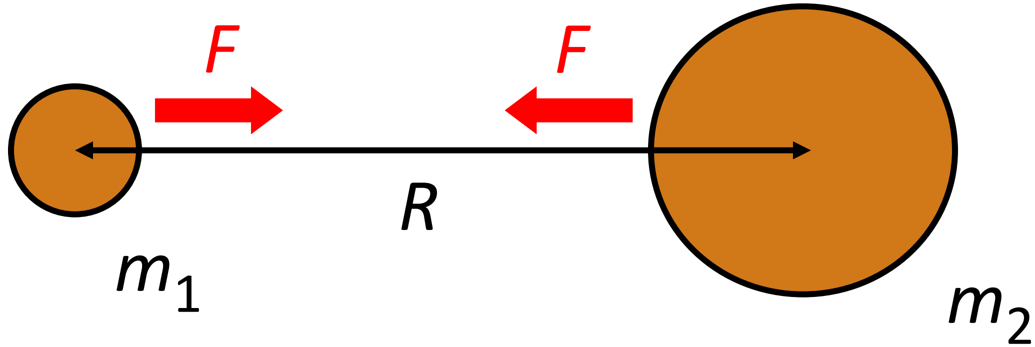
Newton's Universal Law of Gravitation



$$F = G\frac{m_1 m_2}{R^2}$$

$$G = 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$$ gravitational constant

Force is ***inversely proportional to the square*** *of distance*

Clicker Question

A person in a falling elevator weighs

A. Nothing.
B. Much less than normal, but not zero.
C. The same as when standing in the street.
D. More than normally.
E. Who cares, he's as good as dead anyway.

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Astronauts, orbiting the Earth, feel weightless. Are they weightless?

A. No. The force of gravity is pulling them downward.
B. No. The centrifugal force is pulling them outward.
C. Yes, because they are "falling" around the Earth.
D. Yes, because they are outside Earth's field of gravity.
E. Yes. The (outward pointing) centrifugal force of circular motion cancels the (inward) force of gravity.

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Clicker Question

What are the forces acting on an astronaut orbiting the Earth?

A. His weight is canceled by the centrifugal force, so he is weightless.
B. Since he is weightless, there are no forces.
C. His weight provides the centripetal force.
D. Gravity pulls him down. In addition, the centripetal force also pulls him down.
E. His weight is canceled by the centripetal force.

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A child twirls a rock tied to a string in a circle above her head. What are the forces acting on the rock?

A. The tension in the string, and gravity.
B. The tension in the string, and the centripetal force.
C. Gravity, the tension in the string, and the centripetal force.
D. Gravity, the centripetal force, and the centrifugal force.
E. Since the speed is constant, the net force is zero.

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The centripetal force is the net force acting on the rock (string tension + gravity).

Quick review:
Four kinds of proportion

- Directly proportional to…
- Inversely proportional to…
- Proportional to the square of…
- Inversely proportional to the square of…

y is proportional to x

$$y \propto x$$

If x is doubled, what happens to y?

It is doubled.

If x is tripled, what happens to y?

It is tripled.

$$y \propto \text{inverse of } x$$

$$y \propto 1/x$$

If x is doubled, what happens to y?

It is halved.

If x is tripled, what happens to y?

It is reduced to 1/3 of its original value.

$$y \propto \text{square of } x$$

$$y \propto x^2$$

If x is doubled, what happens to y?

It is quadrupled.

If x is tripled, what happens to y?

It is increased by a factor of 9.

$y \propto$ inverse square of x

$$y \propto 1/x^2$$

If x is doubled, what happens to y?

It is reduced to 1/4 of its value.

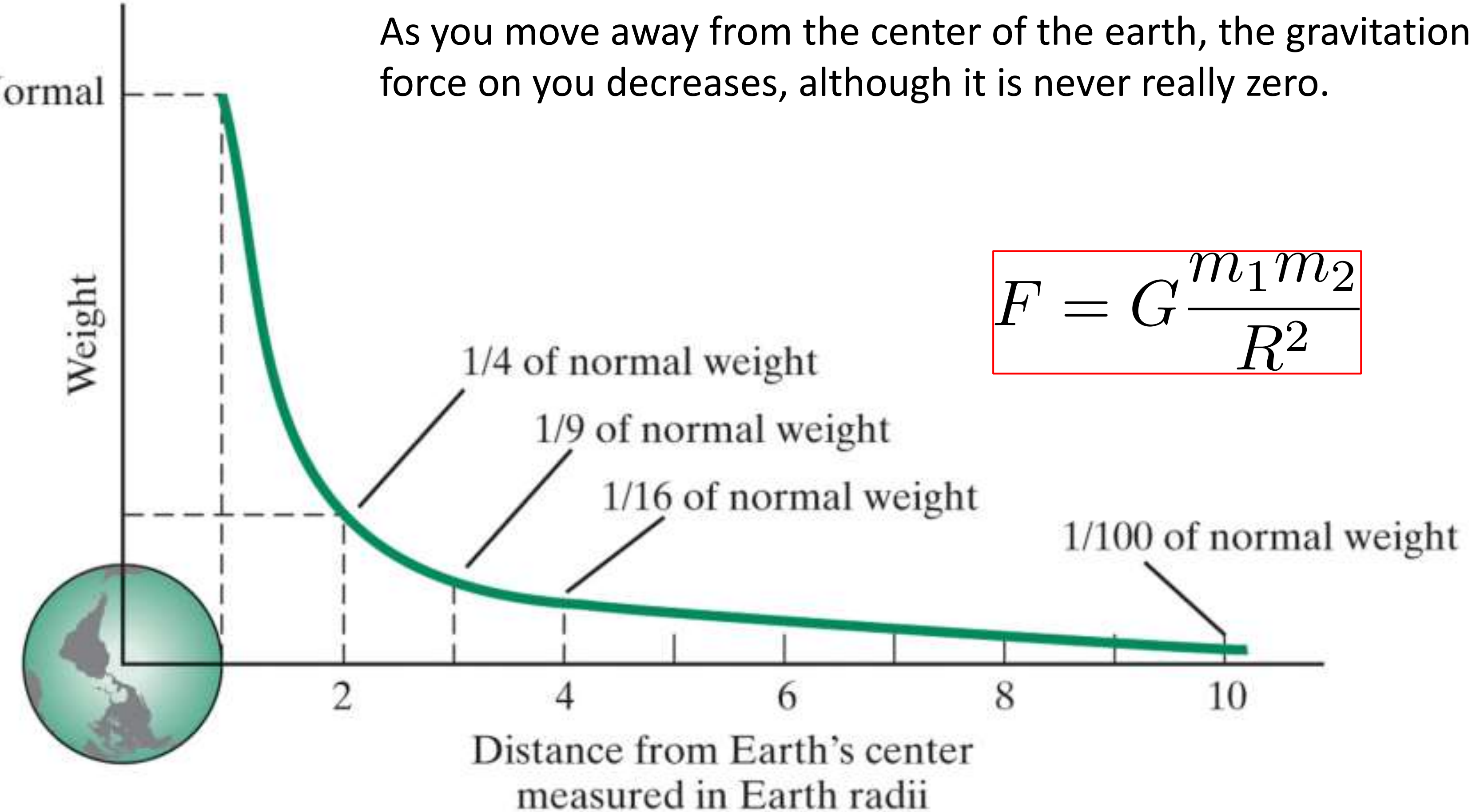
If x is tripled, what happens to y?

It is reduced to 1/9 of its value.

Newton's universal law of gravitation

As you move away from the center of the earth, the gravitational force on you decreases, although it is never really zero.

$$F = G\frac{m_1 m_2}{R^2}$$



Clicker Question

Suppose Earth collapsed down to one tenth of its present size (diameter), while retaining all of its present mass. Compared to your present weight, your weight would be:

A. Increased by a factor of 10
B. Increased by a factor of 100
C. Remain the same
D. Decreased by a factor of 1/100
E. Decreased by a factor of 1/10

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Clicker Question

Imagine a planet with a radius of one-half of the Earth's radius, and one quarter of Earth's mass. Compared to your weight on Earth, on this planet your weight would be:

A. Double.
B. Quadrupled.
C. Half.
D. One quarter.
E. The same.

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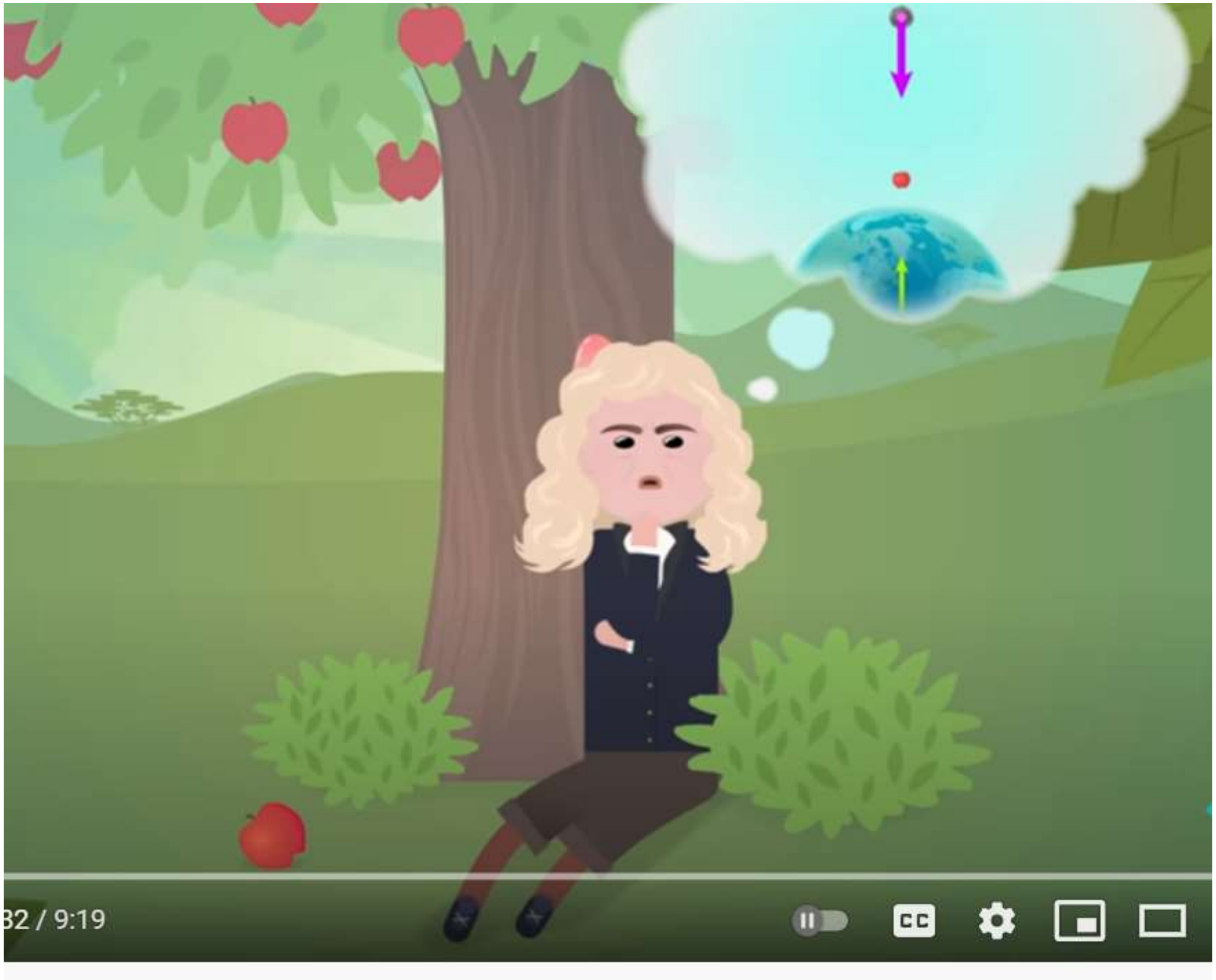
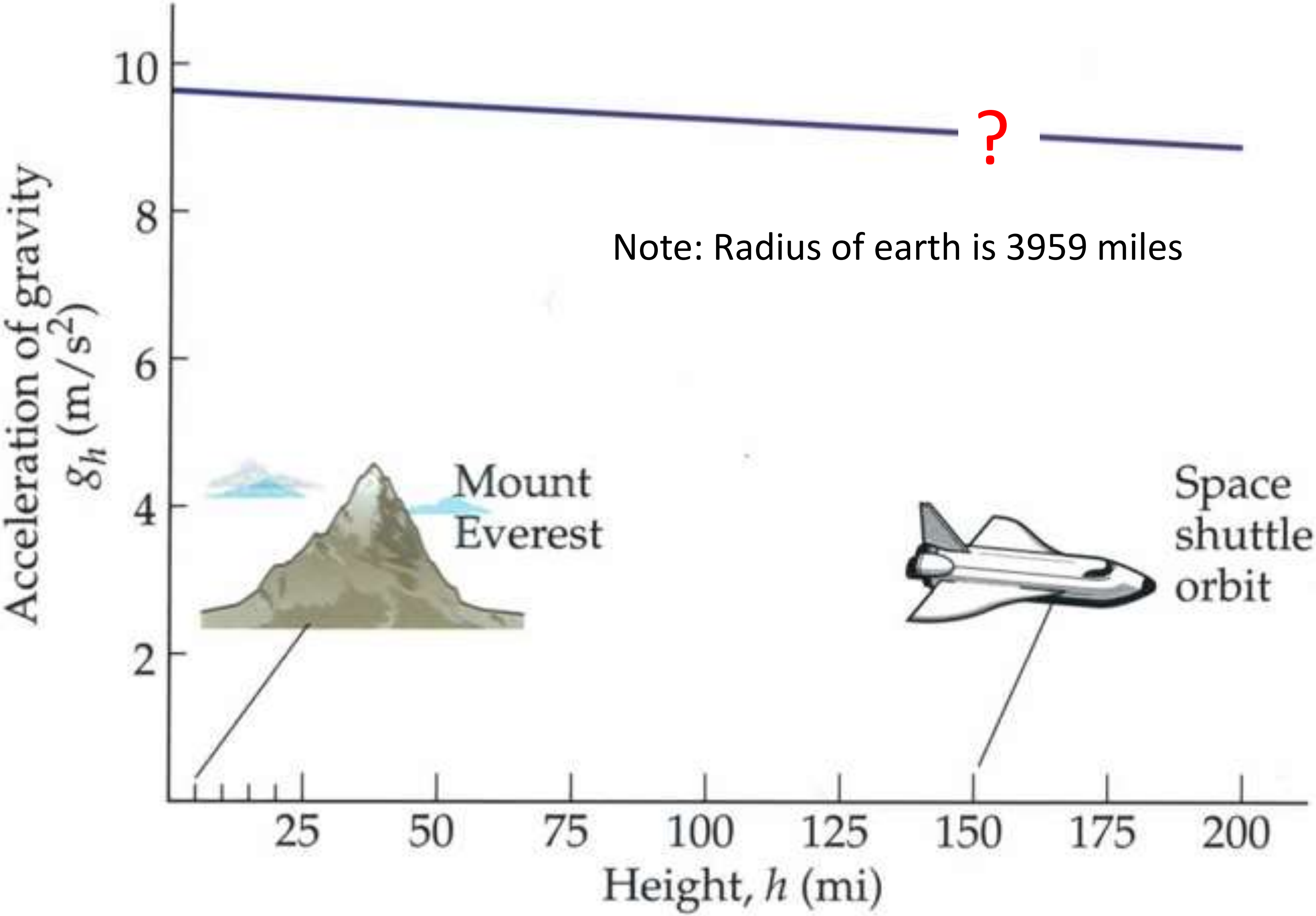
Video



What is the acceleration due to gravity of the space shuttle in orbit?



Solution

- Radius of earth is 3959 miles.
- Space shuttle orbit from center of Earth: 3959 mi + 150 mi = 4109 mi.
- Since $a \propto 1/R^2$, the acceleration due to gravity at the space shuttle is:

$$a_{space\ shuttle} = (3959/4109)^2\ a_{sea\ level} = 0.93 * 9.8 m/s^2 = 9.1 m/s^2$$

Clicker Question

What is true about the gravitational constant in Newton's law?

A. It is larger for large planets than for small planets.
B. It much smaller for tennis balls than for planets.
C. It is zero for everyday objects, as only stars and planets attract each other.
D. It is incredibly large for a neutron star.
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An astronaut's weight on Earth is 600 N. How many Newtons does she weigh while in a spacecraft orbiting at an altitude of one Earth radius above the surface?

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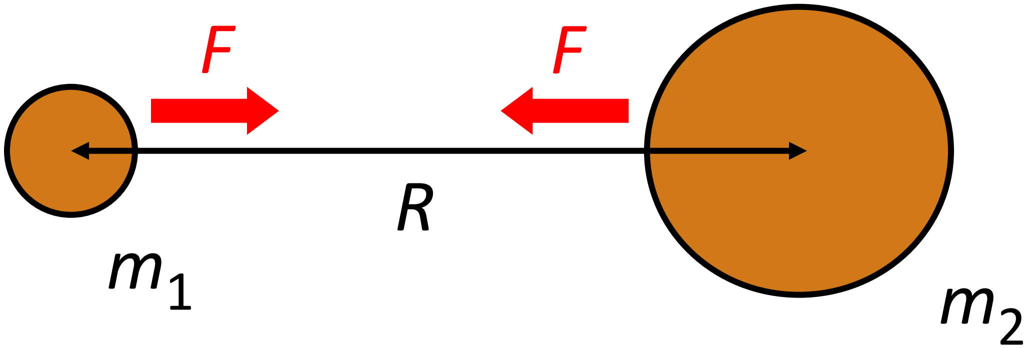
$$F = G\frac{m_1 m_2}{R^2}$$

Double distance → weight reduced by one quarter!

F = 150 N

Newton's Hypothesis:
All matter attracts all other matter via some universal law of gravitation

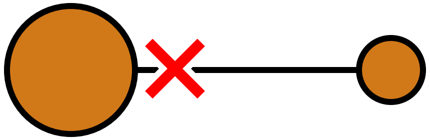


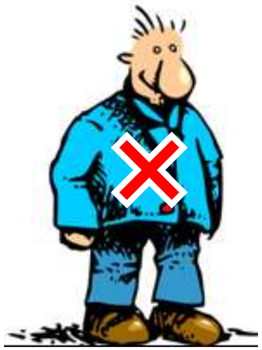
Newton guessed:

- Forces are ***equal*** and opposite
- Forces are linearly proportional to masses
- Forces decrease with distance
- Distance should be measured from body center to body center

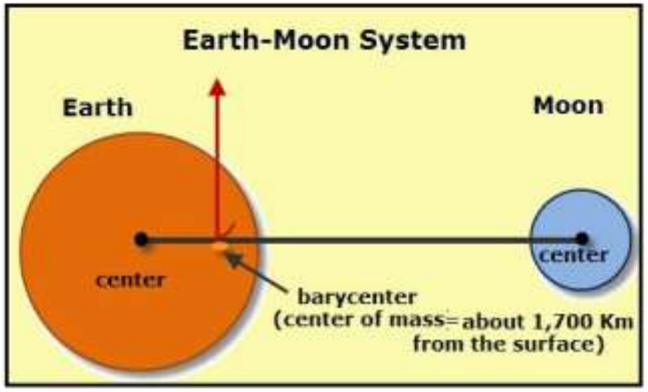
Center of Mass

- Each object has a “center of mass”





- Distance between objects is measured between centers of mass.
- Earth – Moon system:



Acceleration due to gravity

- We found that all objects fall at the surface of the earth in the same way
- Namely, acceleration is

  $g = 10\ \text{m/s}^2$ (well, really $9.8\ \text{m/s}^2$)

- Why?

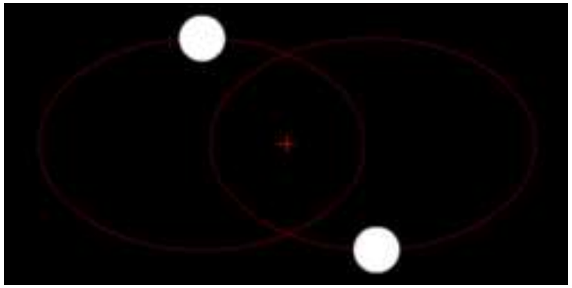
$$F = G\,\frac{m_1\, m_2}{R^2}$$

$$\cancel{m_1}\, a_1 = G\,\frac{\cancel{m_1}\, m_2}{R^2}$$

$$a_1 = G\,\frac{m_2}{R^2}$$

$$g = G\,\frac{m_{\text{Earth}}}{R_{\text{Earth}}^{2}} = 9.8\ \text{m/s}^2$$

Newton’s Law of Gravity also explains:

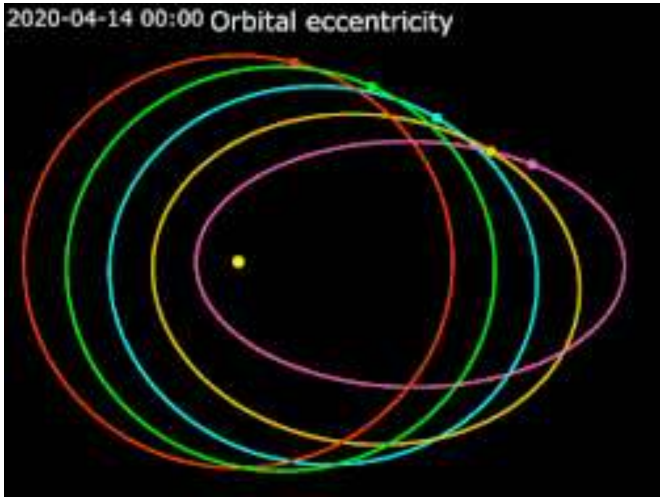


- Kepler’s Laws of planetary orbits:
  - How the period of rotation depends on distance
  - Orbits are elliptical
  - More rapid motion when nearer focus
- Predicted existence of Neptune!
- Why the ocean tides occur

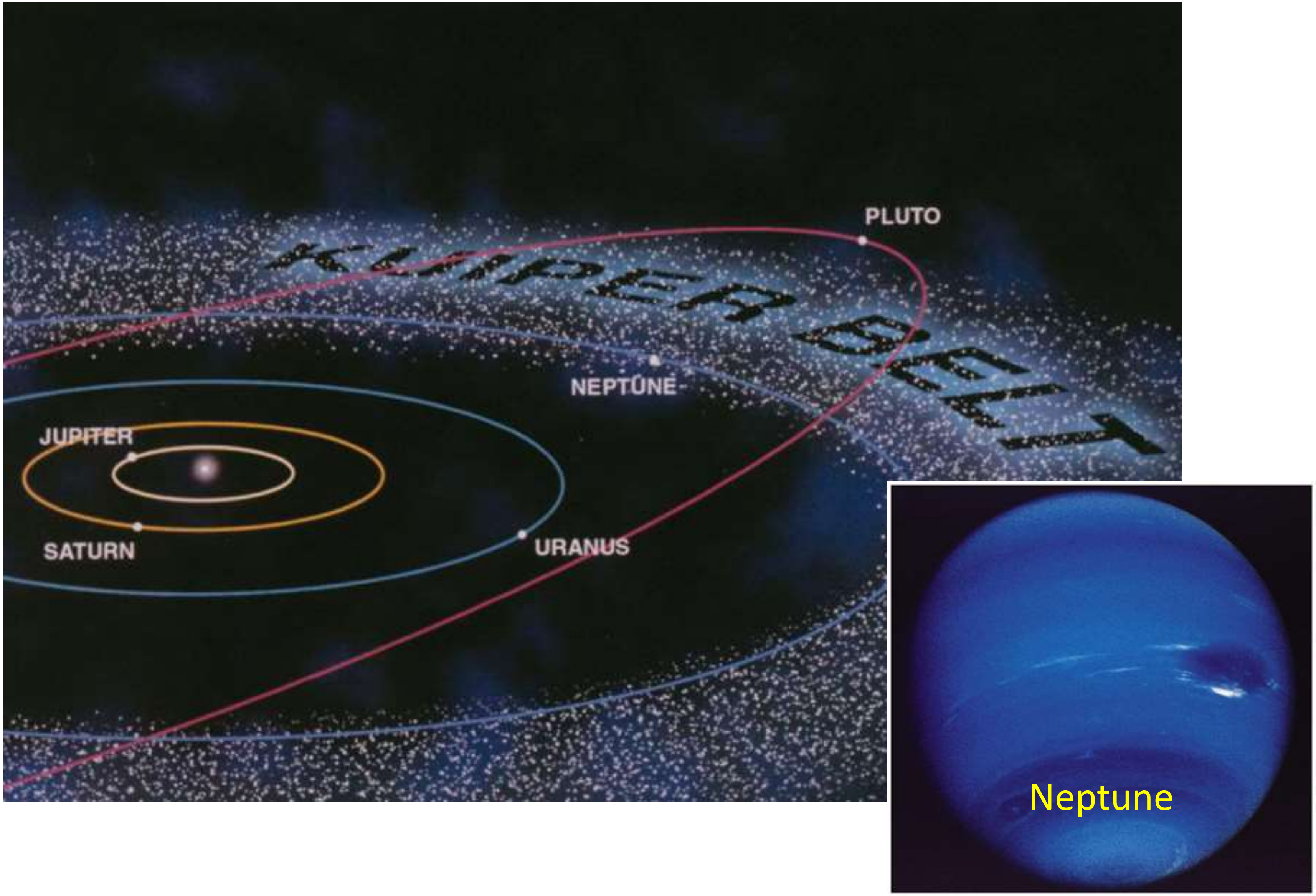
Elliptical orbits

Other force laws would not result in elliptical orbits.

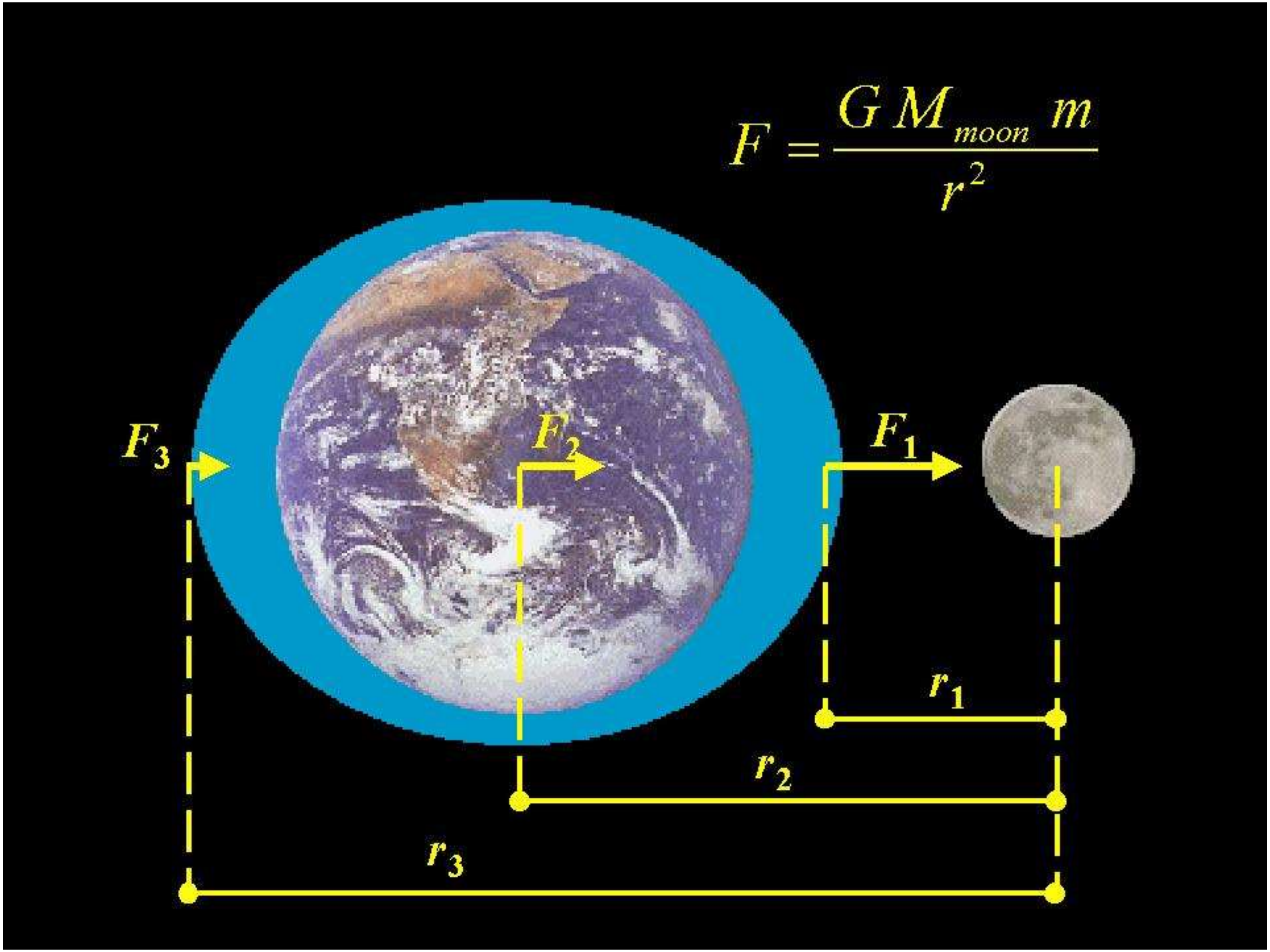
For example, if the force is proportional to 1/R instead of $1/R^2$, the path does not retrace itself.



Orbit of Uranus $\Rightarrow$ Existence of Neptune

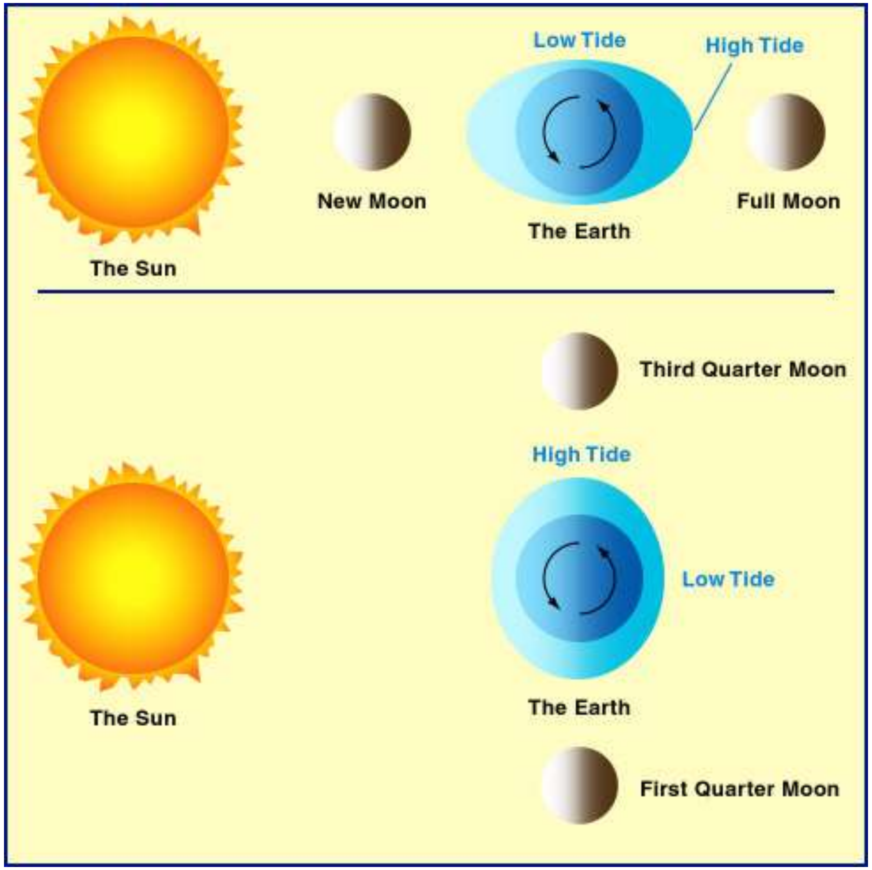


Newton's Law of Gravity Explains the Ocean Tides



Ocean Tides

Video



spring tide

neap tide

In Newton's time:

Unknown

$$g = G \frac{m_{\text{Earth}}}{R_{\text{Earth}}^{2}}$$

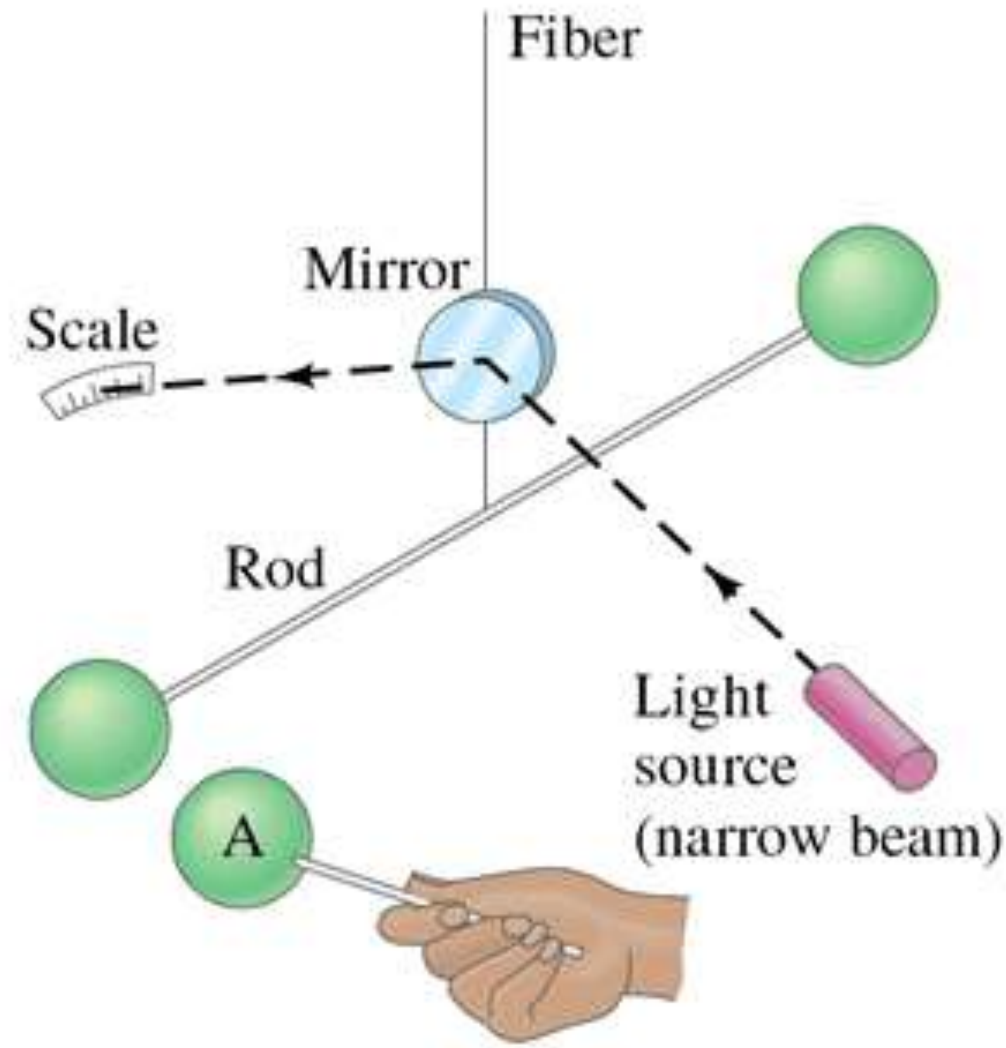
Known

Rearrange:

$$g\,(R_{earth})^2 = G\,m_{Earth}$$

- Newton did not know G and m_{Earth} separately.
- He could now figure out their product, though.
- How can we determine what they are separately?

Cavendish Experiment: Direct measurement of *G*



Henry Cavendish
1797

Cavendish Experiment Video

(skip to 11:20 for time lapse footage)