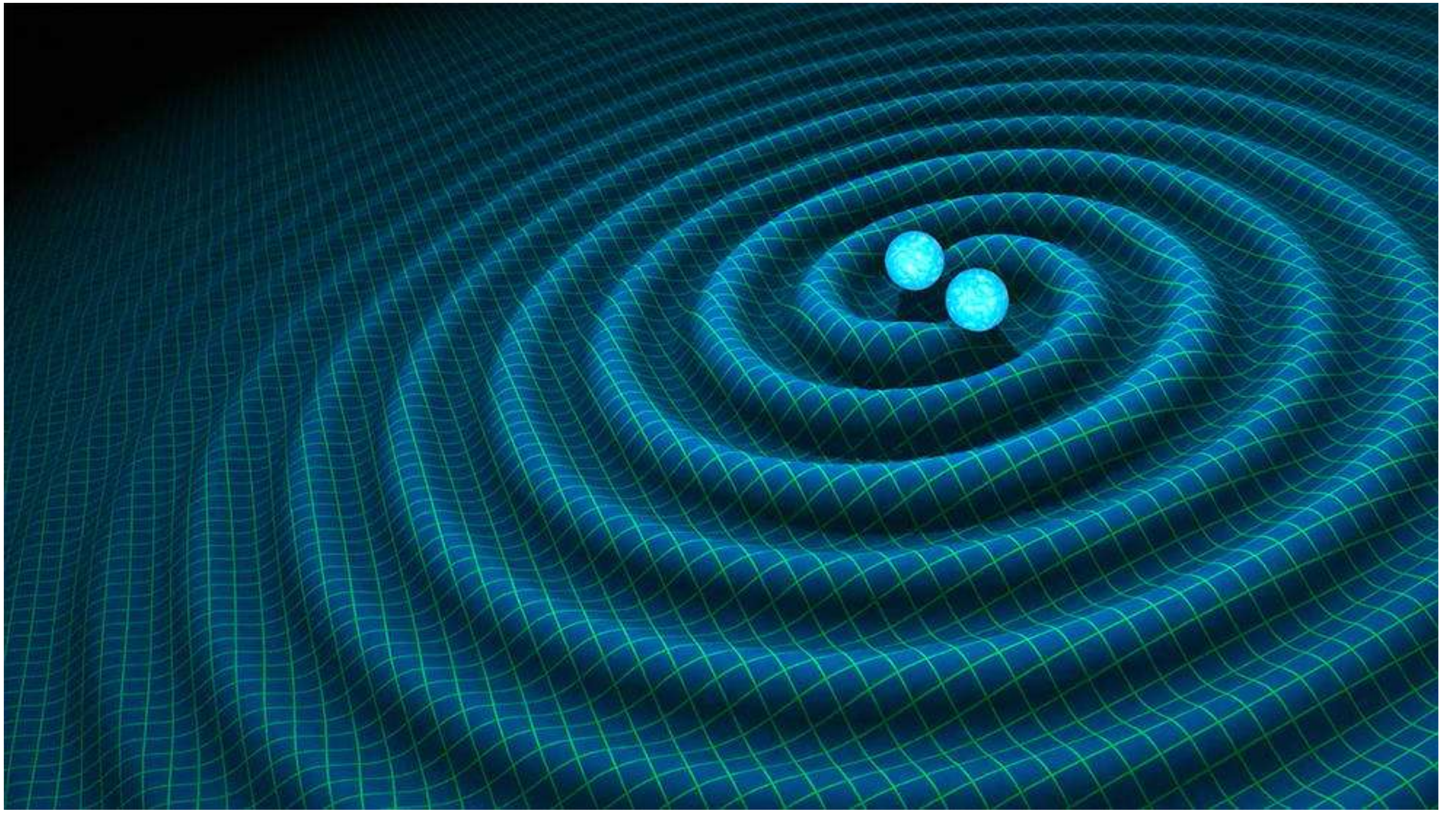



# Views of fundamental forces

| Electricity | Magnetism | Gravity |
|---|---|---|
| Like charges repel | Like poles repel | Masses attract |
| Charge creates E-field<br>E-field applies force to charge | Currents create B-field<br>B-field applies force to current | Mass distorts space-time…<br>changing path of second mass |

# Gravitational Waves

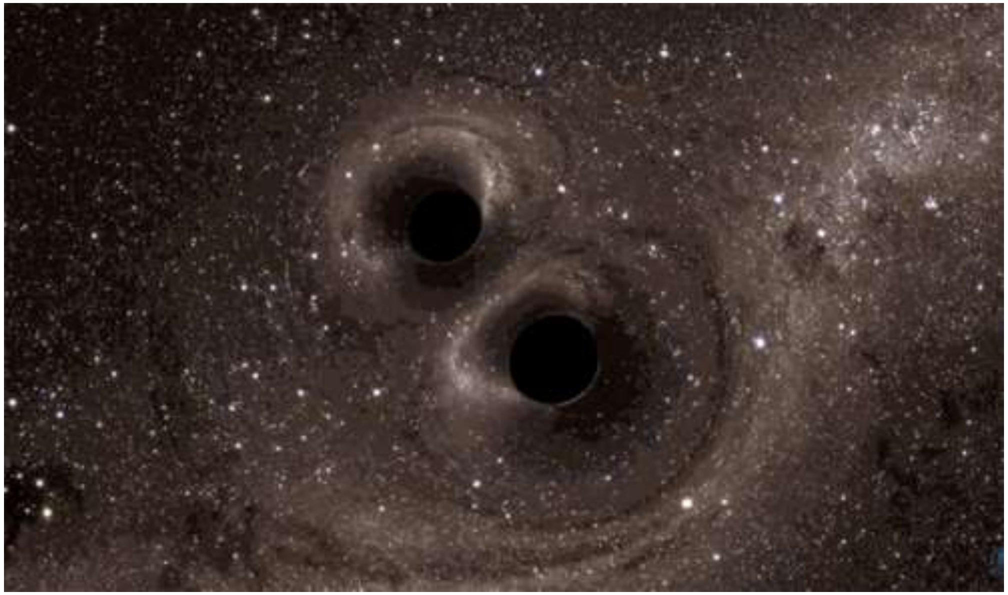
- General Relativity predicts that the effects of gravity travel at the speed of light.
- Two orbiting masses (stars, black holes, etc.) will radiate outward “curvature ripples” in spacetime: **gravitational waves**.



# Detection of Gravitational Waves!

- Detected by the LIGO experiment in 2015, 100 years after Einstein's prediction, in his historic paper on General Relativity!

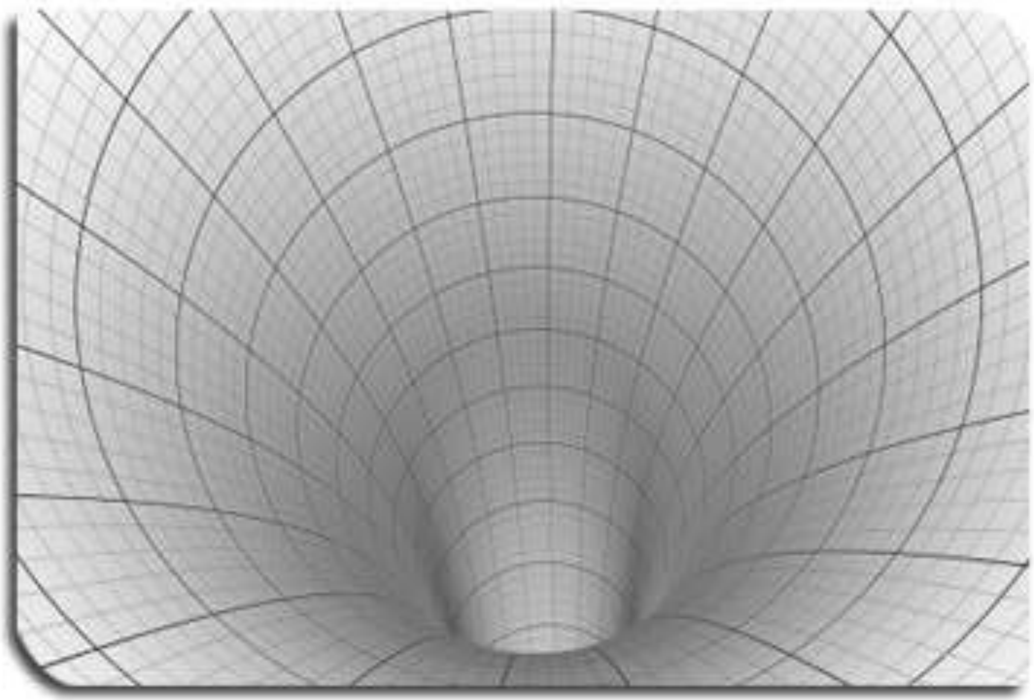
- NYT Video



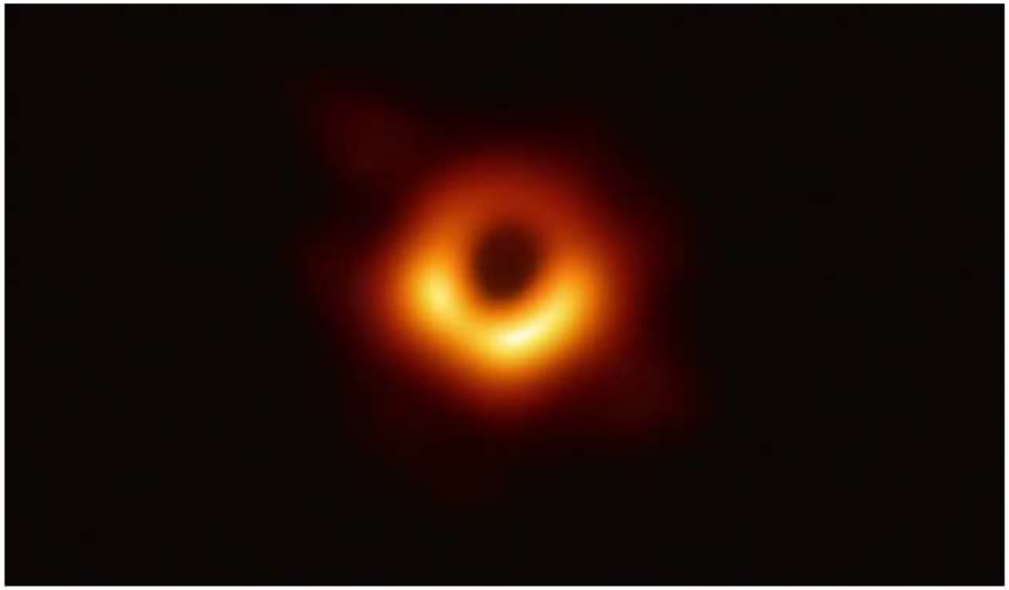
**Black holes:**

When the gravitational collapse causes spacetime curvature to become infinite:

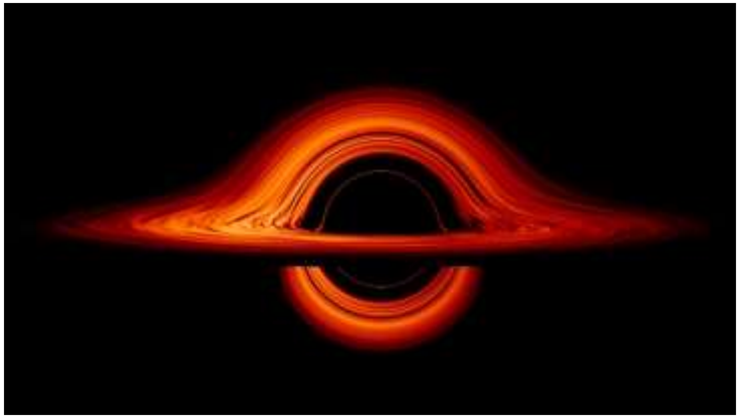
Singularity!



# First direct image of black hole from Event Horizon Telescope!



The Event Horizon Telescope (EHT) – a planet-scale array of eight ground-based radio telescopes – captured an image of the black hole at the center of Messier 87, a massive galaxy in the nearby Virgo galaxy cluster. This black hole resides 55 million light-years from Earth and has a mass 6.5-billion times that of the Sun. (nsf.gov/nsf.gov)



Simulated image of matter falling into black hole

https://www.cnn.com/2019/04/10/opinions/black-hole-ring-of-light-glow-lincoln/index.html?no-st=1554983559
https://www.washingtonpost.com/graphics/2019/national/amp-stories/black-hole-photo-explained/
https://www.youtube.com/embed/0RxitCeuokI

Clicker

Which is correct?

A) Special Relativity is “special” because it applies only to objects that move close to the speed of light.

B) Special Relativity is “special” because it applies only to elementary particles.

C) General Relativity is “general” because it applies to all inertial reference frames.

D) General Relativity is “general” because it applies even to accelerating reference frames.

E) General Relativity was a British military officer in the First World War.

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Which is true, according to the geometrical interpretation of general relativity?

A) All masses cause a warping of space and time around them.

B) All masses move according to the warping due to other masses.

C) Gravitation is not really a force - it is just the effect of the warping on the path of objects.

D) General relativity is hard to grasp intuitively because it requires you to think about warped four-dimensional space-times.

E) All of the above.

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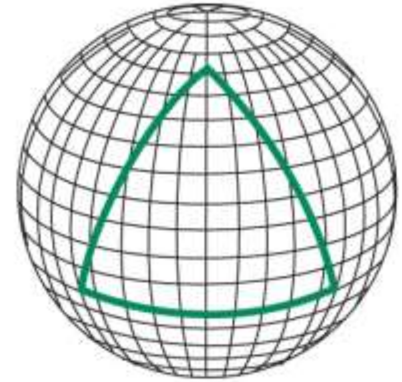
# Cosmology and the Big Bang

Closed



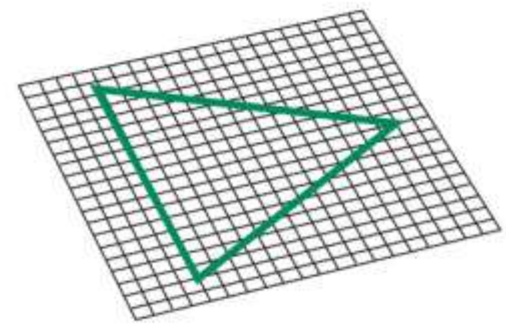
(a) Closed geometry

Does the universe as a whole have a shape?

There are three possibilities for the overall geometry of the universe.

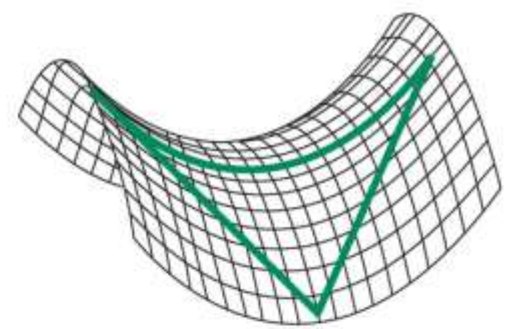
1. The first is a closed geometry, like the surface of a sphere.
2. The second is a flat geometry.
3. The final is an open geometry, which can be visualized as a saddle.

Flat



(b) Flat geometry

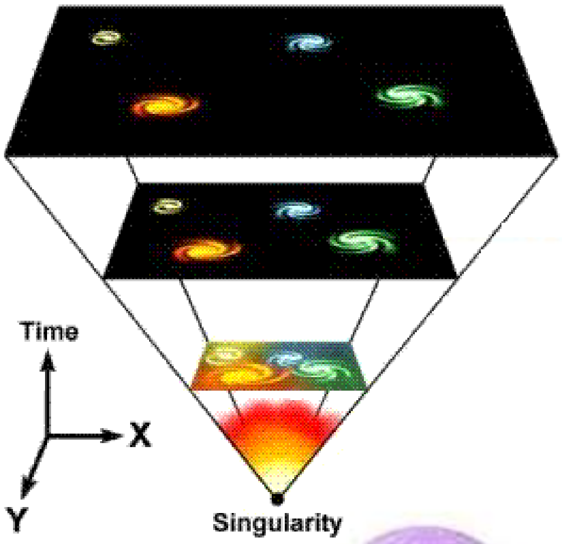
Open

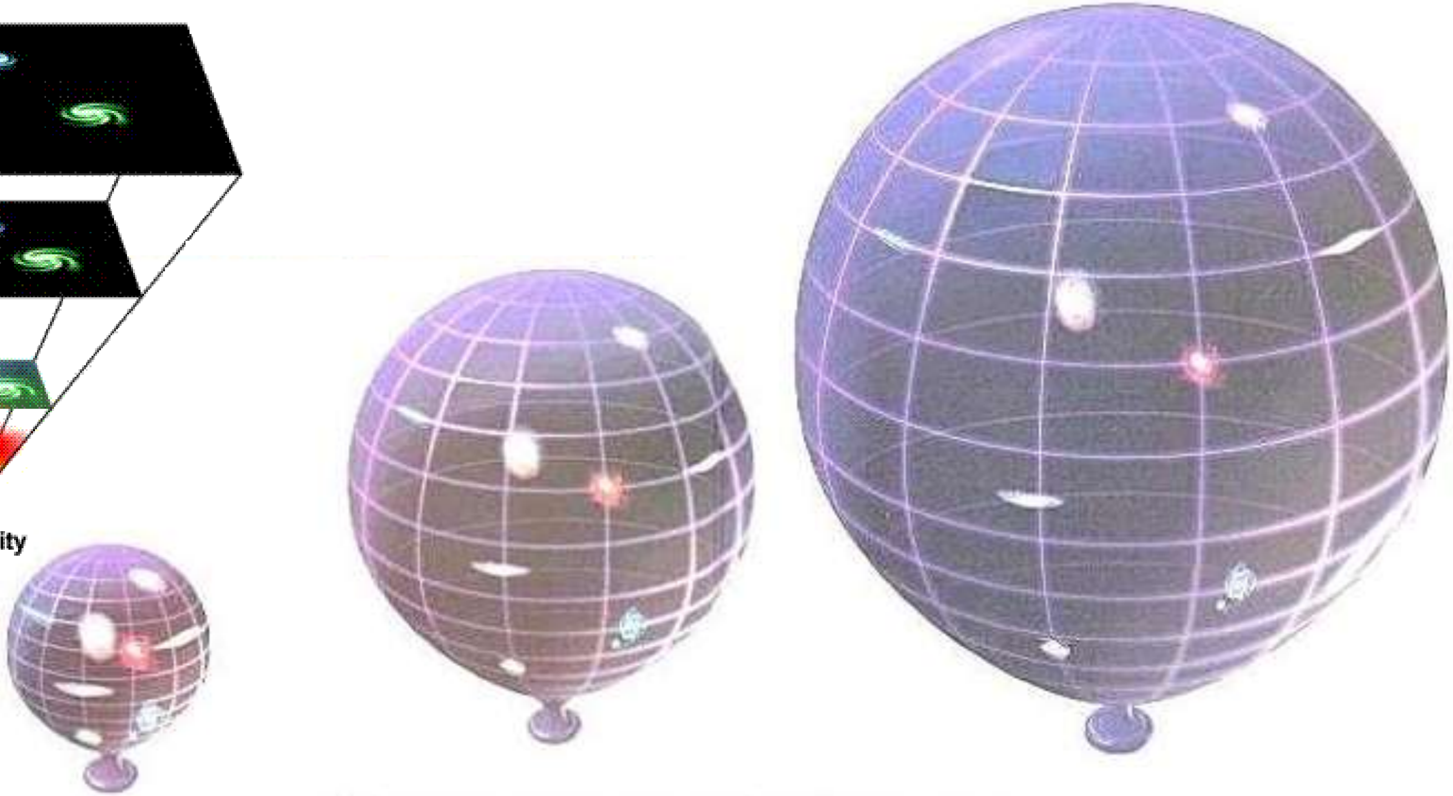


(c) Open geometry

# Expanding Universe

So, which shape does our universe have?



Actually, best evidence $\Rightarrow$ universe is flat

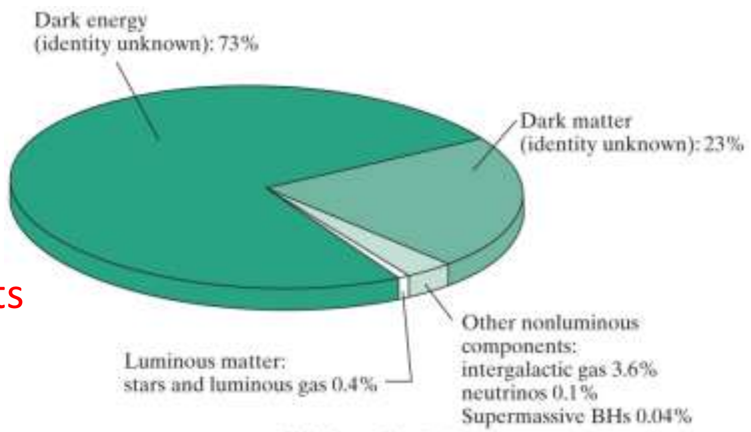
But it still expands similarly to what is shown above

And there is strong evidence that it is accelerating!

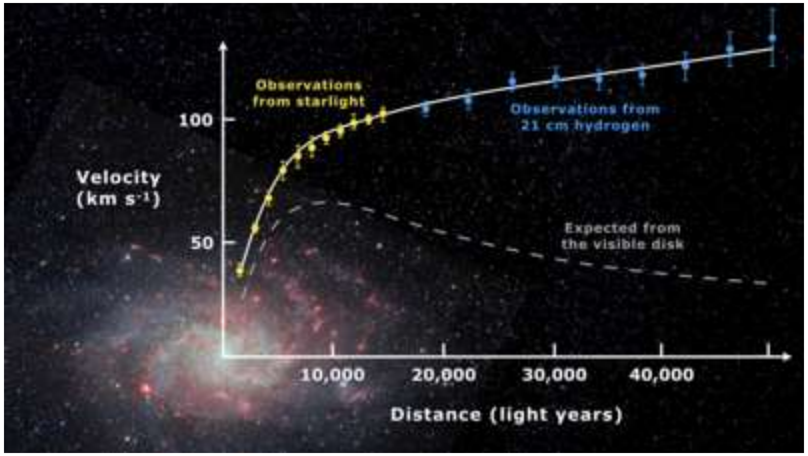
(Nobel prize in physics in 2011)

# Dark Matter

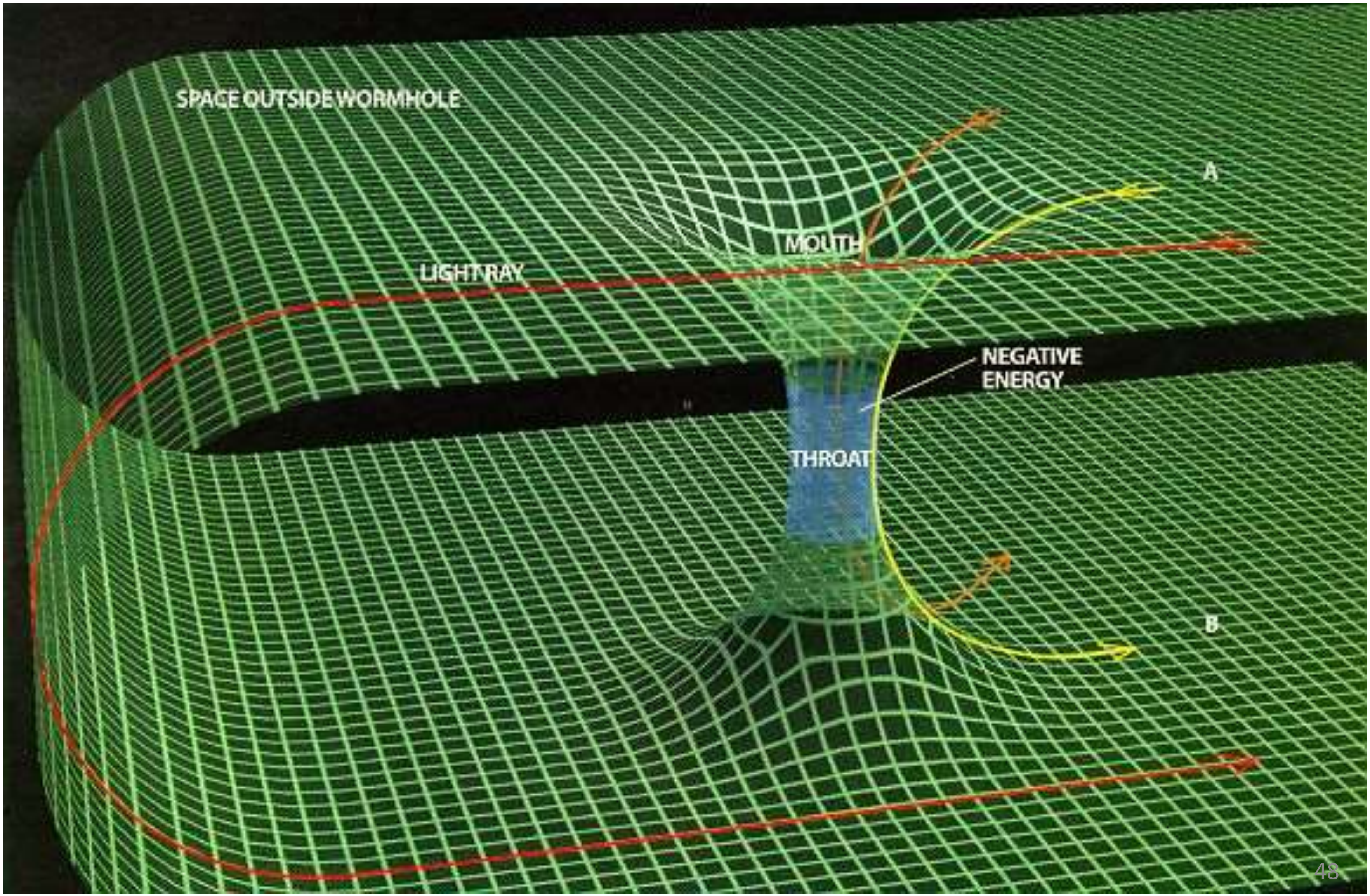
- There are many different forms of matter in the universe; besides the familiar protons, neutrons, and electrons there are neutrinos and black holes.
- But it seems that there is another form of matter, none of the above, that does not interact with electromagnetic radiation but can be detected due to its gravitational interactions. This is called dark matter.
- Dark matter, although we don't know what it is, comprises most of the mass of the universe
  - We can tell this by studying gravitational properties of galaxies.
  - Another way of detecting dark matter is by the way it distorts the light coming from distant galaxies.



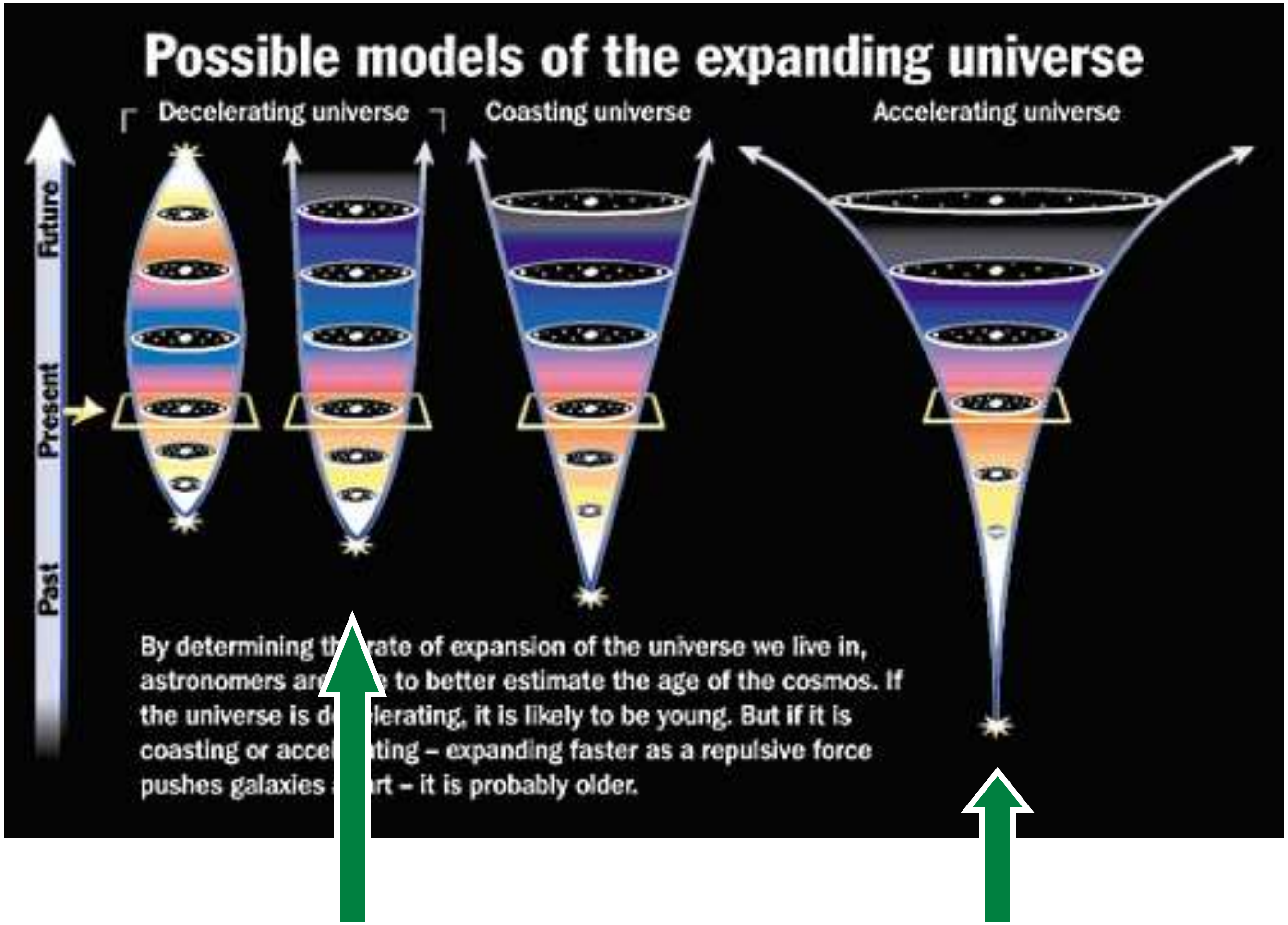



# Wormhole (speculation)



# Expanding Universe




Like this in early universe

Like this more recently